MPhil in Advanced Computer Science
Set Theory for Computer Science

Leader: Glynn Winskel
Timing: Michaelmas
Prerequisites:
Structure: Study group of 8 hours based on notes provided by GW.
As a basic course it should be presented early in Michaelmas over 4 weeks.

AIMS

This module aims to provide fairly rapidly the set theory every computer scientist should know, and as such should be accessible to MPhil students generally.

SYLLABUS

1. Sets and logic: Subsets of a fixed set as a Boolean algebra. Propositional logic and its models. Validity, entailment, and equivalence of boolean propositions.

2. Relations and functions: Product of sets. Relations, functions and partial functions. Composition and identity relations. Injective, surjective and bijective functions. Direct and inverse image of a set under a relation. Equivalence relations and partitions. Directed graphs and partial orders. Size of sets, especially countability. Cantor's diagonal argument to show the reals are uncountable.

3. Constructions on sets: Russell's paradox. Basic sets, comprehension, indexed sets, unions, intersections, products, disjoint unions, powersets. Characteristic functions. Sets of functions. Lambda notation for functions. Cantor's diagonal argument to show powerset strictly increases size. An informal presentation of the axioms of Zermelo-Fraenkel set theory and the axiom of choice.

4. Inductive definitions: Using rules to define sets. Reasoning principles: rule induction and its instances; induction on derivations. Applications. Inductive definitions as least fixed points. Tarski's fixed point theorem for monotonic functions on a powerset. Maximum fixed points and coinduction.

5. Well-founded induction: Well-founded relations and well-founded induction. Examples. Constructing well-founded relations, including product and lexicographic product of well-founded relations. Applications. Well-founded recursion.

6. Inductively defined properties and classes: Ordinals and transfinite induction. Fraenkel-Mostowski set theory and nominal sets. (1-2 lectures by AP)

OBJECTIVES

On completing this course, students should be able to

- understand and be able to use the language of set theory; prove and disprove assertions using a variety of techniques.
- understand the formalization of basic logic (validity, entailment and truth) in set theory.
- define sets inductively using rules, formulate corresponding proof principles, and prove properties about inductively-defined sets.
- understand Tarski's fixed point theorem and the proof principles associated with the least and greatest fixed points of monotonic functions on a powerset;.
- understand and apply the principle of well-founded induction, including transfinite induction (on ordinals).
- possess some understanding of nominal set theory and its applications.

COURSEWORK

Exercises will be provided each week.

ASSESSMENT

A percentage grade (assessor GW) will be given for coursework based on exercises each week.

RECOMMENDED READING

Full notes will be provided. Supplementary reading: Albert Meyer's forthcoming book on Discrete Mathematics.

Last updated: August 2008