Computer Systems Modelling

UNIVERSITY OF CAMBRIDGE

Computer Laboratory

Computer Science Tripos, Part II

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Problem sheet

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Simulation

1. In what contexts is simulation an appropriate technique for performance evaluation? When it is inappropriate?

2. How would you generate random variates for the exponential distribution? Write pseudo code for a simulation component which models the arrival process of customers at a bank, given that the mean arrival rate is 40 customers per hour. What are the important events for this component (the generator).

3. How would you generate random variates from the mixed exponential distribution

$$f(x) = 1/3\lambda_1 \mathrm{e}^{-\lambda_1 x} + 2/3\lambda_2 \mathrm{e}^{-\lambda_2 x}$$

   where $\lambda_1 = 5, \lambda_2 = 10$, which describes the joint arrival process of two separate exponentially distributed streams of customers to a single queue?

4. Suppose that a simulation is constructed to estimate the mean response time, in milliseconds, of an interactive computer system. A number of repetitions are performed with the following results (measured in milliseconds): 4.1, 3.6, 3.1, 4.5, 3.8, 2.9, 3.4, 3.3, 2.8, 4.5, 4.9, 5.3, 1.9, 3.7, 3.2, 4.1, 5.1.

   Calculate the sample mean and variance of these results and hence derive a 95% confidence interval for the mean response time. You may wish to make use of the fact that $\mathbb{P}(Z > 1.96) > 0.025 > \mathbb{P}(Z > 1.97)$ where Z is a random variable with the unit normal distribution.

5. An experiment is performed to estimate the performance of a $M/M/1$ system with FIFO queueing. The response times of each of 1000 successive customers are recorded and the sample mean ($\overline{X}$) and sample variance (S^2) of these numbers are calculated.

   A naïve student believes that $100(1-\alpha)\%$ confidence bounds for the mean can be derived by the expression

$$\overline{X} \pm \frac{z_{\alpha/2} S}{\sqrt{1000}}$$

   where $\mathbb{P}(Z > z_{\alpha/2}) = \alpha/2$ and Z is a standard Normal random variable.

   What mistake has this student made? Would their technique be valid if they were estimating the service time of this queue?

6. Using the inverse transform method show that

$$X = \left\lfloor \frac{\log(U)}{\log(1-p)} \right\rfloor + 1$$

   has a geometric distribution with parameter p where U has the $U(0,1)$ distribution.

7. Describe a procedure for the generation of the first T time units of a Poisson process of fixed rate λ.

8. Describe the variance reduction technique based on antithetic variables and give an example of how it is used.
9. For the variance reduction technique based on control variates derive the optimal choice of $c = c^*$ and its associated variance given in the lectures.
10. 2007 Paper 7 Question 10
11. 2006 Paper 7 Question 5
12. 2005 Paper 8 Question 16
13. 2003 Paper 8 Question 15

Queueing theory

1. In what situations does queuing theory provide appropriate techniques for performance evaluation? When does it not?

2. Show for the M/M/1 queue that the probability that there are n or more customers in the system is given by ρ^n.

   Use this result to find a service rate μ such that, for given λ, n, α where $0 < \alpha < 1$, the probability of n or more customers in the system is given by α.

   Find a value of μ for an M/M/1 queue for which the arrival rate is 10 customers per second, and subject to the requirement that the probability of 3 or more customers in the system is 0.05

3. Using the steady-state distribution of the number of jobs in a $M/M/1$ queueing system given in lectures derive the first and second moments of this distribution and hence the variance of the number of jobs present. Describe what happens as the load ρ increases.

4. Given an $M/M/1/K$ queue with $\lambda = 10$, $\mu = 12$ and $K = 15$ over what proportion of time are customers rejected from the queue? What is the effective arrival rate? What is the effective utilization of the server?

5. Repairing a computer takes 4 stages in sequence, namely removing the lid, finding the faulty part, replacing it, and reassembling the machine. Each step is independent and exponentially distributed with mean 3 minutes. What is the coefficient of variation of the repair time? Construct a Markov chain model of this system, assuming an infinite population of machines.

6. A closed queueing network (shown below) comprises two $M/M/1$ nodes, **A** and **B** between which n identical jobs circulate. The nodes have service rates μ_A and μ_B respectively. Upon completion at **A**, a job moves to **B** with probability p and otherwise it remains at **A**. Similarly, upon completion at **B**, a job moves to **A** with probability q.

   Derive a Markov-chain model of this system, explaining what each state in the model signifies and what transition rates exist between states.

7. 2007 Paper 9 Question 11

8. 2006 Paper 8 Question 11

9. 2005 Paper 7 Question 5

10. 2004 Paper 8 Question 16
11. 2003 Paper 7 Question 5
12. 2002 Paper 8 Question 14
13. 2002 Paper 7 Question 8

Case studies

1. Describe the major differences between queueing networks and stochastic Petri nets? What are their pros and cons as modeling formalisms?
2. What are the major steps in the workload characterization process?
3. What are the advantages of queueing Petri net models?
4. Sketch the process of modeling a system for performance prediction purposes.
5. Describe how models are validated?