Failure of full abstraction

$\exists \; M_1, M_2 \in PCF_\tau$

s.t $\quad M_1 \cong_{ctx} M_2$

but $\quad [\![M_1]\!] \neq [\![M_2]\!]$

We will look at higher types τ, say $\tau = \tau_1 \to \tau_2$

$$M_1 \cong_{ctx} M_2 : \tau_1 \to \tau_2$$

iff $\forall N.\; M_1 N \Downarrow V$ iff $M_2 N \Downarrow V$

$[\![M_1]\!]([\![N]\!]) = [\![M_1 N]\!] = [\![V]\!] \qquad [\![V]\!] = [\![M_2 N]\!] = [\![M_2]\!]([\![N]\!])$

$$[\![M_1]\!] \neq [\![M_2]\!] : [\![\tau_1]\!] \to [\![\tau_2]\!]$$

iff $\quad \exists d \in [\![\tau_1]\!].\; [\![M_1]\!]\, d \neq [\![M_2]\!]\, d$

We look for non-definable d's!

$\forall \tau.\ \bot \in [\![\tau]\!]$

is definable by

$$\Omega = \underline{fix}(\underline{fn}\ x:\tau.\ x)$$

$\Omega \not\Downarrow$

Suppose $\Omega \Downarrow V$
Then there a smallest
derivation of this fact.

$$\dfrac{\dfrac{}{\mathrm{fn}\,x.\,x \Downarrow \mathrm{fn}\,x.\,x} \qquad \dfrac{\vdots}{\overset{\Omega}{\shortparallel}\atop x[\Omega/x] \Downarrow V}}{\dfrac{(\mathrm{fn}\,x.\,x)(\Omega) \Downarrow V}{\Omega \Downarrow V}}$$

and the
analysis

leads to
contradiction.

A non-definable function

$$\underline{por} \in B_\perp \to B_\perp \to B_\perp$$

$$\begin{cases} \underline{por}\,(\underline{true})\,\perp = \underline{true} \\ \underline{por}\,(\perp)\,\underline{true} = \underline{true} \\ \underline{por}\,\underline{false}\,\underline{false} = \underline{false} \end{cases}$$

There is no

$$M \in PCF_{bool \to bool \to bool}$$

s.t.

$$[\![M]\!] = \underline{por}$$

→ operationally.

→ stable model.

$$[\![T_1]\!] : (\mathbb{B}_\perp \to \mathbb{B}_\perp \to \mathbb{B}_\perp) \to \mathbb{B}_\perp$$

$$[\![T_1]\!](f) = \begin{cases} \text{true} & \text{if } f = \underline{por} \\ \perp & \text{otw.} \end{cases}$$

$$[\![T_2]\!](f) = \begin{cases} \text{false} & \text{if } f = por \\ \perp & \text{otw} \end{cases}$$

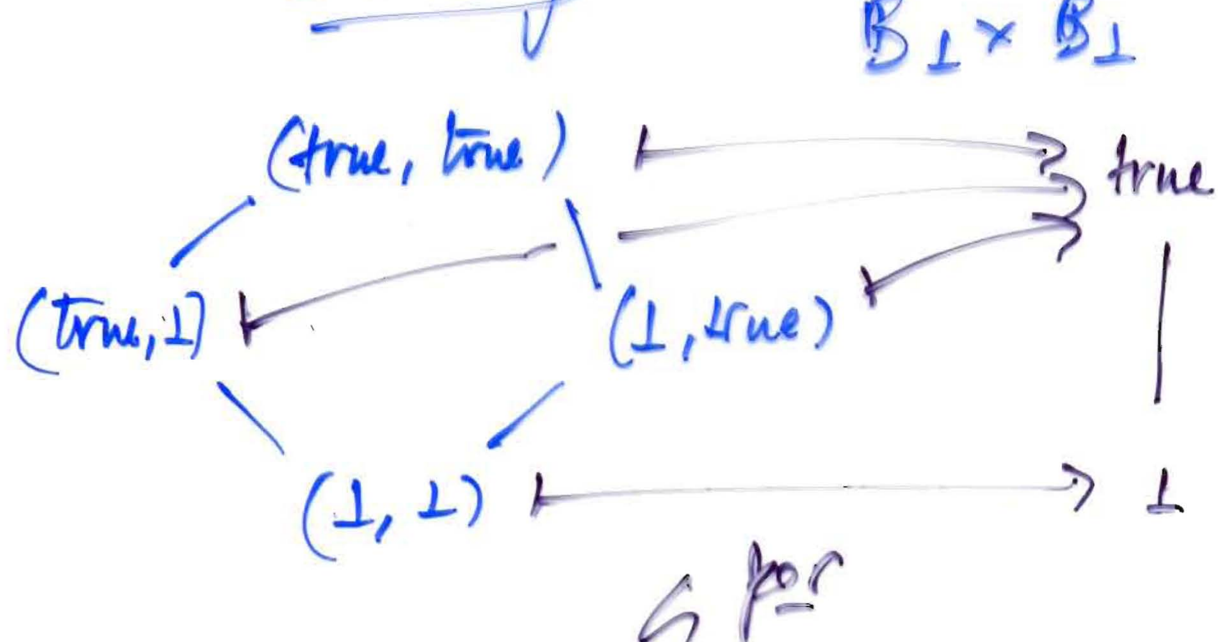
$\forall M : bool \to bool \to bool$

$$T_1(M) \not\Downarrow \quad \& \quad T_2(M) \not\Downarrow$$

and so

$$T_1 \cong_{ctx} T_2$$

Stability



not stable

$$\underline{por} : bool \to bool \to bool$$
$$\wr$$
$$\underline{por} : bool \times bool \to bool$$