Robin Milner, 1934–2010

His work in theorem proving and verification

John Harrison

Intel Corporation

January 28th, 2011 (09:15–09:27)

Invited speaker at TPHOLs 2000?

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From: Robin Milner <Robin.Milner@cl.cam.ac.uk>
To: John Harrison <johnh@ichips.intel.com>
Date: Tue, 25 Jan 2000 11:32:39 +0000

Dear John

Thanks very much for inviting me to speak at
TPHOLs.  I would enjoy it, but the main question
is whether I can offer enough of a perspective on
automated and interactive theorem proving, as I
haven't done any to speak of for 20 years!
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1968: Arrival as a researcher in Swansea

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Cooper is perhaps most famous for the first elementary-time decision procedure for linear integer (Presburger) arithmetic. (In fact, arguably the first for any significant first-order theory.)

1969: Dana Scott's Oxford lectures

That was at the time when Dana Scott produced his famous domain theory. He gave a series of talks then, in '69, and I went over to Oxford and heard him. That was very exciting.
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Dana Scott's influence on semantics is well-known, but his work was also an important factor in the development of interactive theorem proving.

1970: From automated to interactive proving

I wrote an automatic theorem prover in Swansea for myself and became shattered with the difficulty of doing anything interesting in that direction and I still am. I greatly admired Robinson's resolution principle, a wonderful breakthrough; but in fact the amount of stuff you can prove with fully automatic theorem proving is still very small. So I was always more interested in amplifying human intelligence than I am in artificial intelligence.

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Interest in 'interactive' theorem proving was growing at the time, either because

- Abilities of ATP systems had grown but were plateauing
- More interactive computers made it natural/convenient

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The requirements of the LCF system directly motivated many features of ML.

How an LCF-style prover works

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Highly automated or convenient *derived* inference rules can be programmed using these as the basic building-blocks, including support for backward proof via ‘tactics’.

The LCF diaspora

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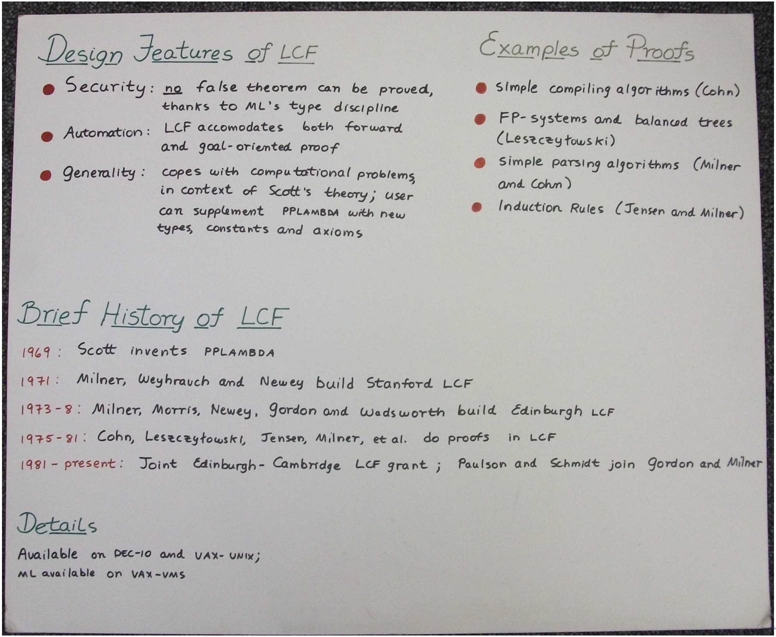
Despite the name, which has stuck, the LCF approach is not tied to the Logic of Computable Functions, and many other LCF-style provers were written over the years.

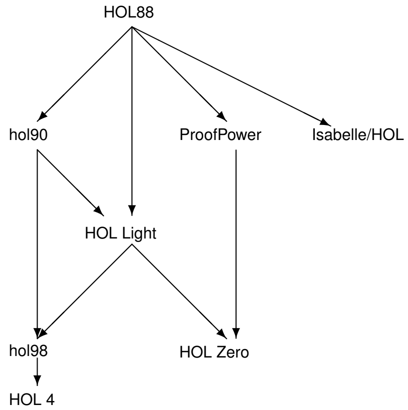
- Cambridge LCF (rationalized system for same LCF logic)
- HOL (higher-order logic based on polymorphic type theory)
- Nuprl (Martin-Löf type theory)
- Coq (the Calculus of Inductive Constructions)
- Isabelle (framework supporting multiple object logics)

Avra Cohn’s LCF poster



The HOL family DAG

The HOL system alone has given rise to numerous different LCF-style implementations of essentially the same logic:



Applications of LCF provers

Several LCF-style systems have been used for major work in formal verification and formalization of mathematics.

- Verification of microprocessors, compilers, floating-point microcode, cryptographic protocols, OS kernels, properties of programming languages and their type systems ....
- Formalization of Jordan Curve Theorem, Prime Number Theorem, 4-Colour Theorem .... Feit-Thompson theorem and Kepler conjecture ('Flyspeck project') in progress.

The ideas that Robin Milner developed almost 40 years ago are central to machine-assisted proof today.

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From: Robin Milner <Robin.Milner@cl.cam.ac.uk>
To: John Harrison <johnh@ichips.intel.com>
Date: Fri, 28 Jan 2000 17:26:21 +0000

Dear John

I've thought a bit more. I believe I can offer, informally, some interesting reminiscences. What I can't do, given my current preoccupation, is to spend very long on that. (Indeed, a whole talk would be too hard.) At present I am working flat out on the theory I mentioned, hence my interest in how machine assistance could help with it. I can't afford too much time away from this task over the coming year -- life is short.