Introduction to Functional Programming

Exercises on structural induction

1. Prove the statements of ***Lecture X*** by structural induction.

2. Let

```
fun foldl f e [] = e
  | foldl f e (h::t) = foldl f ( f(x,e) ) t
```

   (a) For all $f : \alpha * \beta \to \beta$, $b : \beta$, and $\ell_0, \ell_1 : \alpha$ `list`, show that

$$\texttt{foldl}\ f\ b\ (\ell_0 \,\texttt{@}\, \ell_1) \ = \ \texttt{foldl}\ f\ (\texttt{foldl}\ f\ b\ \ell_0)\ \ell_1 \quad : \beta$$

   (b) For $\oplus : \beta * \beta \to \beta$ an associative function show that, for all $b_0, b_1 : \beta$ and $\ell : \alpha$ `list`,

$$\texttt{foldl}\ \oplus\ (b_1 \oplus b_0)\ \ell \ = \ (\texttt{foldl}\ \oplus\ b_1\ \ell) \oplus b_0 \quad : \beta$$

3. Let

```
fun foldr f e [] = e
  | foldr f e (h::t) = f( h , foldr f e t )
```

   (a) For all $\ell_0, \ell_1 : \alpha$ `list`, show that

$$\texttt{foldr}\ (\texttt{op::})\ \ell_0\ \ell_1 \ = \ \ell_1 \,\texttt{@}\, \ell_0 \quad : \alpha\ \texttt{list}$$

   (b) For $\otimes : \beta * \beta \to \beta$ an associative function and $e : \beta$ such that $\otimes(e, x) = x$ for all $x : \beta$, show that

$$(\texttt{foldr}\ \otimes\ e\ \ell) \otimes b \ = \ \texttt{foldr}\ \otimes\ b\ \ell$$

   and

$$\texttt{foldr}\ (\texttt{fn}(l, b) \Rightarrow \texttt{foldr}\ \otimes\ b\ l)\ e\ \ell \ = \ \texttt{foldr}\ \otimes\ e\ (\texttt{map}\ (\texttt{foldr}\ \otimes\ e)\ \ell) \quad : \beta$$

   for all $b : \beta$ and $\ell : \beta$ `list`.