Two Signature Schemes

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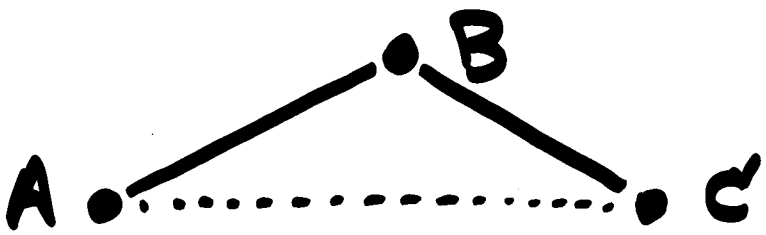
Outline

- Review signatures, RSA, security, ...
- Prefix aggregation scheme: can compute $\sigma(x)$ from $\sigma(x0)$ and $\sigma(x1)$ (joint with Suresh Chari & Tal Rabin)

- Transitive signature scheme: can compute $\sigma(A,C)$ from $\sigma(A,B)$ and $\sigma(B,C)$ (joint with Silvio Micali)



Digital Signatures

- **Key generation** → public key PK
  → secret key SK

- **Signing procedure**

  given message m

  signature is $\sigma(m, SK)$

  (or $\sigma(m)$ when SK understood)

- **Verification procedure**

  $V(m, PK, s) = \text{true}$ **iff**

  $s = \sigma(m, SK)$

Digital Signature Security

A digital signature scheme is secure if adversary can not forge signature for any new message m, even if adversary knows PK (but not SK) & can first obtain valid signatures for any messages (other than m) that he wishes.

Basic RSA

- **Keygen:** p, q large primes

  $n = p \cdot q$

  e public exponent

  d secret exponent

  $e \cdot d \equiv 1 \pmod{(p-1)\cdot(q-1)}$

  $PK = (n, e)$ $SK = d$

- **Signing:**

$$\sigma(m) = m^d \pmod{n}$$

- **Verification:**

$$s^e \stackrel{?}{\equiv} m \pmod{n}$$

RSA is multiplicative

- $\sigma(x) \cdot \sigma(y) = \sigma(xy)$
- This can be useful! (or dangerous!)

  E.g. to get your blind signature on m:

  - I give you $m \cdot r^e$ to sign
  - Your sig $= (m \cdot r^e)^d = m^d \cdot r$
  - I divide by r to get $m^d = \sigma(m)$

  You have no idea what m is.

- Useful for e-cash, voting.

"Signature Algebras"

- For what other operations on message space can we find corresponding signature scheme, such that $\sigma(x)$ and $\sigma(y)$ can be combined (by anyone) to obtain $\sigma(x \underline{\text{op}} y)$?

- Note: need to modify def. of security

Open Problem

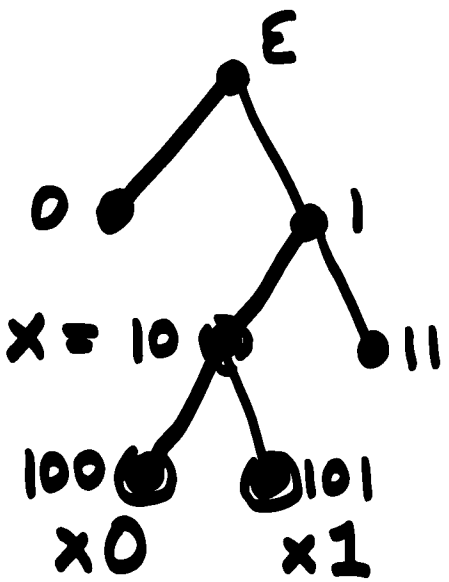
Let xy denote concatenation.

Is there a signature scheme σ such that anyone can compute $\sigma(xy)$ from $\sigma(x)$ and $\sigma(y)$?

E.g. combine $\sigma(ababb)$ and $\sigma(aba)$ to get $\sigma(ababbaba)$?

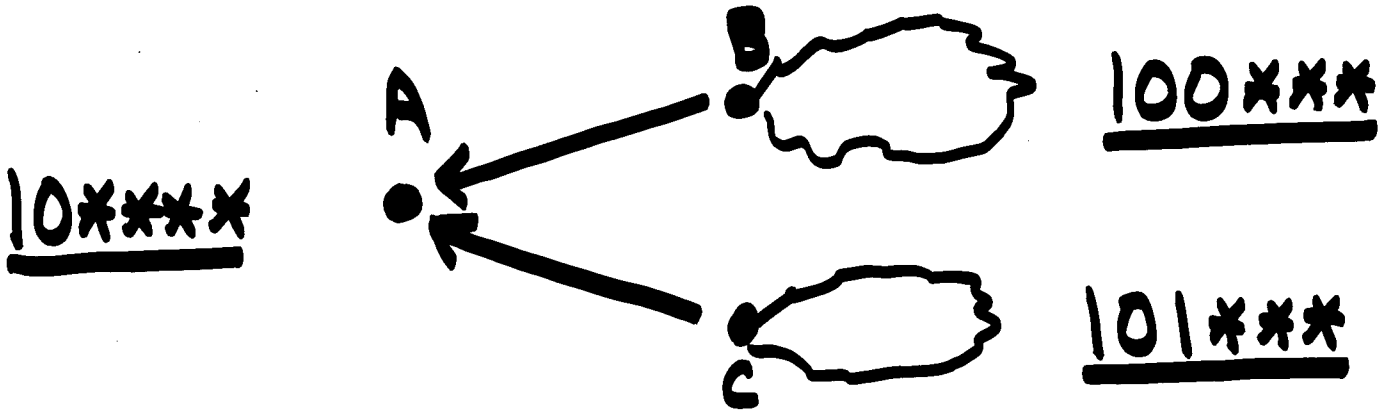
Prefix Aggregation

Is there a signature scheme σ such that anyone can compute $\sigma(x)$ from $\sigma(x0)$ and $\sigma(x1)$?



Signatures of both children $\Rightarrow$ signature of parent

Motivation: Routing



B says: "I can route to IP addresses of the form 100***"

C says: "I can route to IP addresses of the form 101***"

⇒

A says: "I can route to IP addresses of the form 10****"

Scheme

- Uses basic RSA
- Let H be a hash fn onto $\mathbb{Z}_n^*$
- Define label $\lambda(x)$ for each node x:

$$\begin{cases} \lambda(\varepsilon) = H(\varepsilon) \\ \lambda(x0) = H(x0) \\ \lambda(x1) = \lambda(x)/\lambda(x0) \end{cases}$$

so $\underline{\lambda(x) = \lambda(x0) \cdot \lambda(x1) \bmod n}$

- Define $\sigma(x) = \lambda(x)^d \bmod n$

so $\underline{\sigma(x) = \sigma(x0) \cdot \sigma(x1) \bmod n}$

- Fact that you can compute $\sigma(x1)$ from $\sigma(x)$ and $\sigma(x0)$ not a problem in this application!

Security

- Assume it is hard to compute $\sigma(m) = m^d \pmod{n}$ given n, e, and m. (Basic RSA)
- Then it is hard for adversary to forge signature $\sigma(x)$ in scheme, even if adversary can adaptively ask for signatures on other nodes, assuming $\sigma(x)$ not implied by what has been asked for and relation $\sigma(x) = \sigma(x0) \cdot \sigma(x1)$

Open Problems

- Do "AND" in a clean way, so that $\sigma(A)$ can be computed from $\sigma(B)$ and $\sigma(C)$.

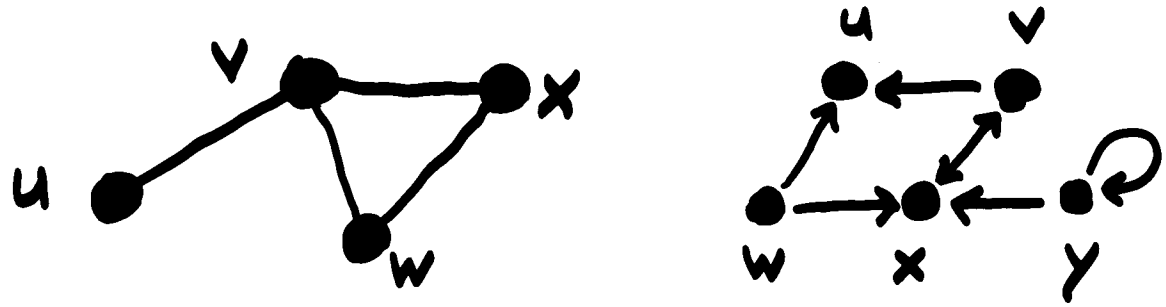
(But $\sigma(B)$ is not computable from $\sigma(A)$ and $\sigma(C)$.)

- Do "OR"

- Do formulae & circuits built from AND's and OR's.

Signing Graphs

- A <u>graph</u> $G = (V, E)$ has a finite set V of vertices and a set $E \subseteq V \times V$ of edges. (May be directed or undirected)

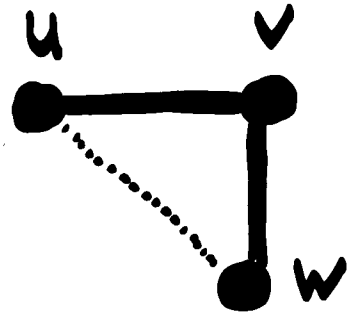


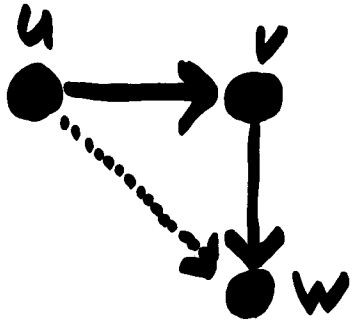
- Graphs are widely used representation.
- We are interested in secure (authenticated, signed) representation of graphs when graph has certain properties.

Transitive Closure

- A graph is transitively closed if

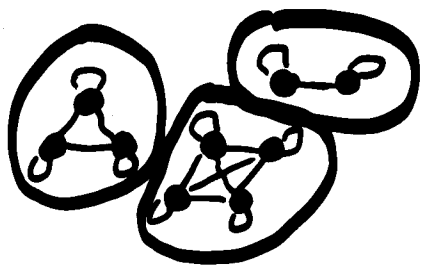
$$\frac{(u,v) \in E \;\&\; (v,w) \in E}{\Rightarrow \; (u,w) \in E}$$



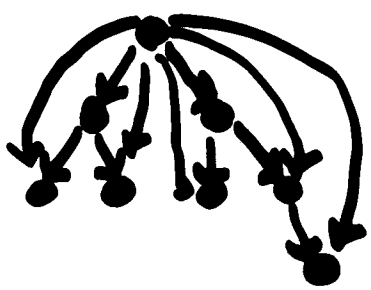


- Many graphs are naturally closed transitively (& reflexively):

administrative domains



chain of command

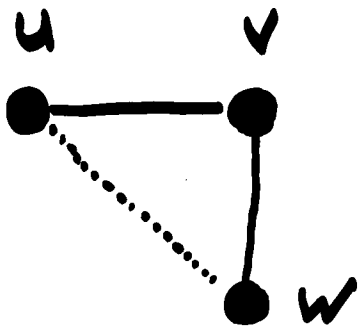


Transitive Signature Schemes

- A transitive signature scheme is a way of signing vertices ($\sigma(v)$) and edges ($\sigma(u,v)$) such that given

  $\sigma(u,v)$

  and $\sigma(v,w)$

  one can compute $\sigma(u,w)$

- Imagine some issuer signs various vertices and edges over time...
- Inferred signature $\sigma(u,w)$ should be indistinguishable from an original issuer sig.
- Provides efficiency for issuer & verifier.

TSS for undirected graphs

- $p = 2q+1$ large prime

  g, h elements of Z_p^* of order q

  $\log_g(h)$ infeasible to compute (DLP)

- For vertex i, issuer computes

$$\boxed{v_i = g^{x_i} h^{y_i} \pmod p}$$

  v_i public (& signed), x_i & y_i secret, random

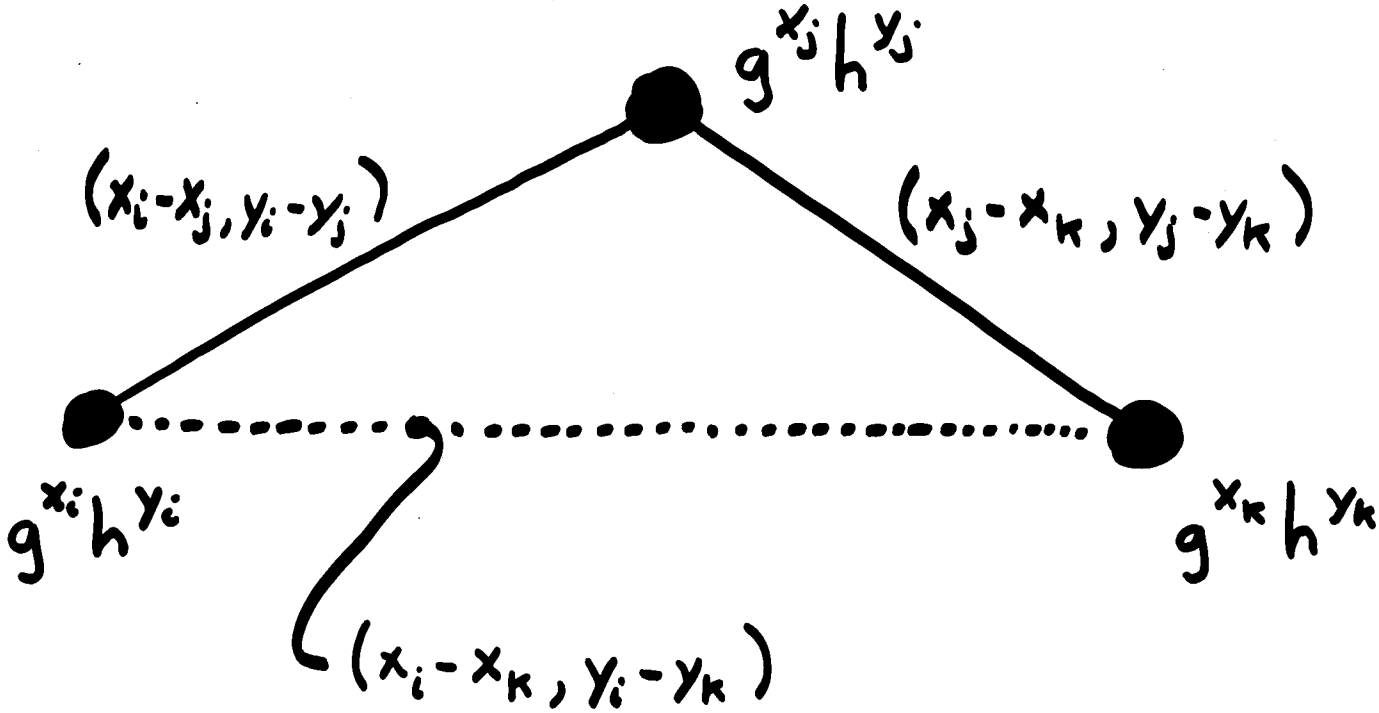
- For edge (i,j), issuer's sig is

$$(\Delta x, \Delta y) = (x_i - x_j, y_i - y_j) \bmod q$$

- Verify edge:

$$v_i / v_j \stackrel{?}{=} g^{\Delta x} h^{\Delta y} \pmod p$$

Transitivity



$$= (x_i - x_j, y_i - y_j) + (x_j - x_k, y_j - y_k)$$

$$\sigma(i,k) = \sigma(i,j) + \sigma(j,k)$$

Security

Theorem: Assuming that discrete logarithm problem is hard, an adversary can not forge a signature on an edge not already signed and not implied by transitivity, even if he can adaptively request edge signatures first.

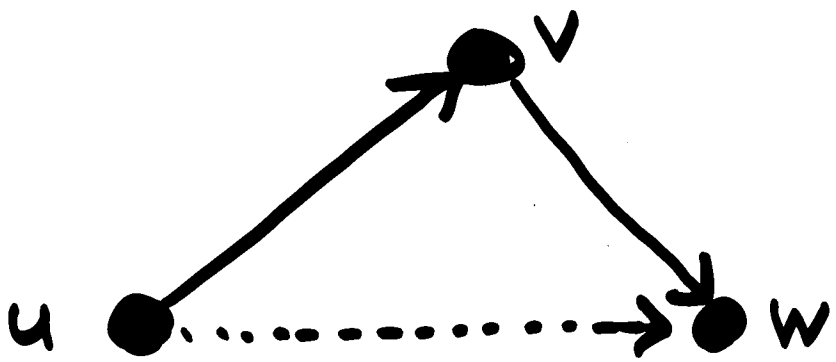
Proof sketch: Given DLP instance $\log_g(h) \bmod p$, simulate adversaries view. Can answer all signature requests *without* knowing $\alpha = \log_g(h)$.

Representations unknown to adversary by multiples of $(-\alpha, 1)$, since $g^{-\alpha}h = 1$. But

$$g^{\Delta x} h^{\Delta y} = g^{\Delta x'} h^{\Delta y'} \Rightarrow \alpha = \frac{\Delta x - \Delta x'}{\Delta y' - \Delta y}$$

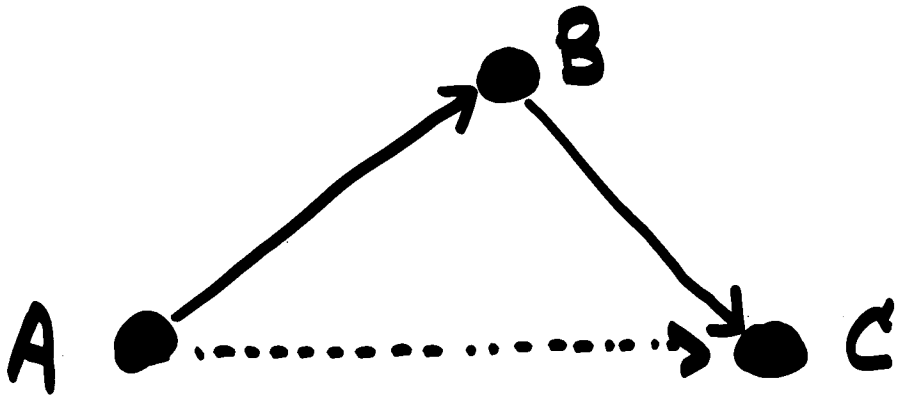
Open Problem I

Find a secure directed transitive signature scheme.



Open Problem II

Assume vertices = public keys



Find a TSS such that only B can create $\dot{\sigma}(A,C)$ from $\sigma(A,B)$ and $\sigma(B,C)$.

(Delegation; SPKI/SDSI tuple reduction)