Complexity Theory
Easter 2010
Suggested Exercises 2

1. Given a graph $G = (V, E)$, a set $U \subseteq V$ of vertices is called a *vertex cover* of G if, for each edge $(u, v) \in E$, either $u \in U$ or $v \in U$. That is, each edge has at least one end point in U. The decision problem V-COVER is defined as:

   given a graph $G = (V, E)$, and an integer K, does G contain a vertex cover with K or *fewer* elements?

   (a) Show a polynomial time reduction from IND to V-COVER.

   (b) Use (a) to argue that V-COVER is NP-complete.

2. The problem of four dimensional matching, 4DM, is defined analogously with 3DM:

   Given four sets, W, X, Y and Z, each with n elements, and a set of quadruples $M \subseteq W \times X \times Y \times Z$, is there a subset $M' \subseteq M$, such that each element of W, X, Y and Z appears in exactly one triple in M'.

   Show that 4DM is NP-complete.

3. Given a graph $G = (V, E)$, a *source vertex* $s \in V$ and a *target vertex* $t \in V$, a *Hamiltonian Path* from s to t in G is a path that begins at s, ends at t and visits every vertex in V exactly once. We define the decision problem HamPath as:

   given a graph $G = (V, E)$ and vertices $s, t \in V$, does G contain a Hamiltonian path from s to t?

   (a) Give a polynomial time reduction from the Hamiltonian cycle problem to HamPath.

   (b) Give a polynomial time reduction from HamPath to the problem of determining whether a graph has a Hamiltonian cycle.

   Hint: consider adding a vertex to the graph.