MPhil in Advanced Computer Science
Categorical logic

Leader:	Sam Staton
Timing:	Lent
Prerequisites:	'Introduction to category theory', and some basic knowledge of logic and set theory
Structure:	16 Lectures

AIMS

In this module we will investigate models of constructive logics from the perspective of category theory. The course will be illustrated with examples from theoretical computer science.

SYLLABUS

1. **Simply-typed lambda calculus:** Intuitionistic logic. Propositions as objects in cartesian-closed categories (CCCs): the Curry-Howard correspondence. Examples of CCCs, including presheaf categories. (3L)

2. **First-order logic.** 'Propositions as subobjects'. Finite limits, and Horn logic. Regular categories, and regular logic. Quantifiers as adjoints, and first order logic in pretoposes. Functors between categories. (4L)

3. **Higher-order logic.** Powerobjects and toposes. Properties of toposes. Toposes as set-theories. Sheaf constructions and forcing. (5L)

4. **Dependent types.** Overview and examples of programming in Agda/Coq. (1L: a practical class.)

5. **Categorical type theory.** 'Propositions as arrows': dependent type theories in locally cartesian-closed categories. (3L)

OBJECTIVES

On completion of this module, students should:

- be familiar with various logical formalisms, including dependent type theory;
- be familiar with the basic principles of categorical logic for various fragments of logic.

COURSEWORK

I will hand out exercises, for each section of the course.

PRACTICAL WORK

One practical session will be timetabled for introducing a dependently-typed programming language, either Agda or Coq.

ASSESSMENT

The course will be assessed by a term paper, set and marked by the course lecturer. Marks will be a letter grade (A excellent, B good, C satisfactory, D fail).

RECOMMENDED READING

For preparatory reading, to get a feel for the subject, have a look at

Relating theories of the λ-calculus. D. S. Scott. In: *To H. B. Curry: essays in combinatory Logic, lambda calculus and formalisms.* Edited by R. Hindley and J. Seldin. Academic Press, 1980.

This is a well-developed subject and there are many texts. Here are two recent ones.

Categorical logic. A. M. Pitts. In: *Handbook of Logic in Computer Science, Volume V: Algebraic and Logical Structures.* Edited by S. Abramsky, Dov M. Gabbay, and T. S. E. Maibaum. Oxford University Press, 2001. Available online.

Sketches of an elephant: a topos theory compendium. P. T. Johnstone. Oxford Logic Guides nos 43 and 44. Oxford University Press, 2002. (Portions of Chs A.1, A.2, D.1, D.4, are relevant.)

Paul Taylor's book may be easier on the casual reader.

Practical foundations of mathematics. P. Taylor. Cambridge Studies in Advanced Mathematics, vol 59. Cambridge University Press, 1999. Partly available online, in HTML form.

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