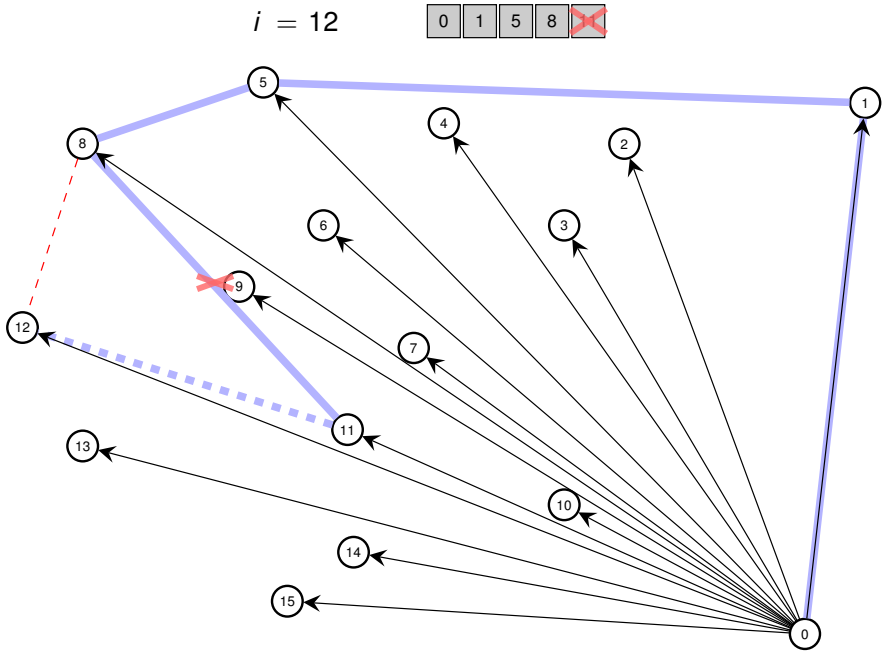


7: Geometric Algorithms

Frank Stajano

Thomas Sauerwald

Lent 2015

UNIVERSITY OF CAMBRIDGE

Outline

Introduction and Line Intersection

Convex Hull

Introduction

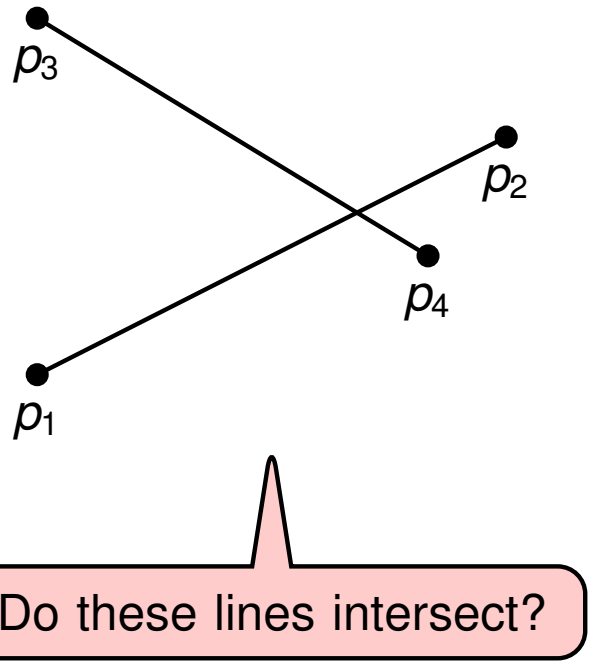
Computational Geometry

- Branch that studies algorithms for geometric problems
- typically, input is a set of points, line segments etc.

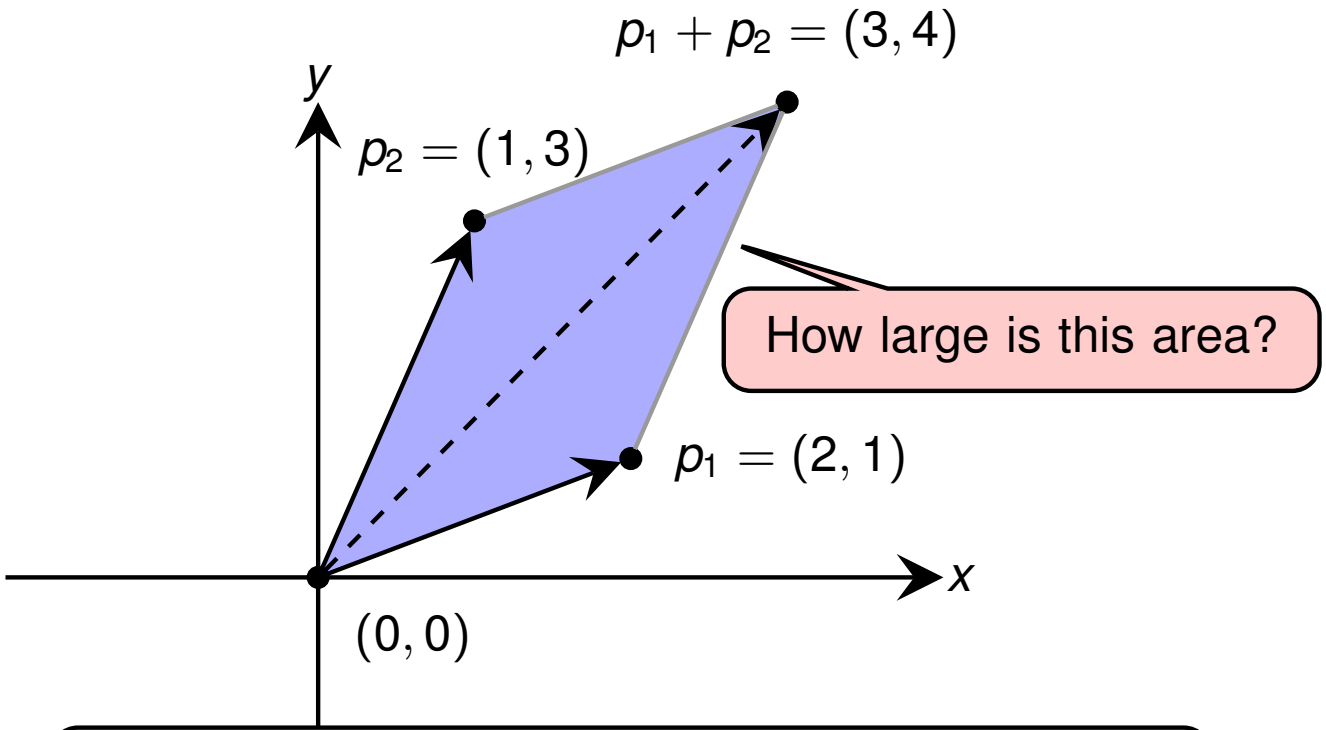
Applications

- computer graphics
- computer vision
- textile layout
- VLSI design

⋮



Cross Product (Area)

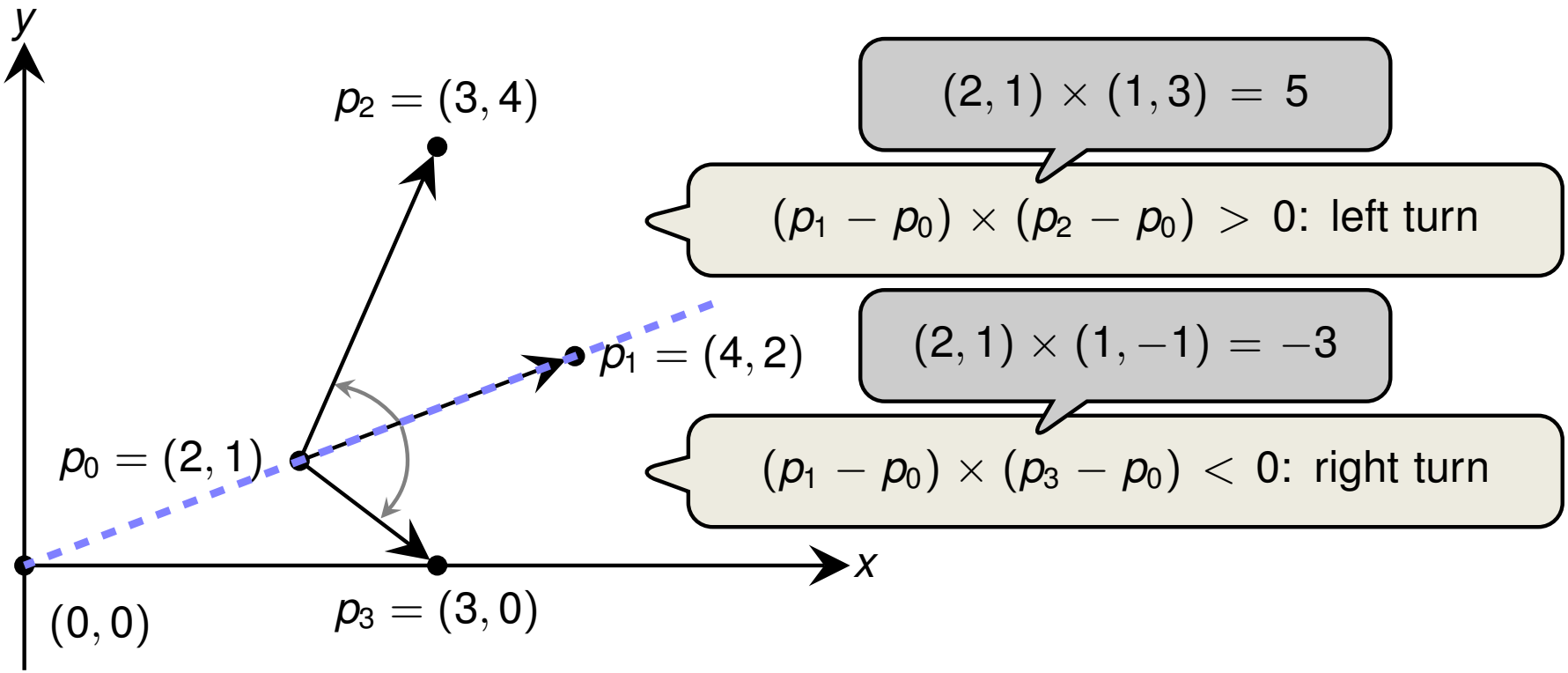


Alternatively, one could take the dot-product (but not used here): $p_1 \cdot p_2 = \|p_1\| \cdot \|p_2\| \cdot \cos(\phi)$.

$$p_1 \times p_2 = \det \begin{pmatrix} x_1 & x_2 \\ y_1 & y_2 \end{pmatrix} = x_1 y_2 - x_2 y_1 = 2 \cdot 3 - 1 \cdot 1 = 5$$

$$p_2 \times p_1 = y_1 x_2 - y_2 x_1 = -(p_1 \times p_2) = -5$$

Using Cross product to determine Turns (2/2)



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Introduction and Line Intersection

Convex Hull

Convex Hull

Definition

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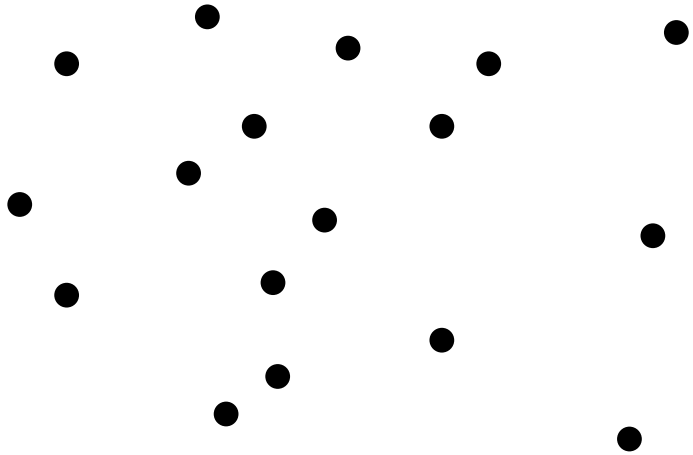
Convex Hull



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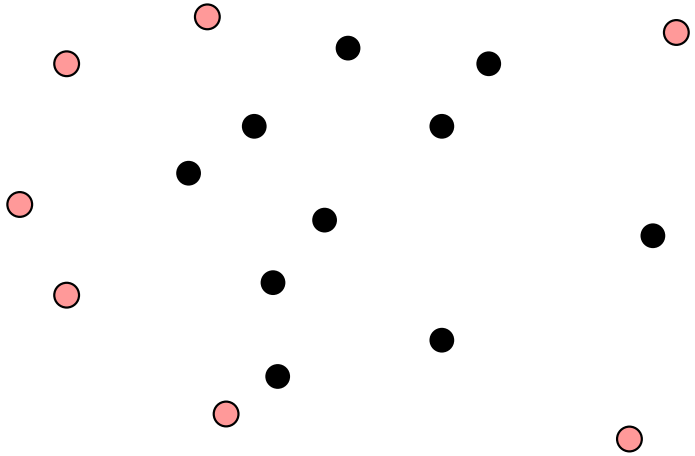

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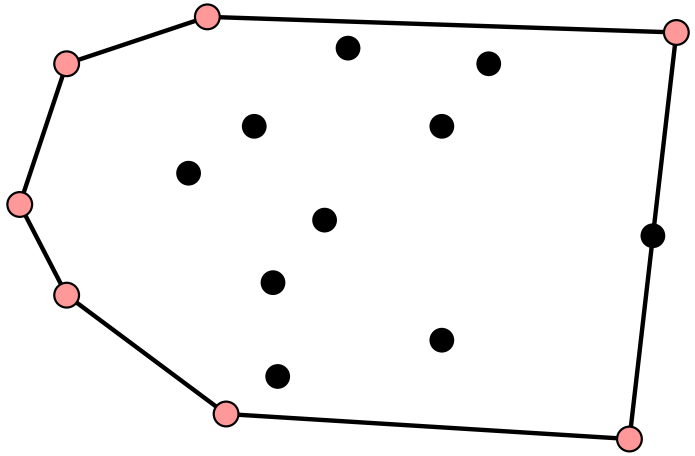

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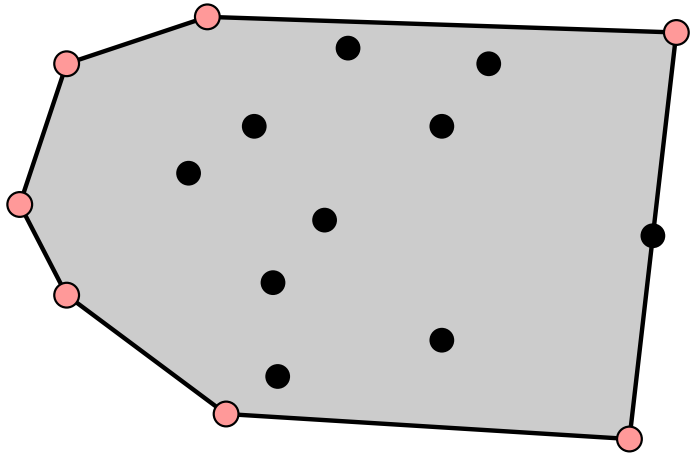

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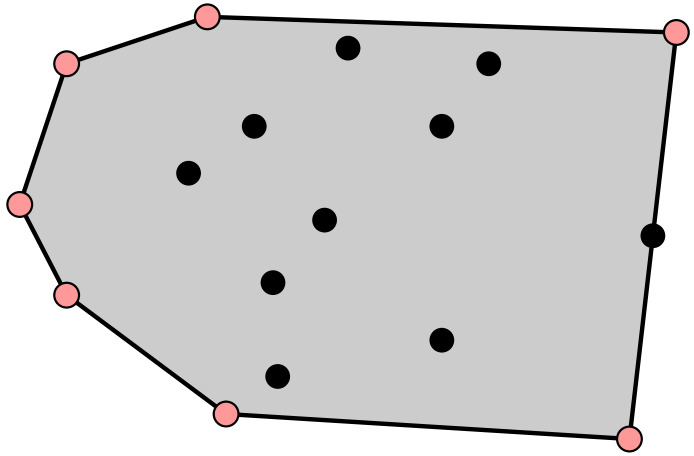

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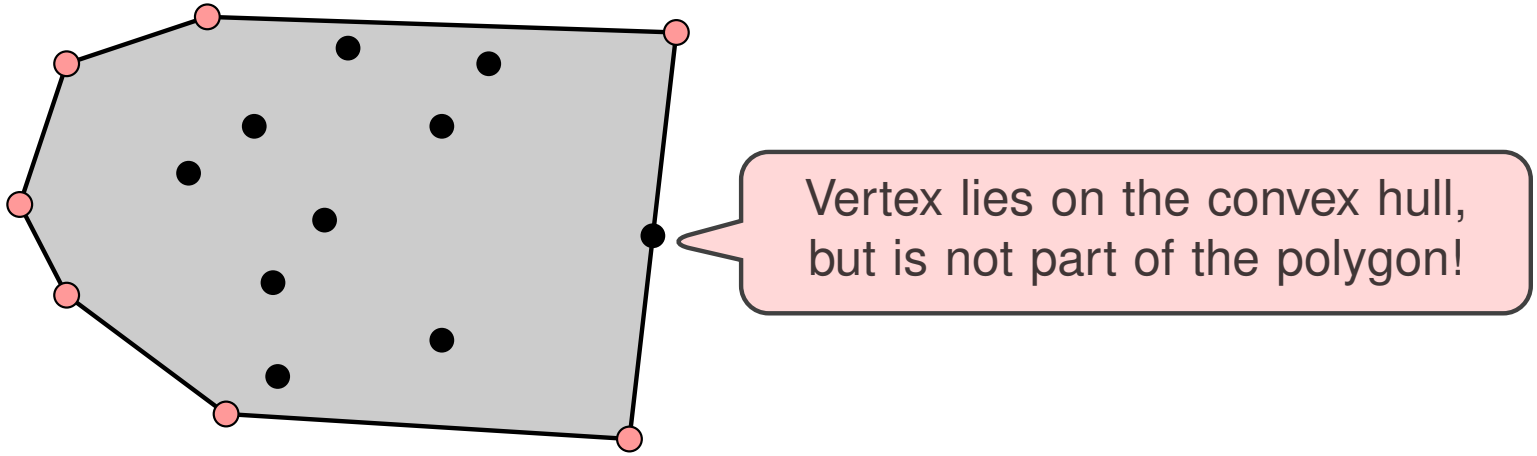
Smallest perimeter fence enclosing the points

Convex Hull



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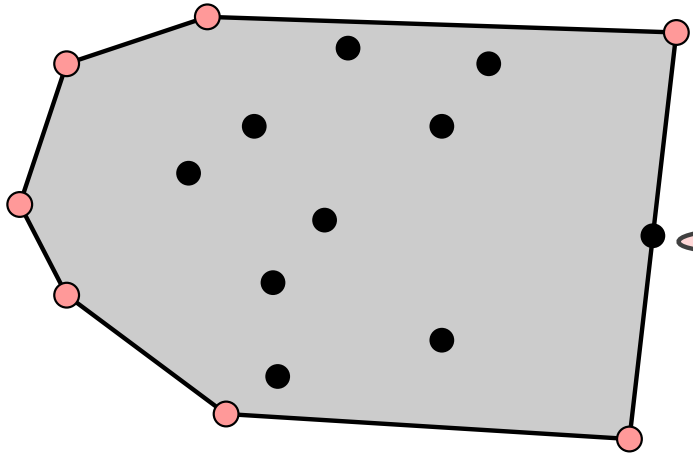

Vertex lies on the convex hull, but is not part of the polygon!

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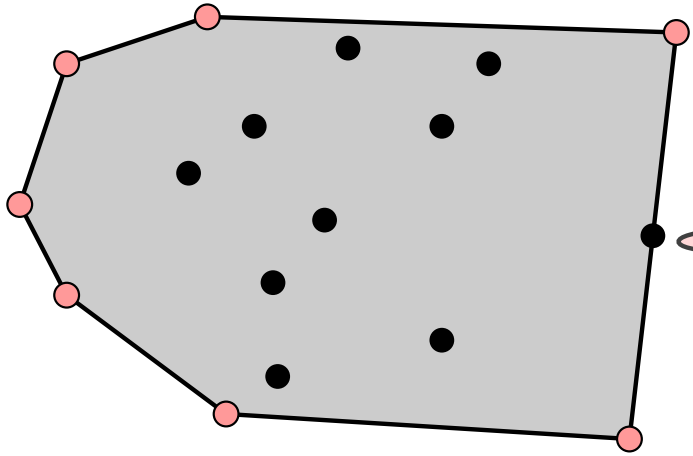
Convex Hull Problem

Convex Hull



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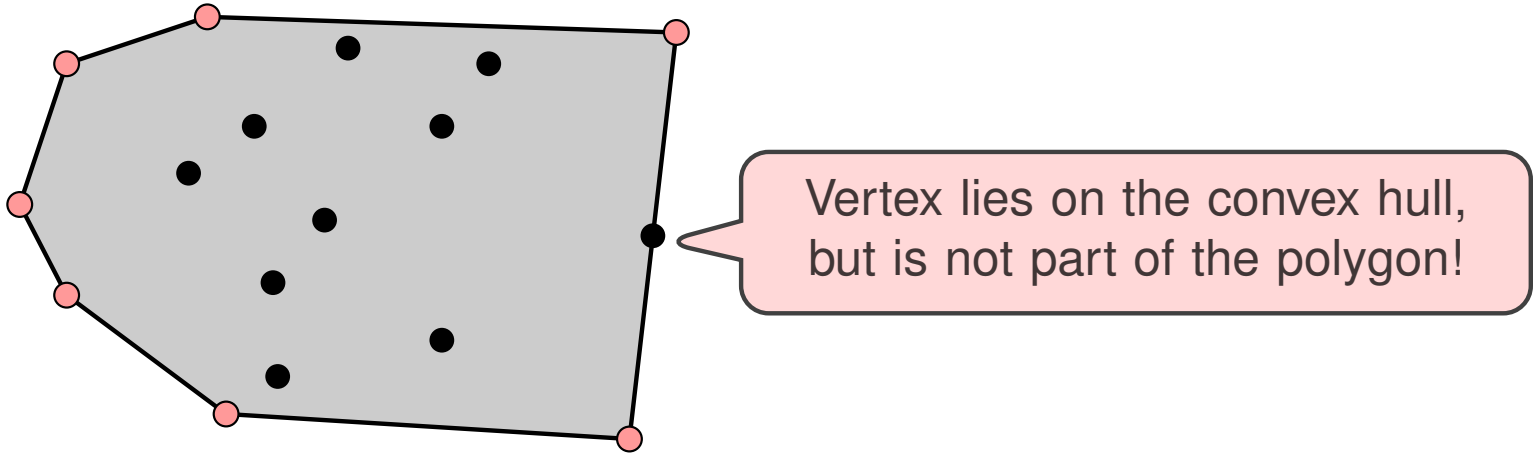
- Input: set of points Q in the Euclidean space

Convex Hull



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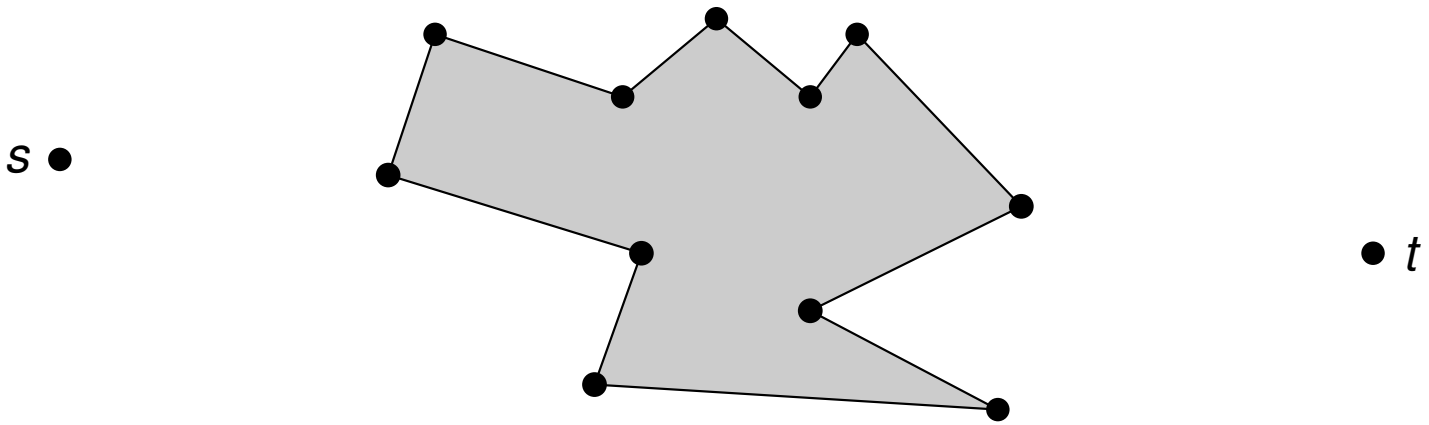
Convex Hull Problem

- Input: set of points Q in the Euclidean space
- Output: return points of the convex hull in counterclockwise order

Application of Convex Hull

Robot Motion Planning

Find shortest path from s to t which avoids a polygonal obstacle.



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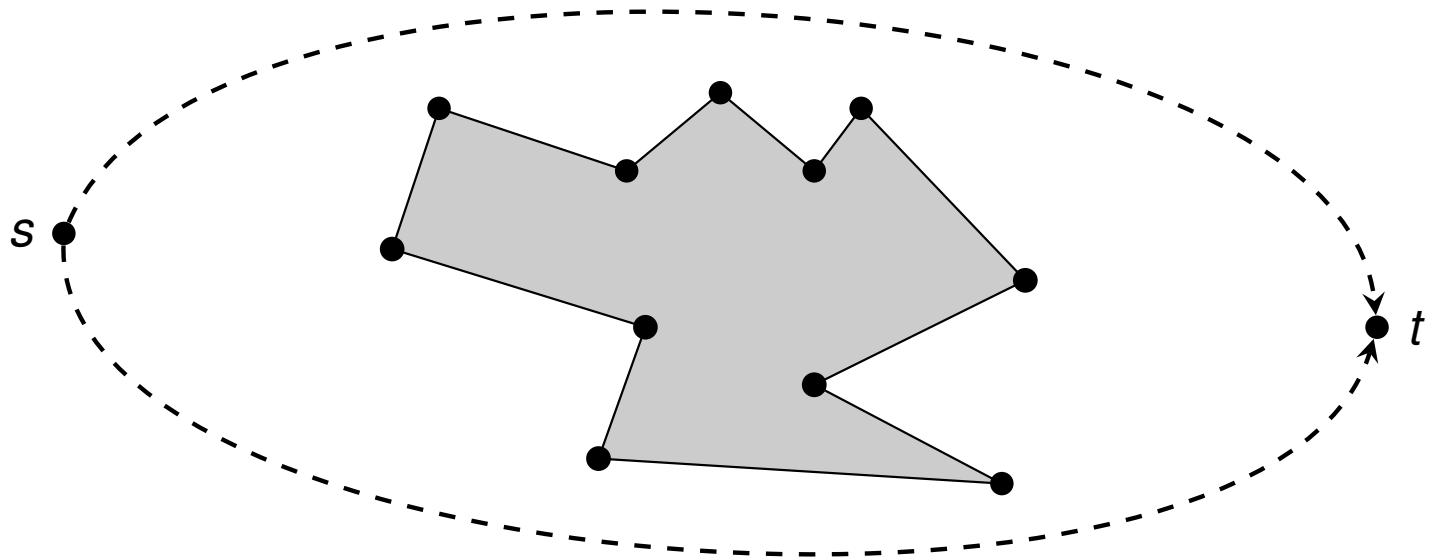

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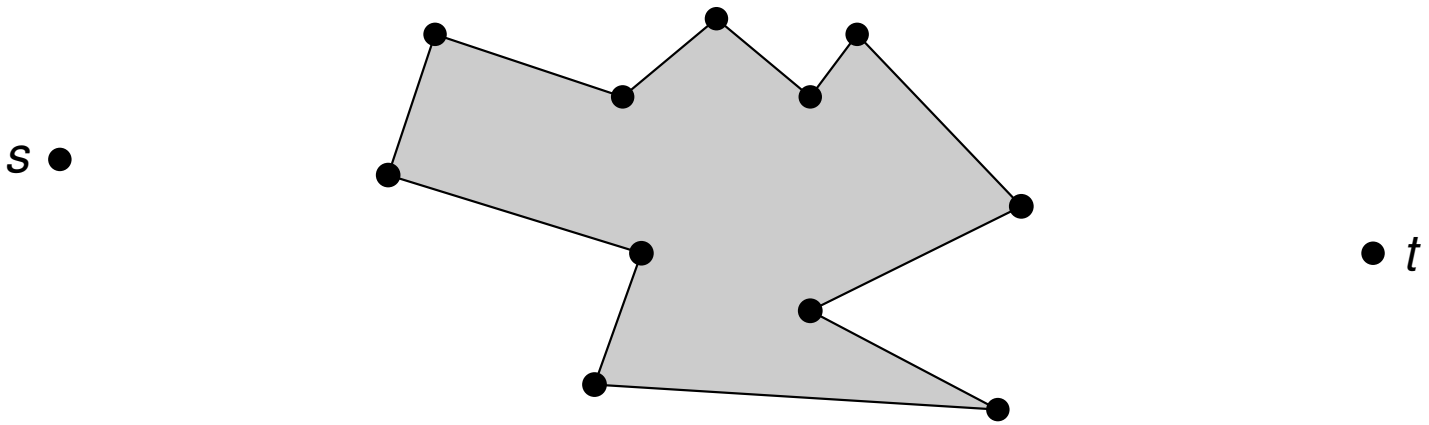

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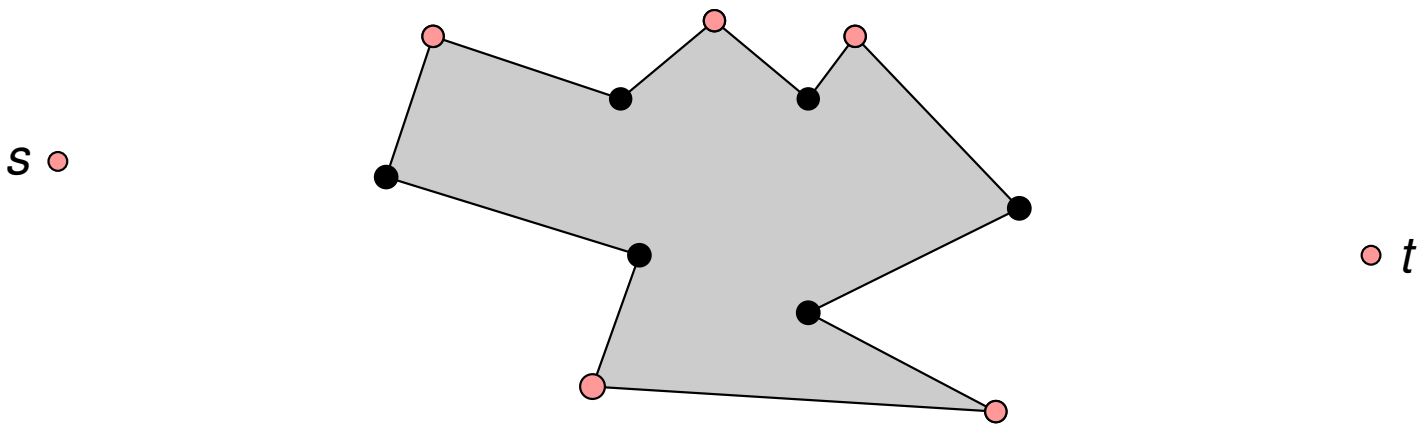

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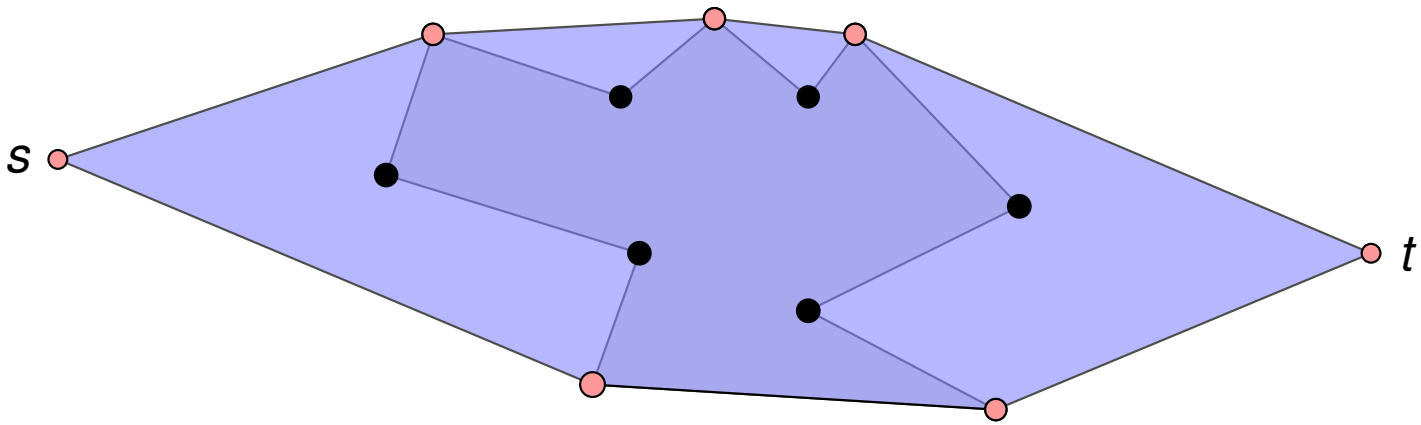

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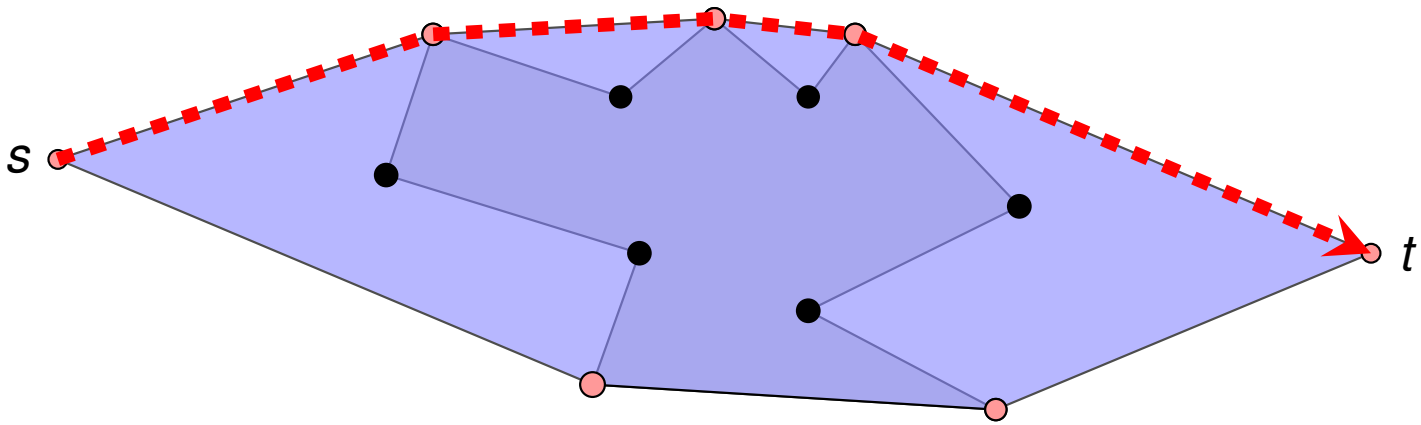

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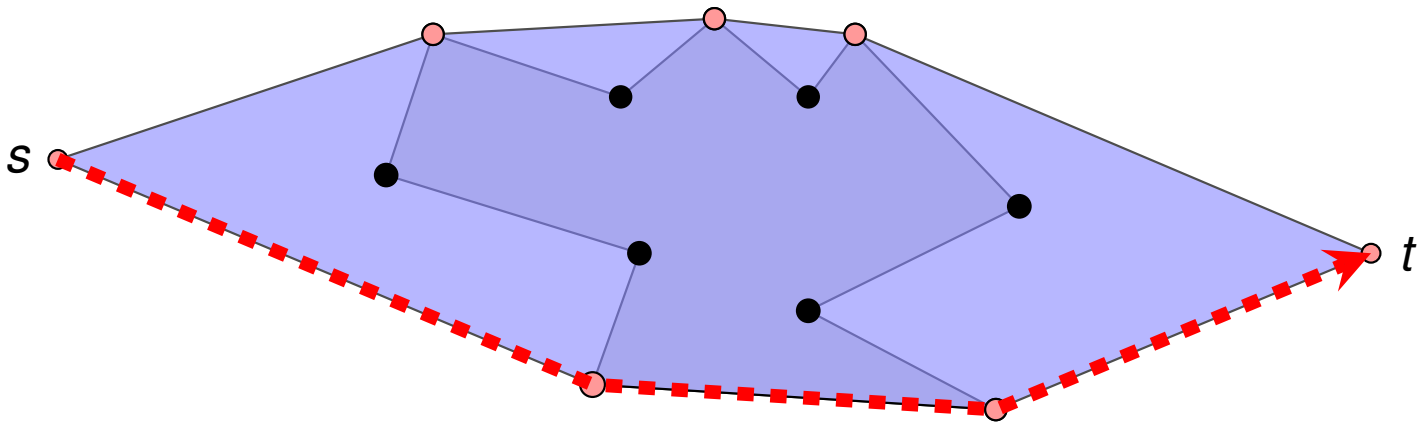
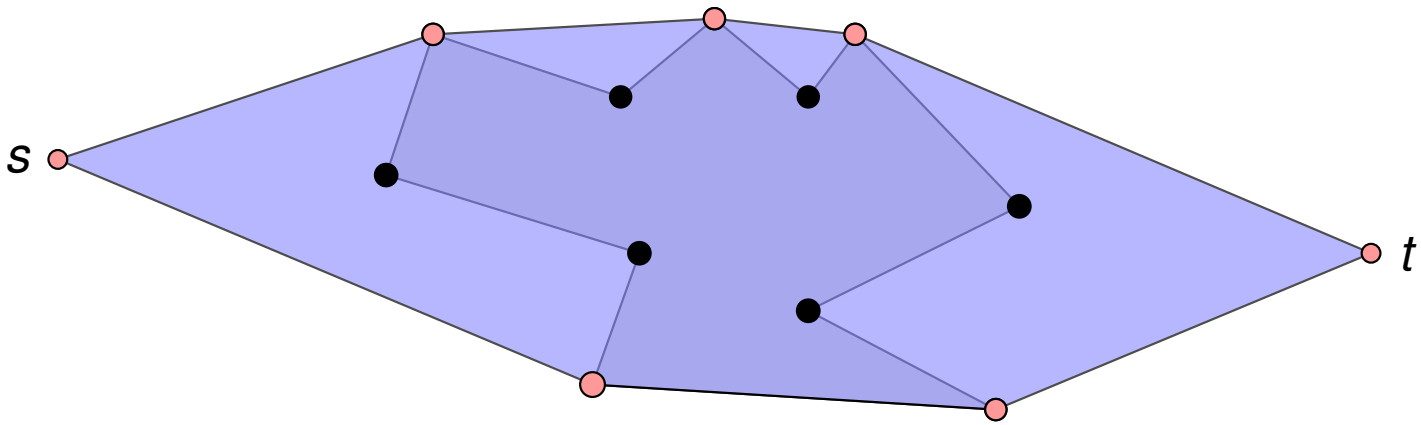


Application of Convex Hull

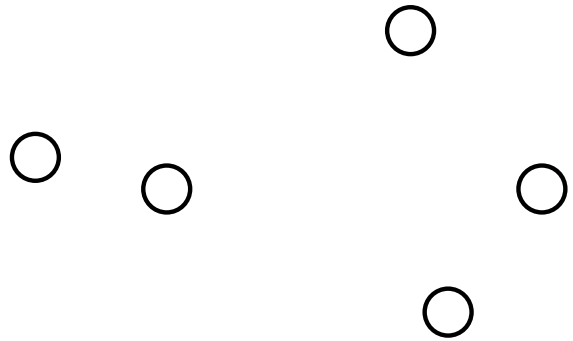
Robot Motion Planning

Find shortest path from *s* to *t* which avoids a polygonal obstacle.

can be solved by computing the Convex hull!



Graham's Scan



Basic Idea

- Start with the point with smallest y-coordinate

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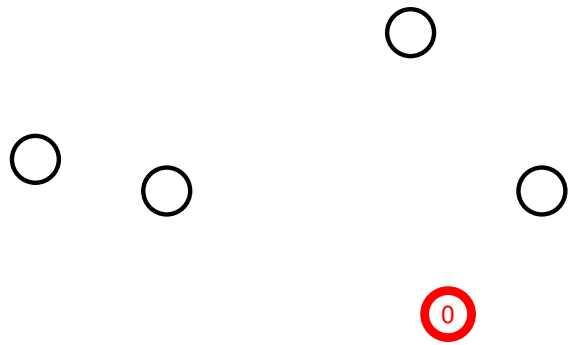

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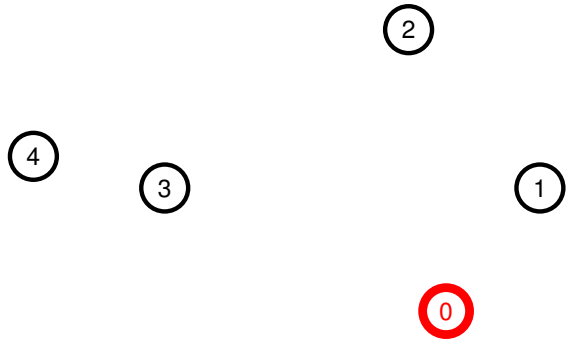

Basic Idea

- Start with the point with smallest y-coordinate
- Sort all points increasingly according to their polar angle

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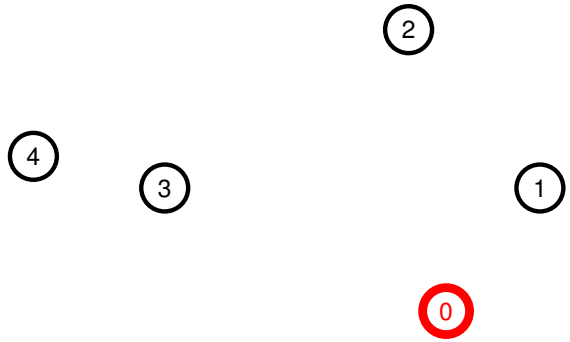

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- Try to add next point to the convex hull

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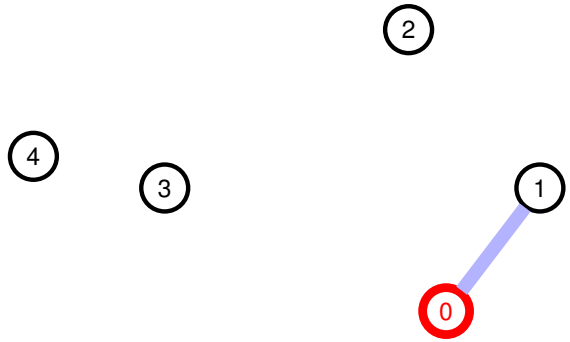

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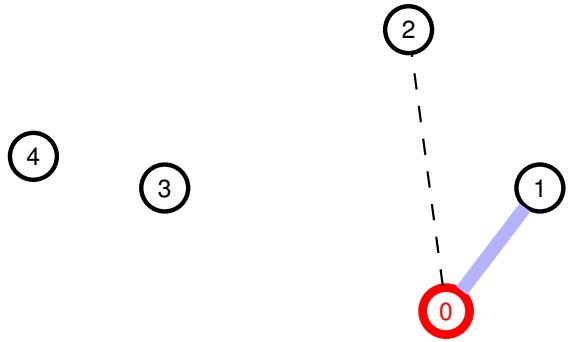

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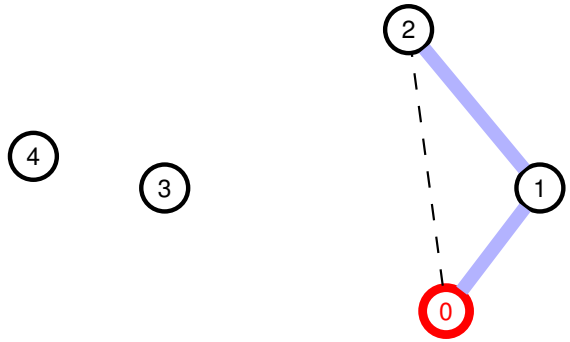

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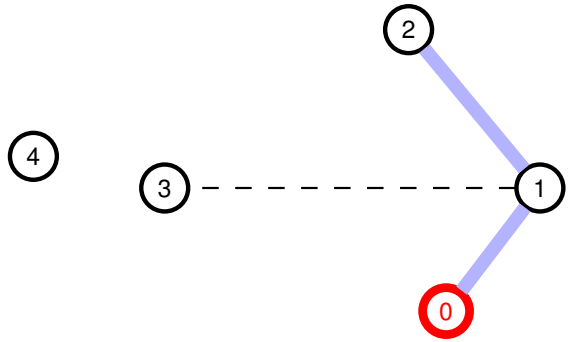

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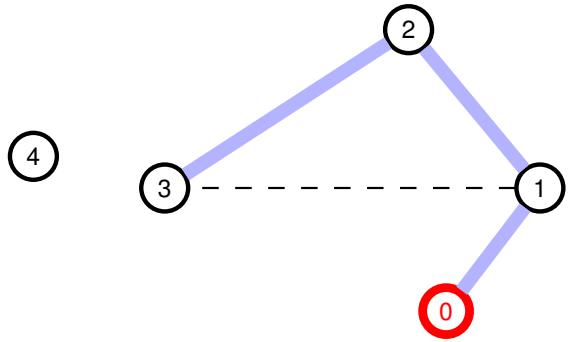

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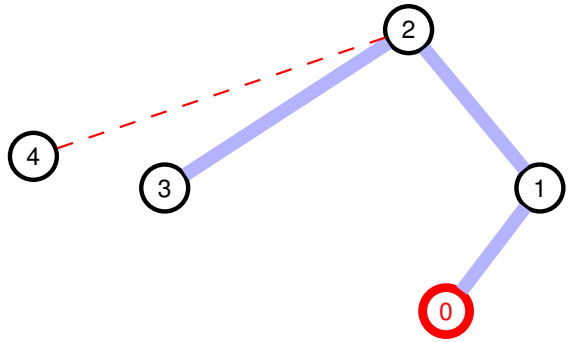

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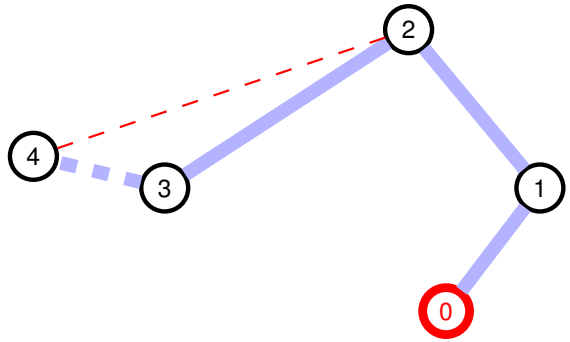

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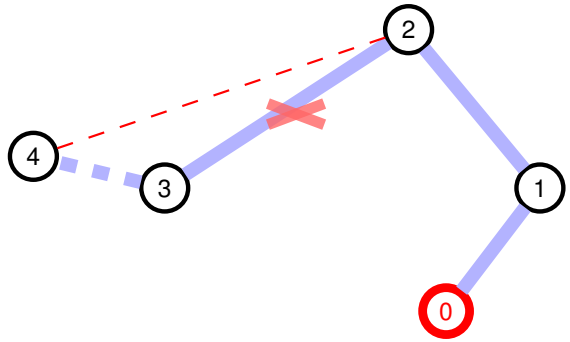

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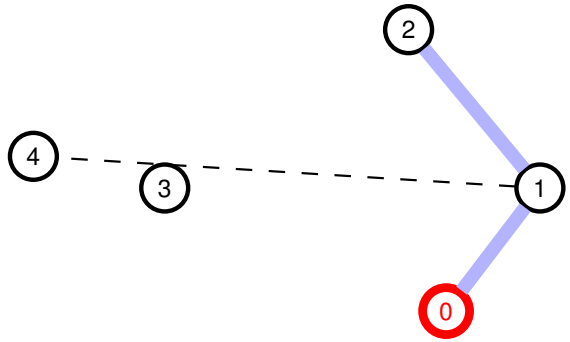

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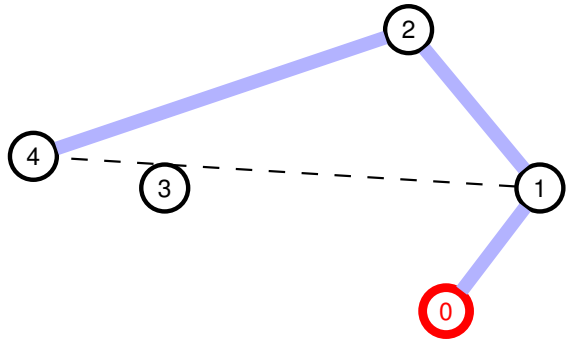

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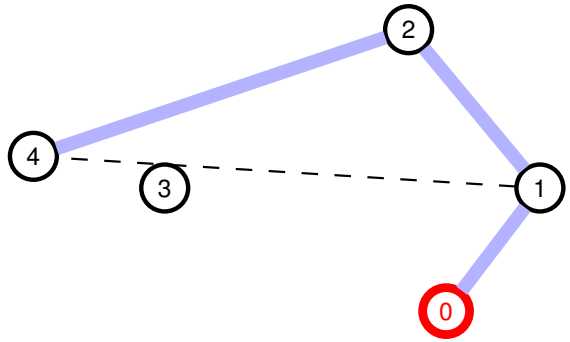

Efficient Sorting by comparing (not computing!) polar angles

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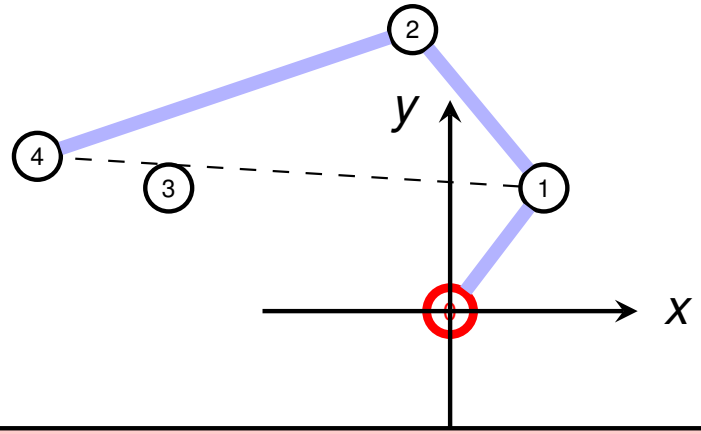

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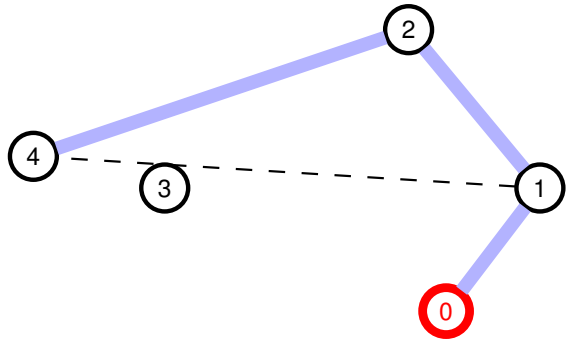

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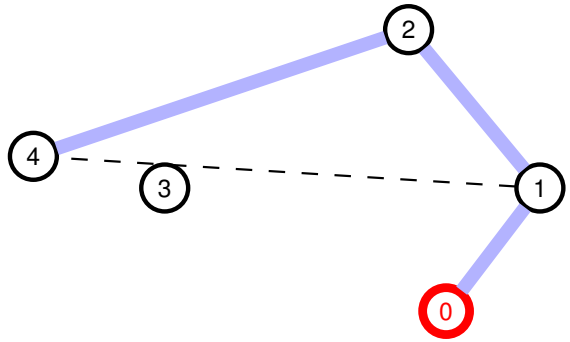

Use Cross Product!

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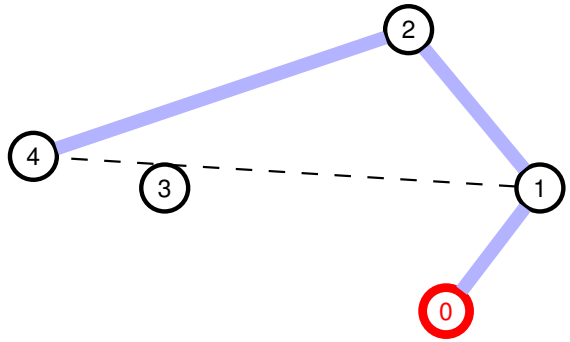
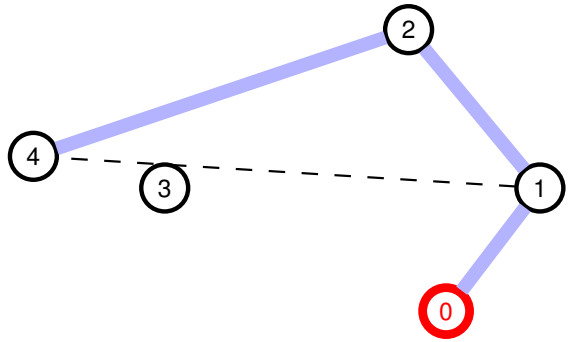



```
 0: GRAHAM-SCAN(Q)
 1:     Let p_0 be the point with minimum y-coordinate
 2:     Let (p_1, p_2, ..., p_n) be the other points sorted by polar angle w.r.t. p_0
 3:     If n < 2 return false
 4:     S = ∅
 5:     PUSH(p_0,S)
 6:     PUSH(p_1,S)
 7:     PUSH(p_2,S)
 8:     For i = 3 to n
 9:         While angle of NEXT-TO-TOP(S),TOP(S),p_i makes a non-left turn
10:             POP(S)
11:         End While
12:         PUSH(p_i,S)
13:     End For
14:     Return S
```

Graham's Scan

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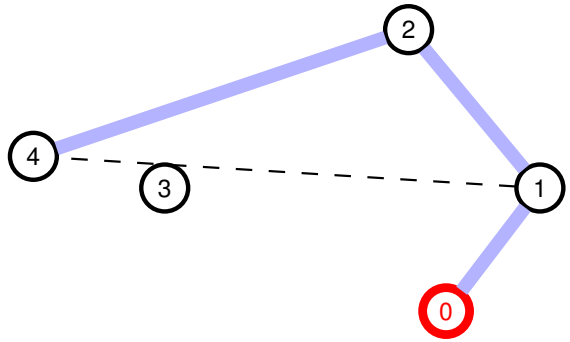
Graham's Scan

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1:      Let p_0 be the point with minimum y-coordinate
2:      Let (p_1, p_2, ..., p_n) be the other points sorted by polar angle w.r.t. p_0
3:      If n < 2 return false
4:      S = ∅
5:      PUSH(p_0,S)
6:      PUSH(p_1,S)
7:      PUSH(p_2,S)
8:      For i = 3 to n
9:          While angle of NEXT-TO-TOP(S),TOP(S),p_i makes a non-left turn
10:             POP(S)
11:         End While
12:         PUSH(p_i,S)
13:     End For
14:     Return S
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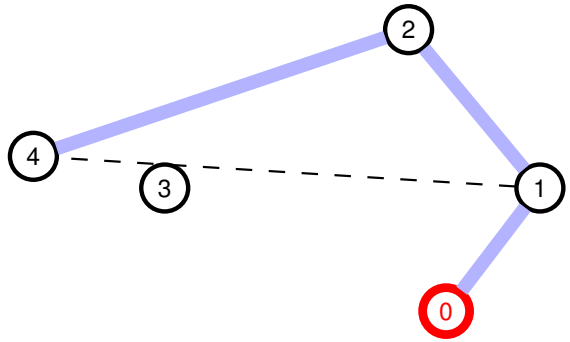
Takes $O(n \log n)$ time

Graham's Scan

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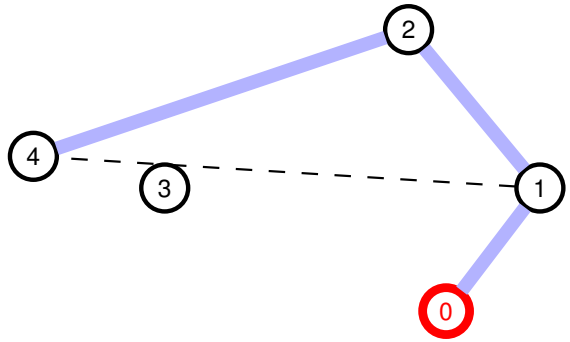


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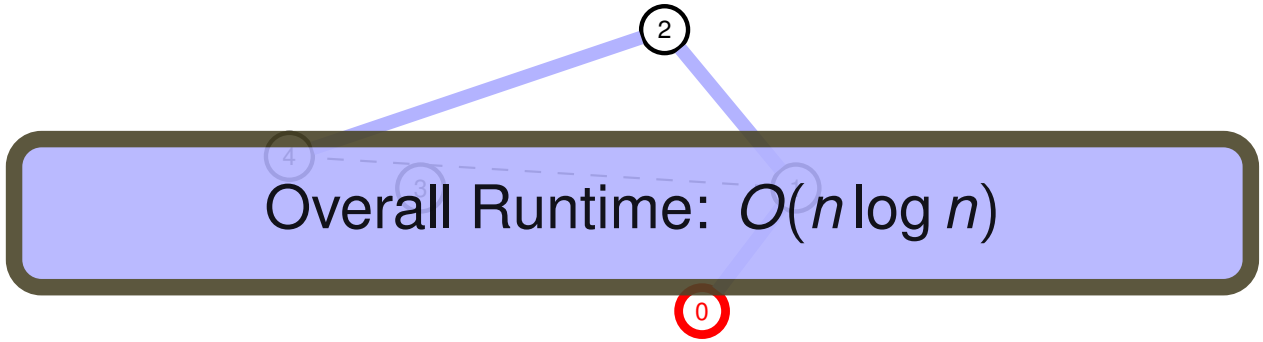


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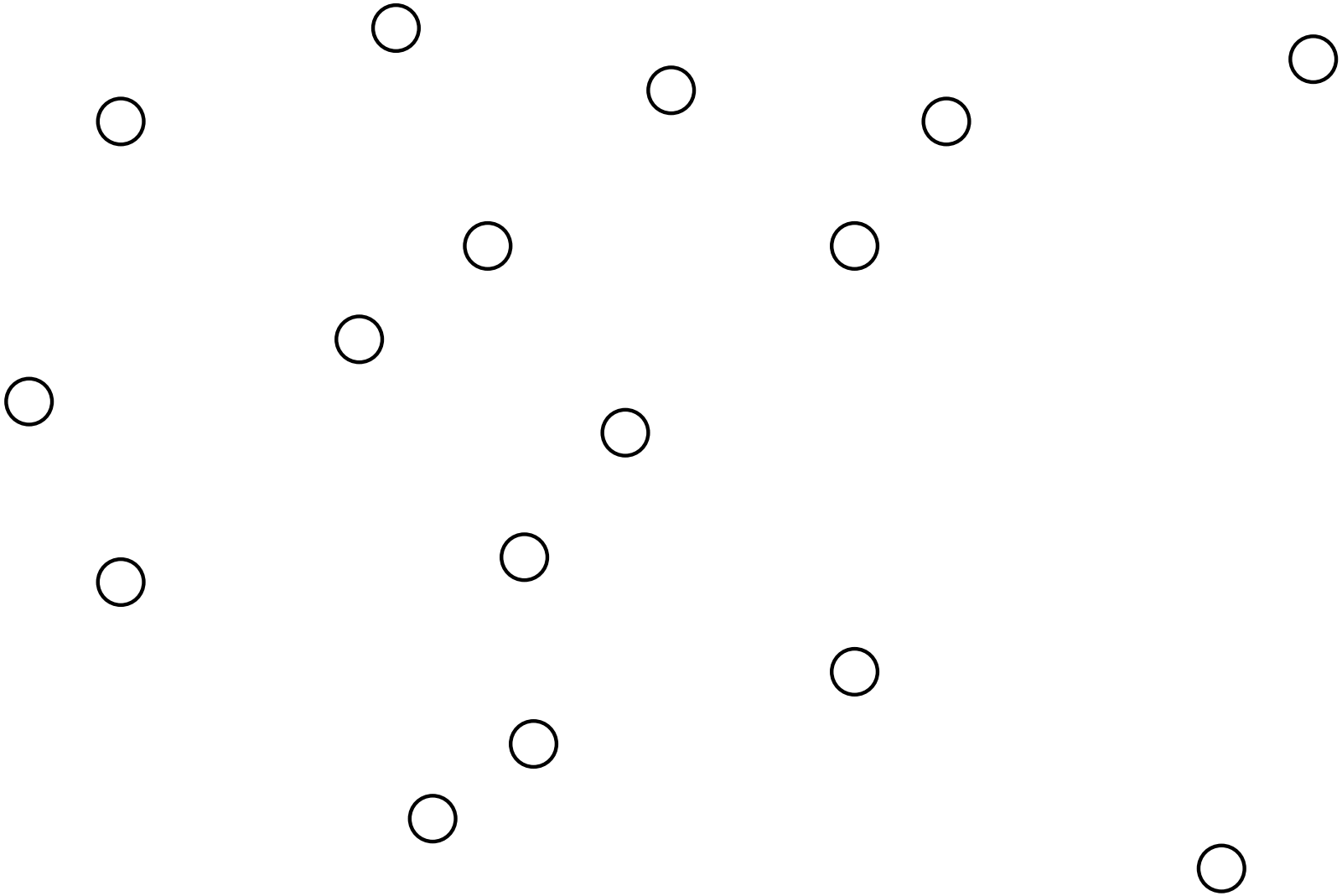

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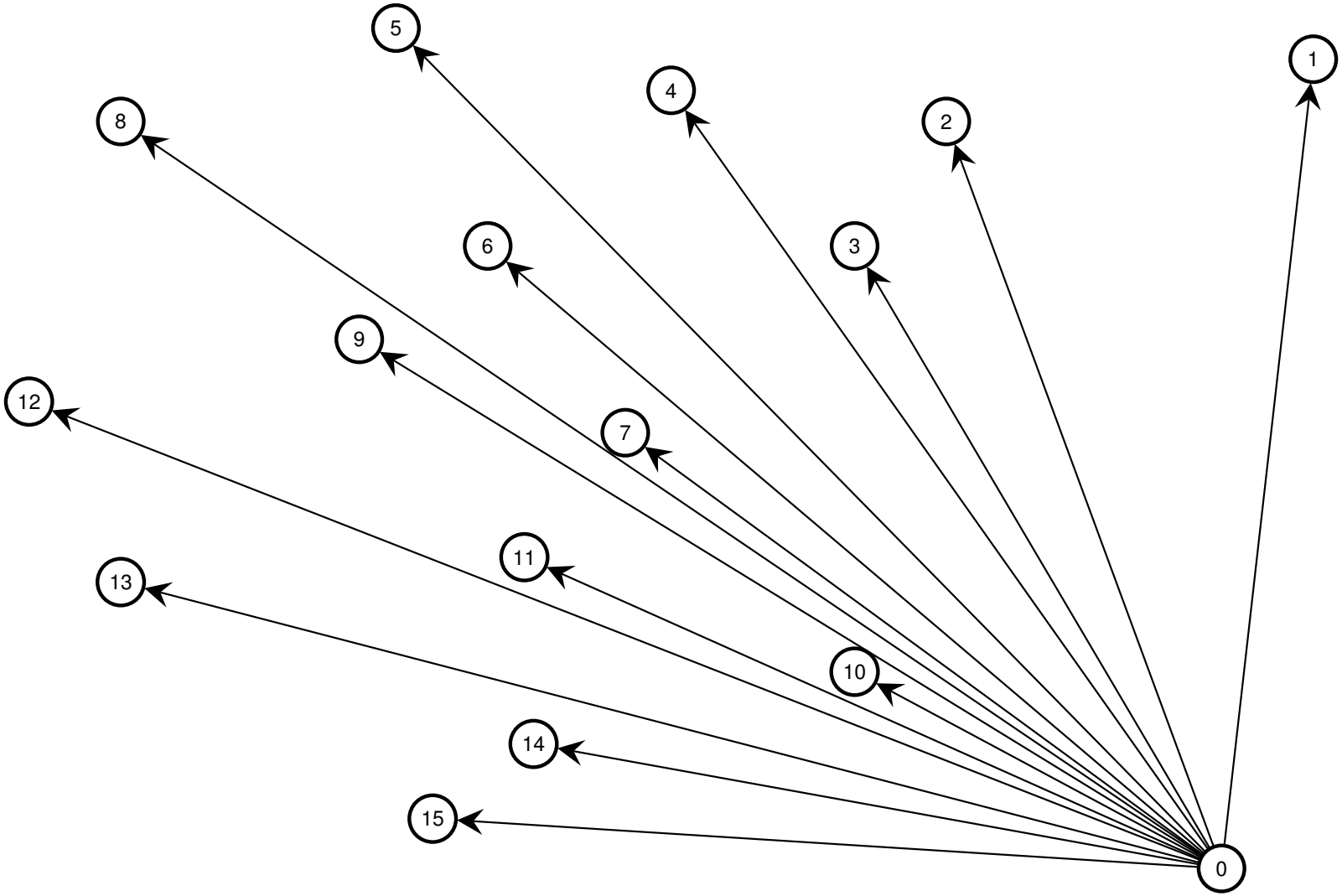

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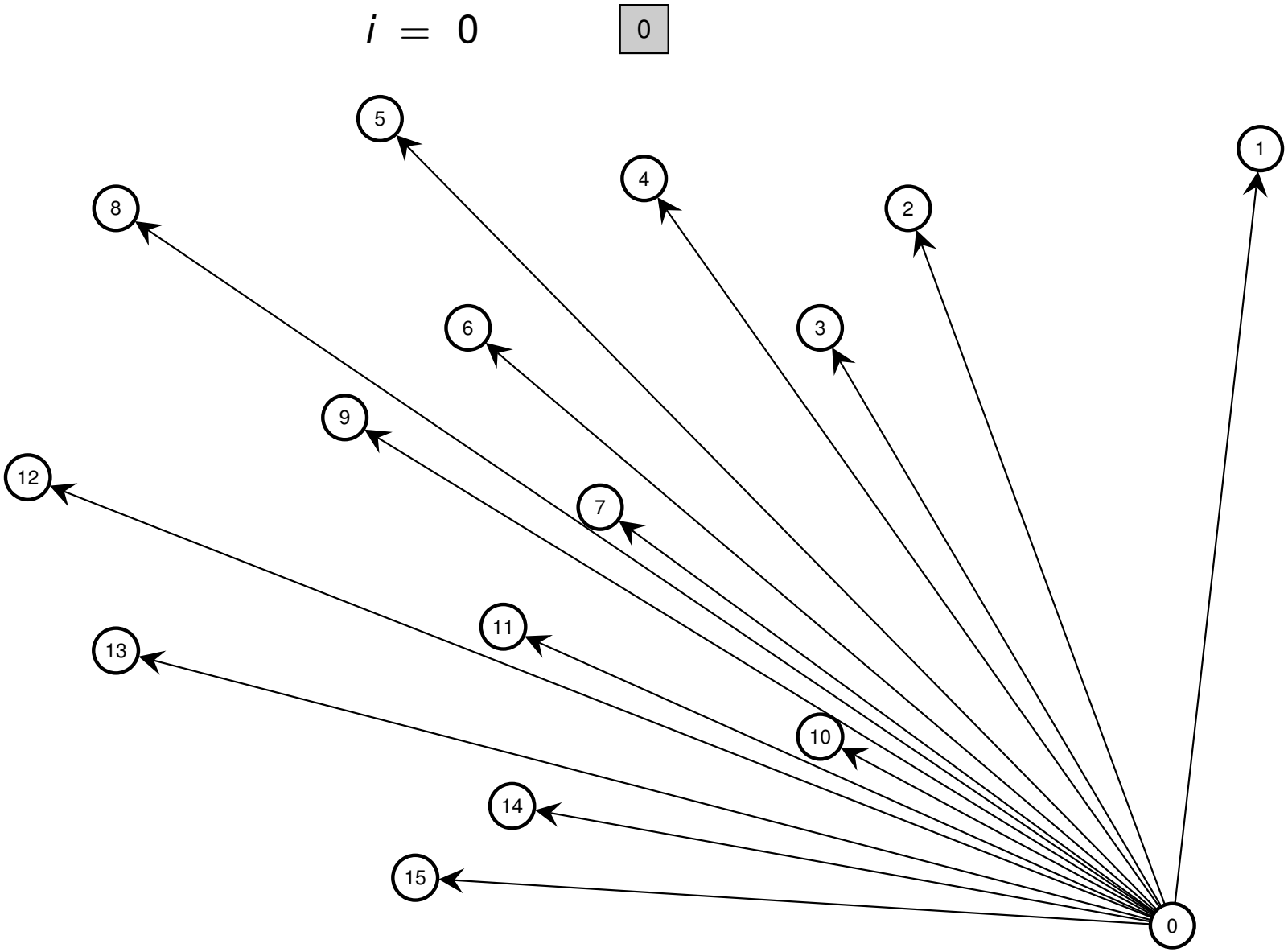

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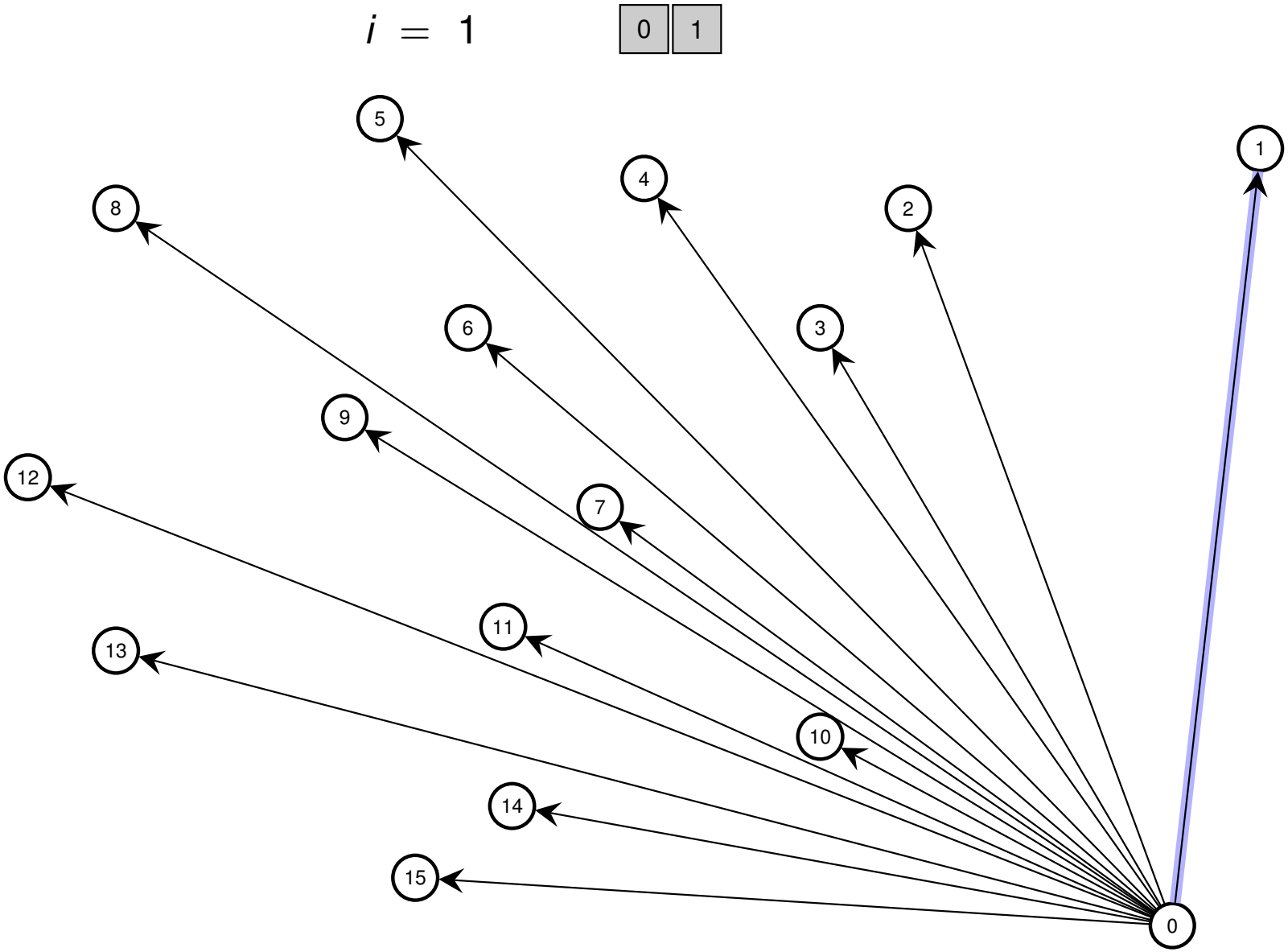

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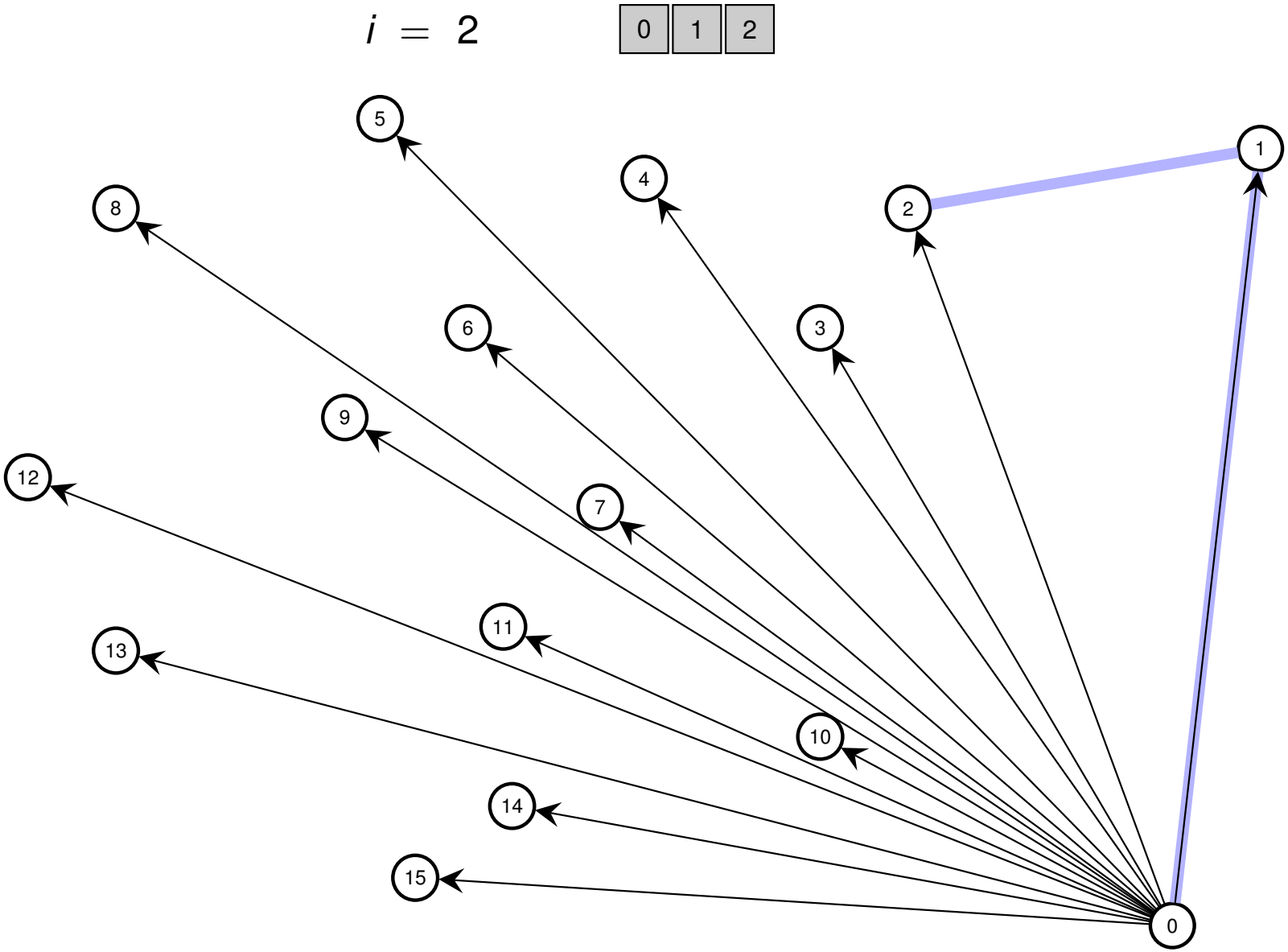
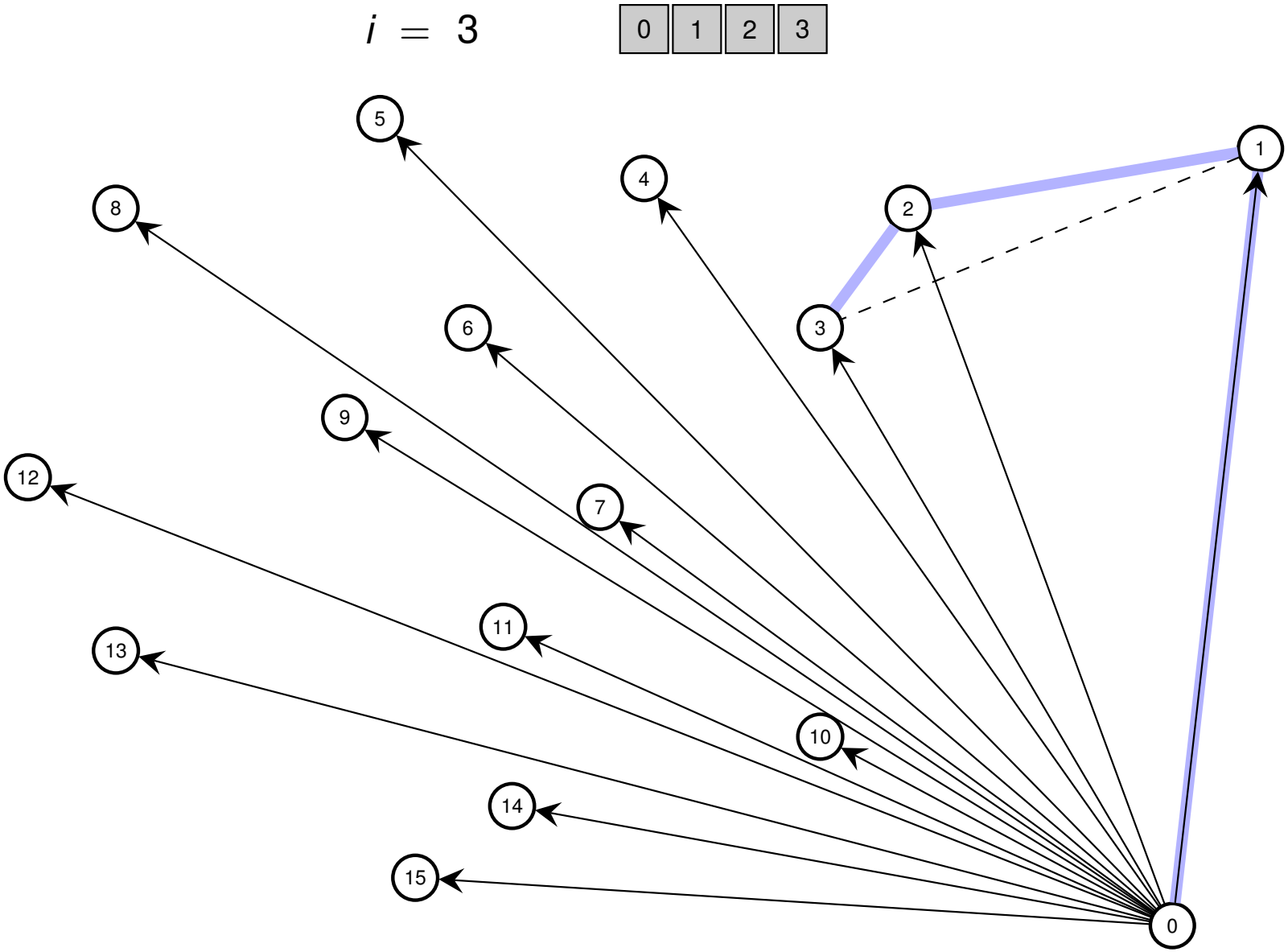
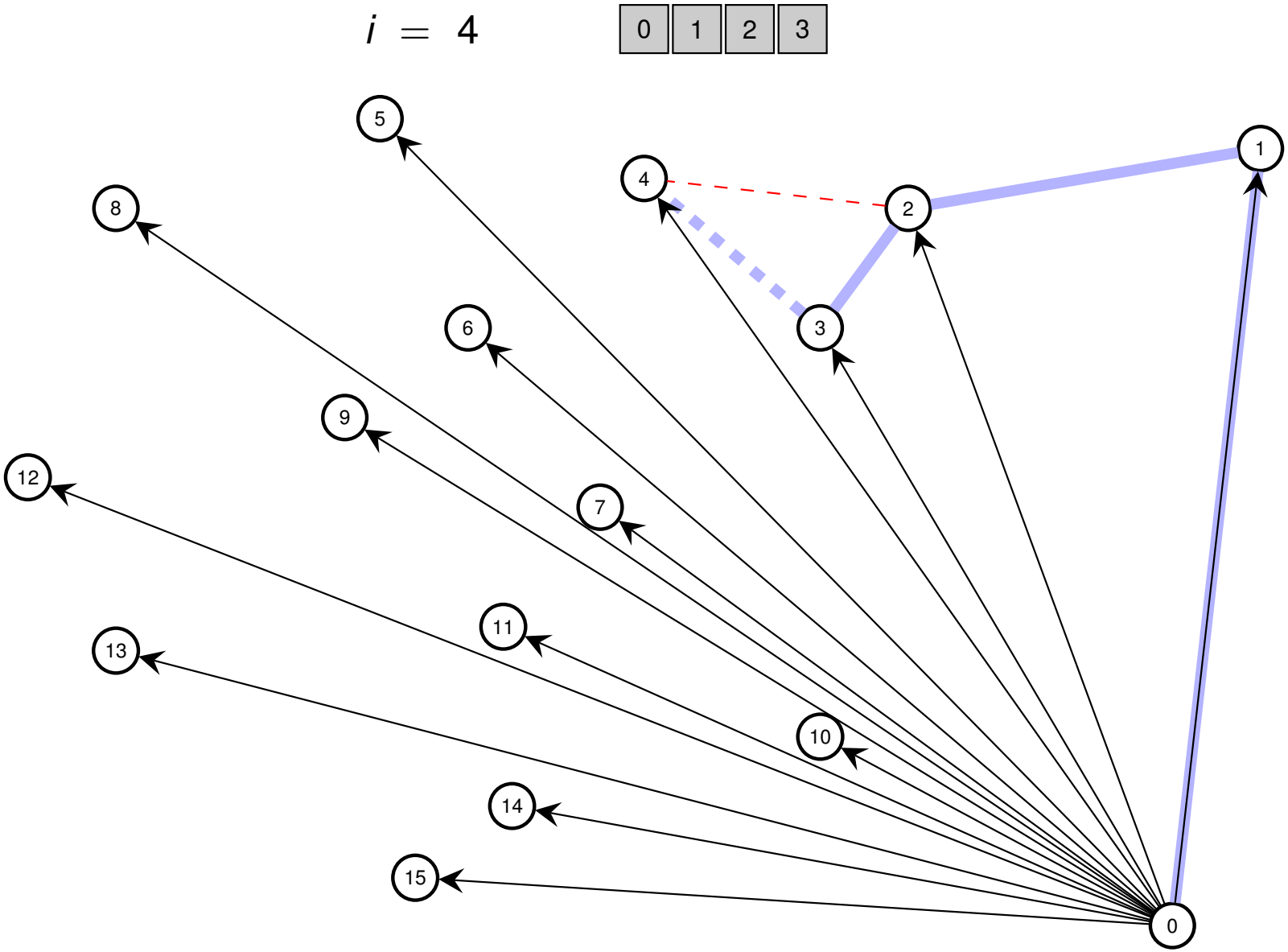


Complete Run of Graham's Scan



Complete Run of Graham's Scan



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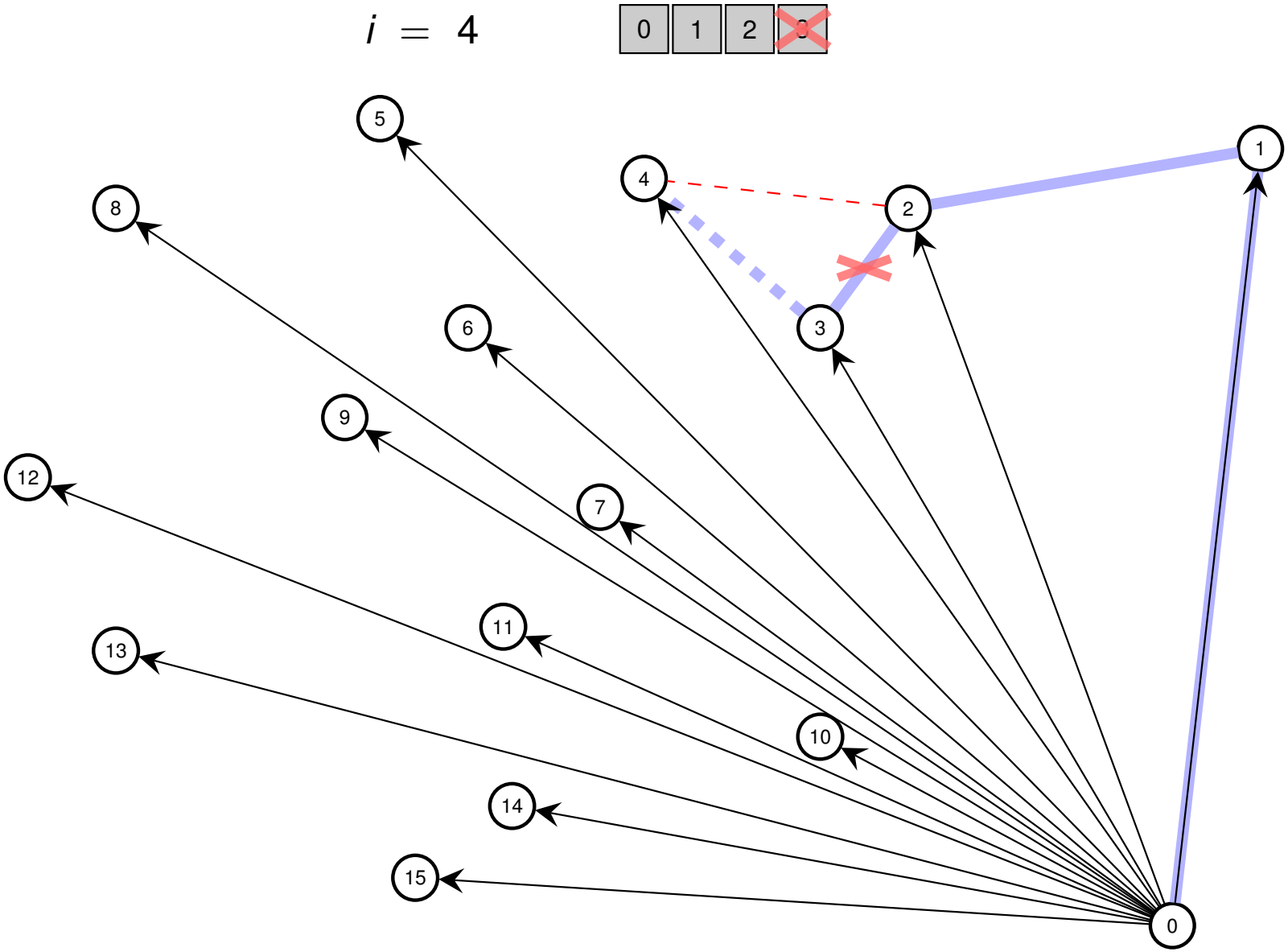
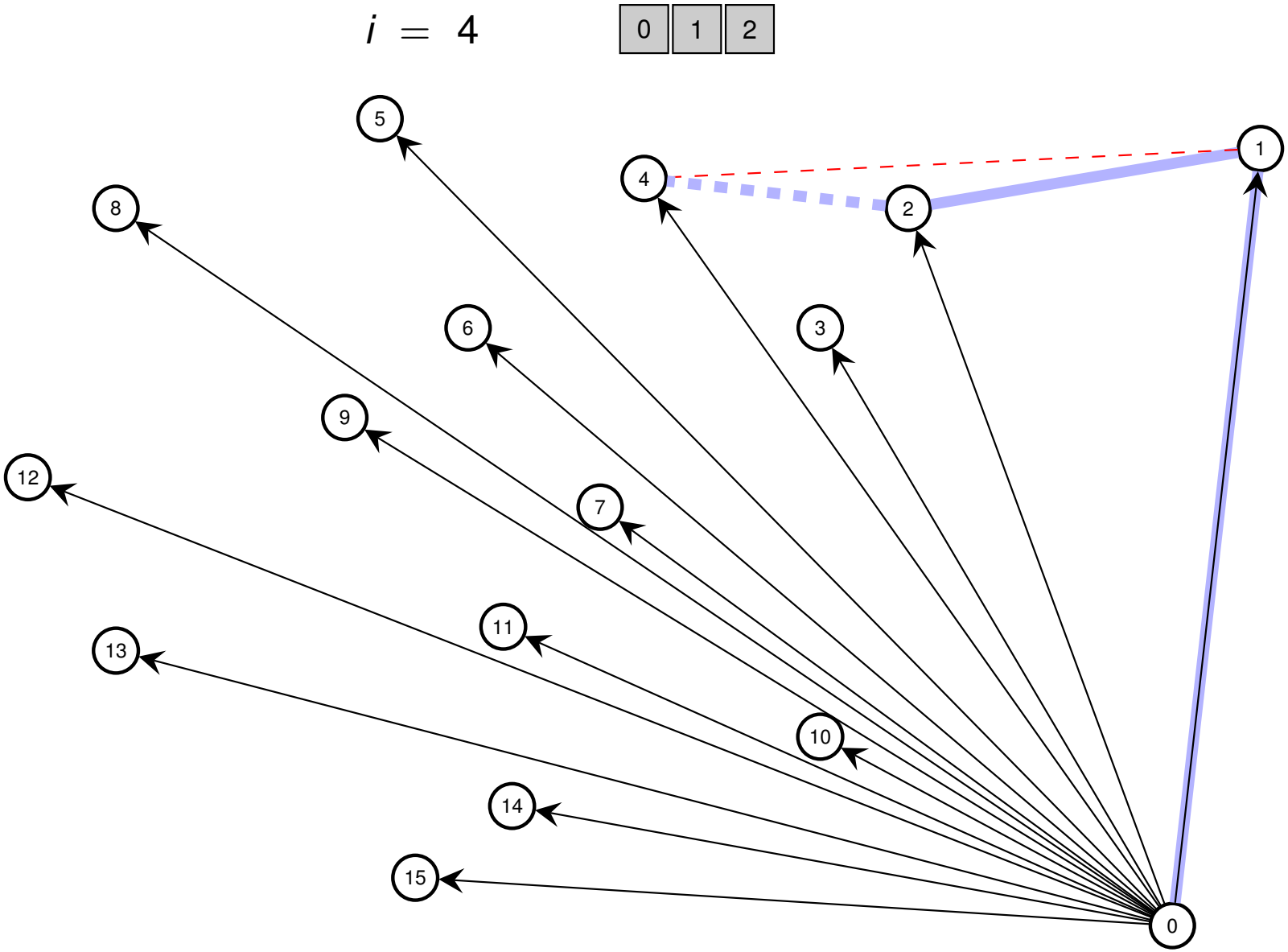
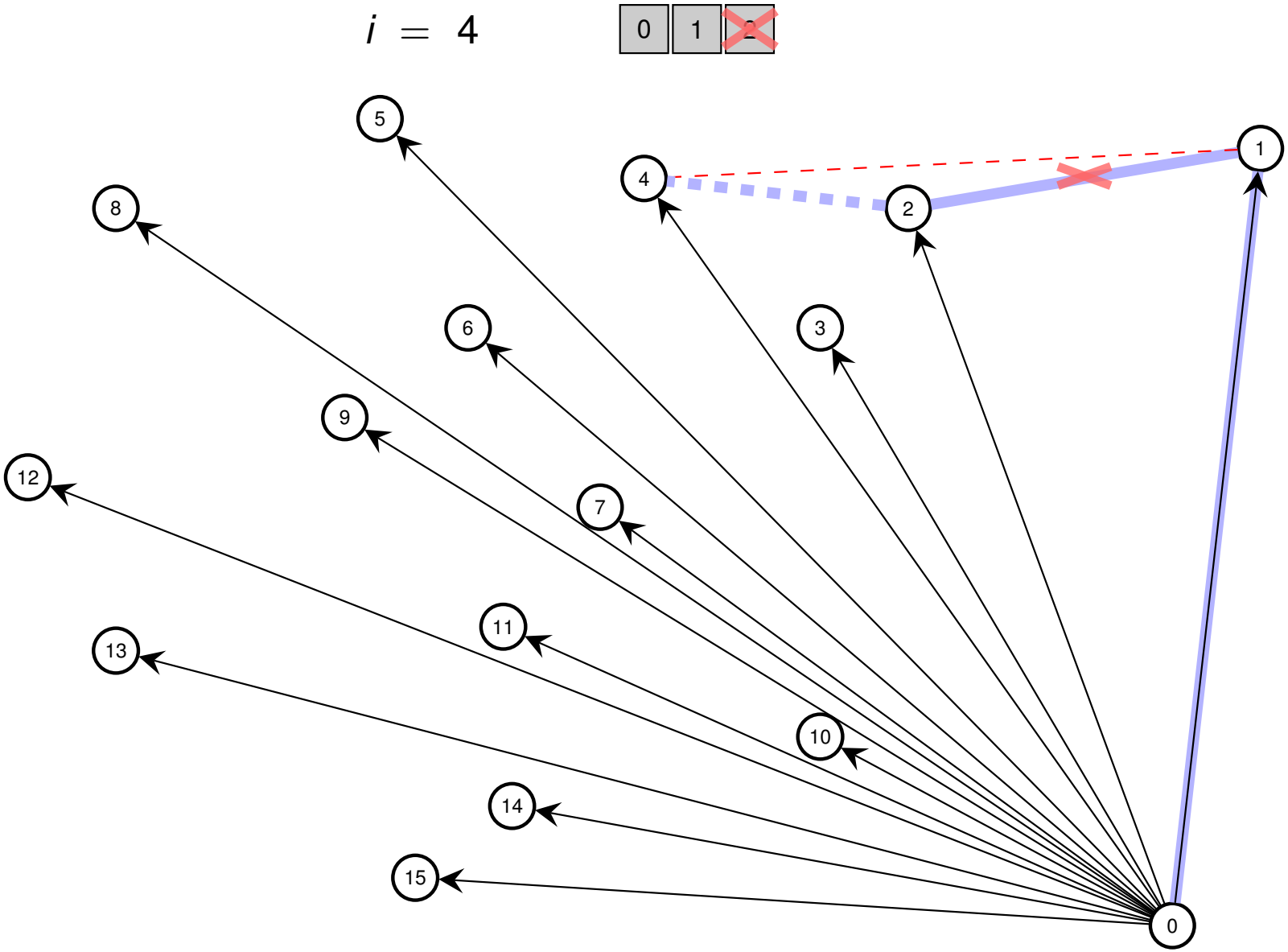


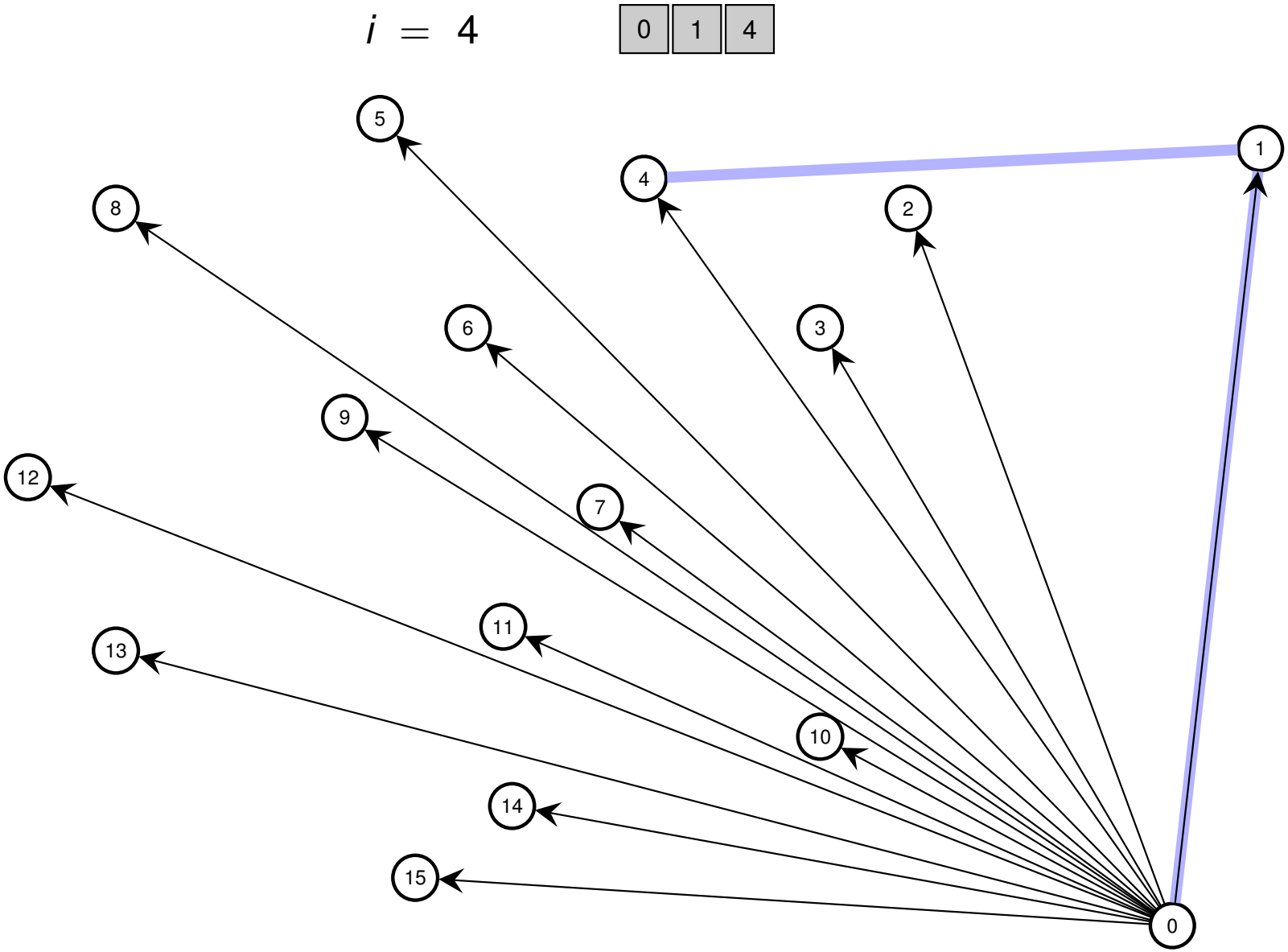
Complete Run of Graham's Scan



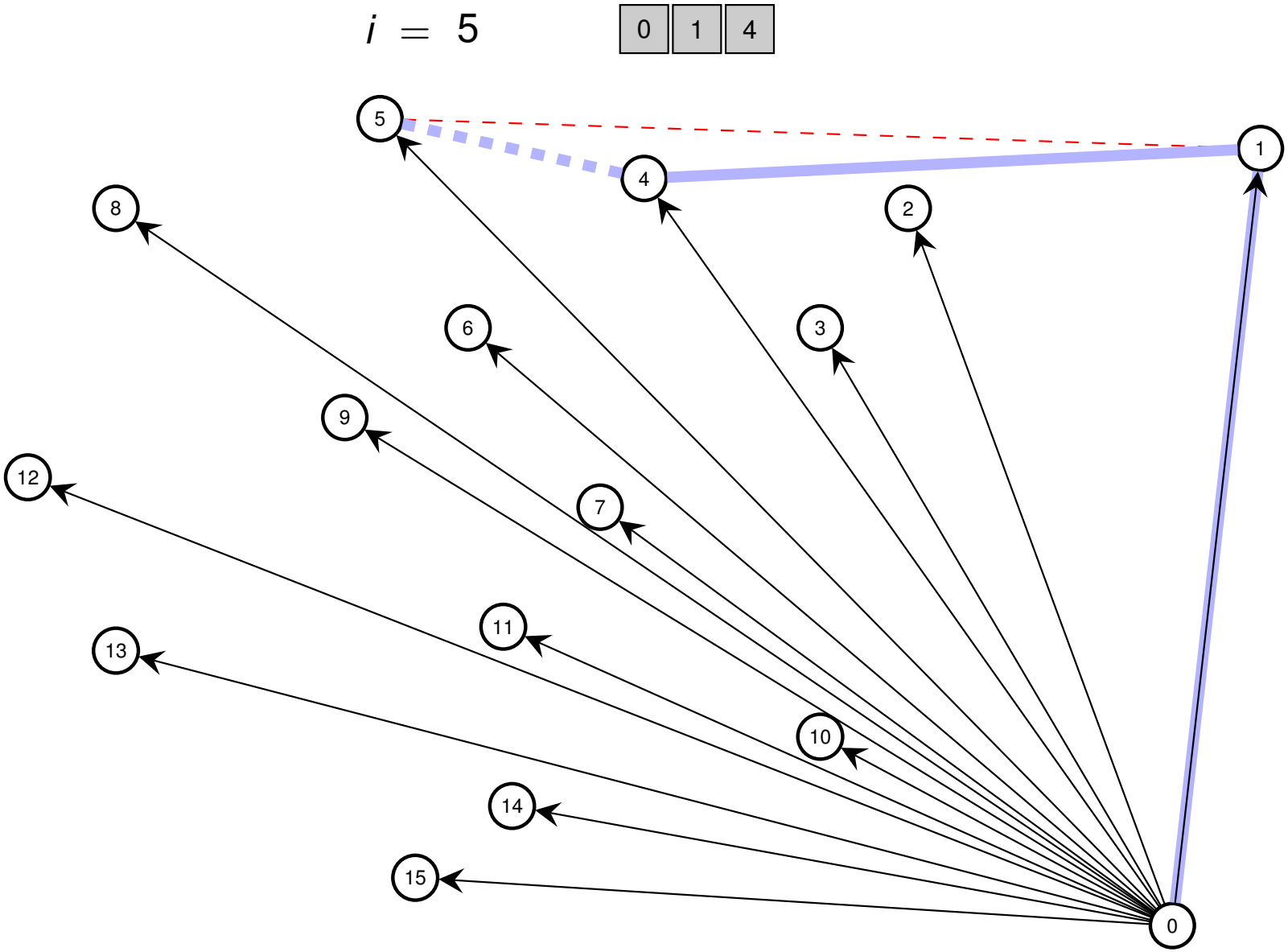
Complete Run of Graham's Scan



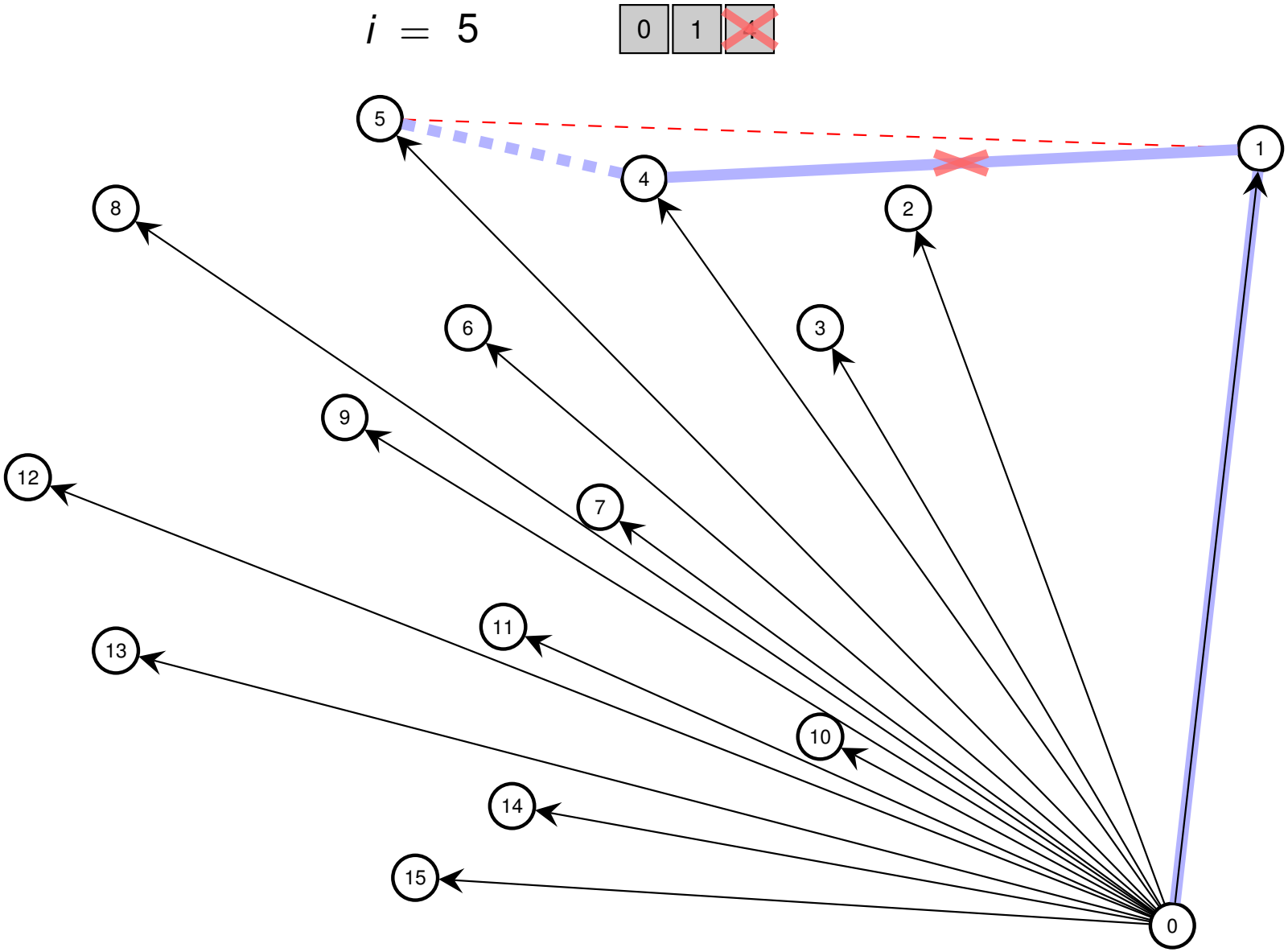
Complete Run of Graham's Scan



Complete Run of Graham's Scan



Complete Run of Graham's Scan



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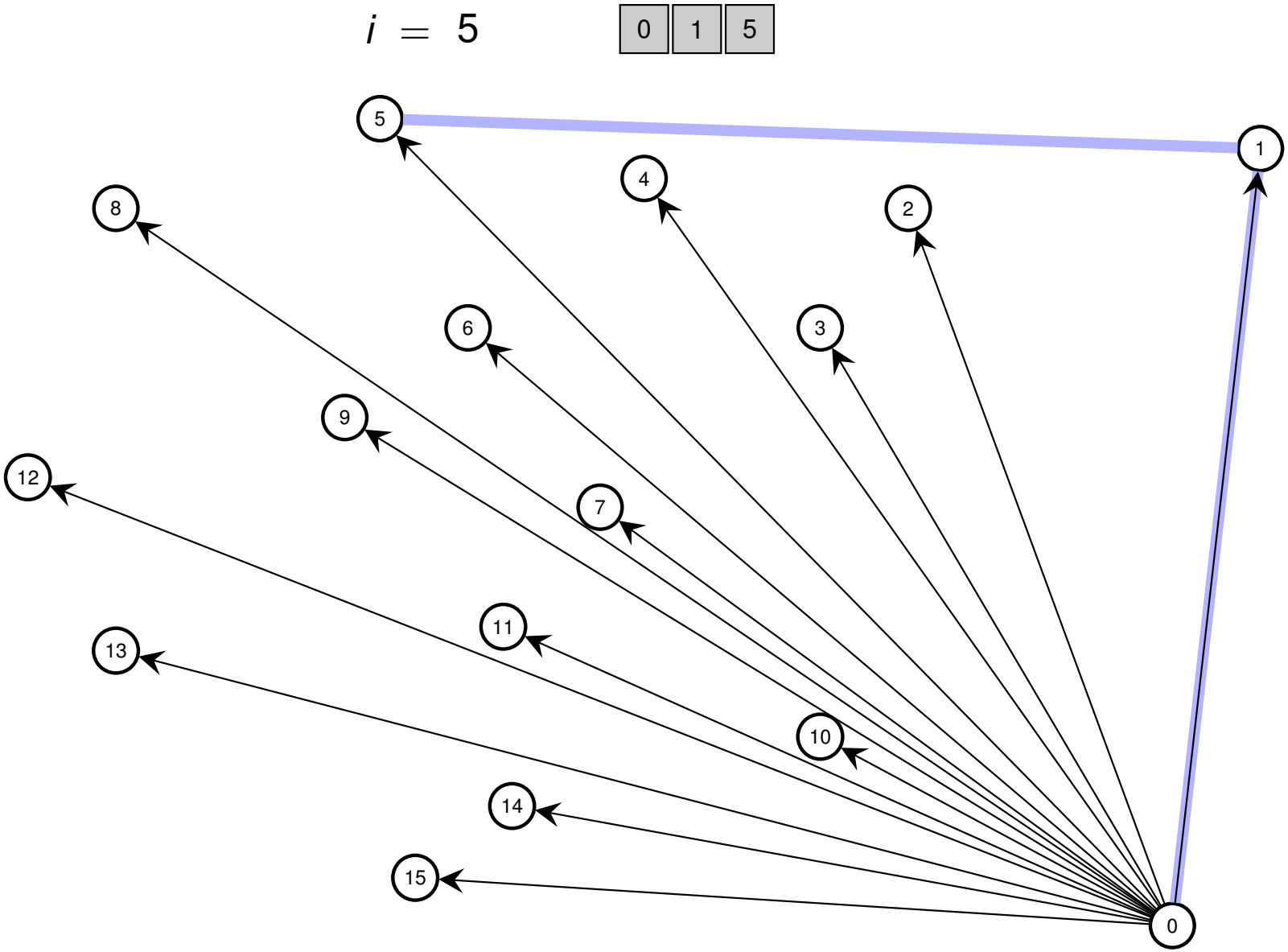

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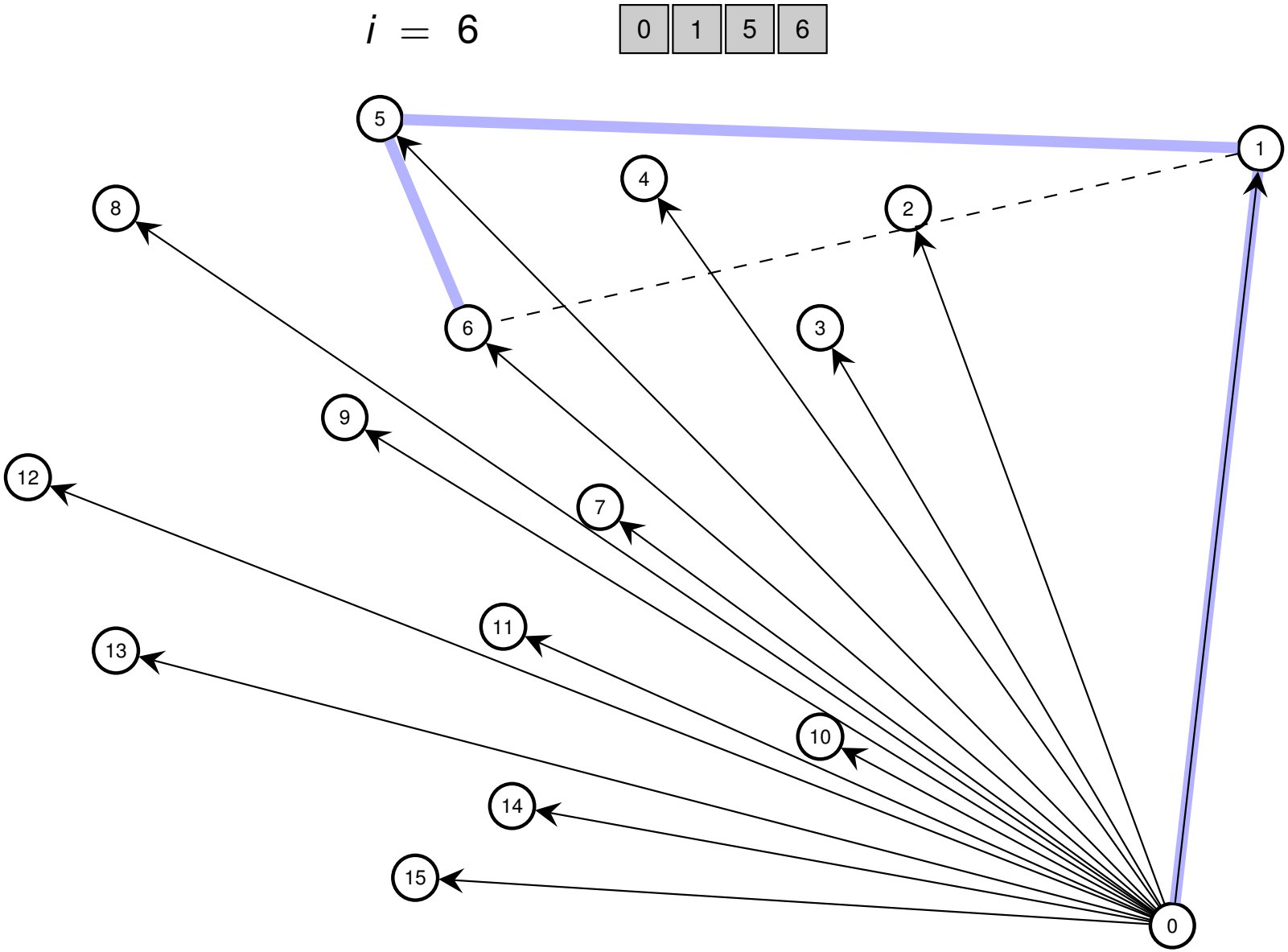

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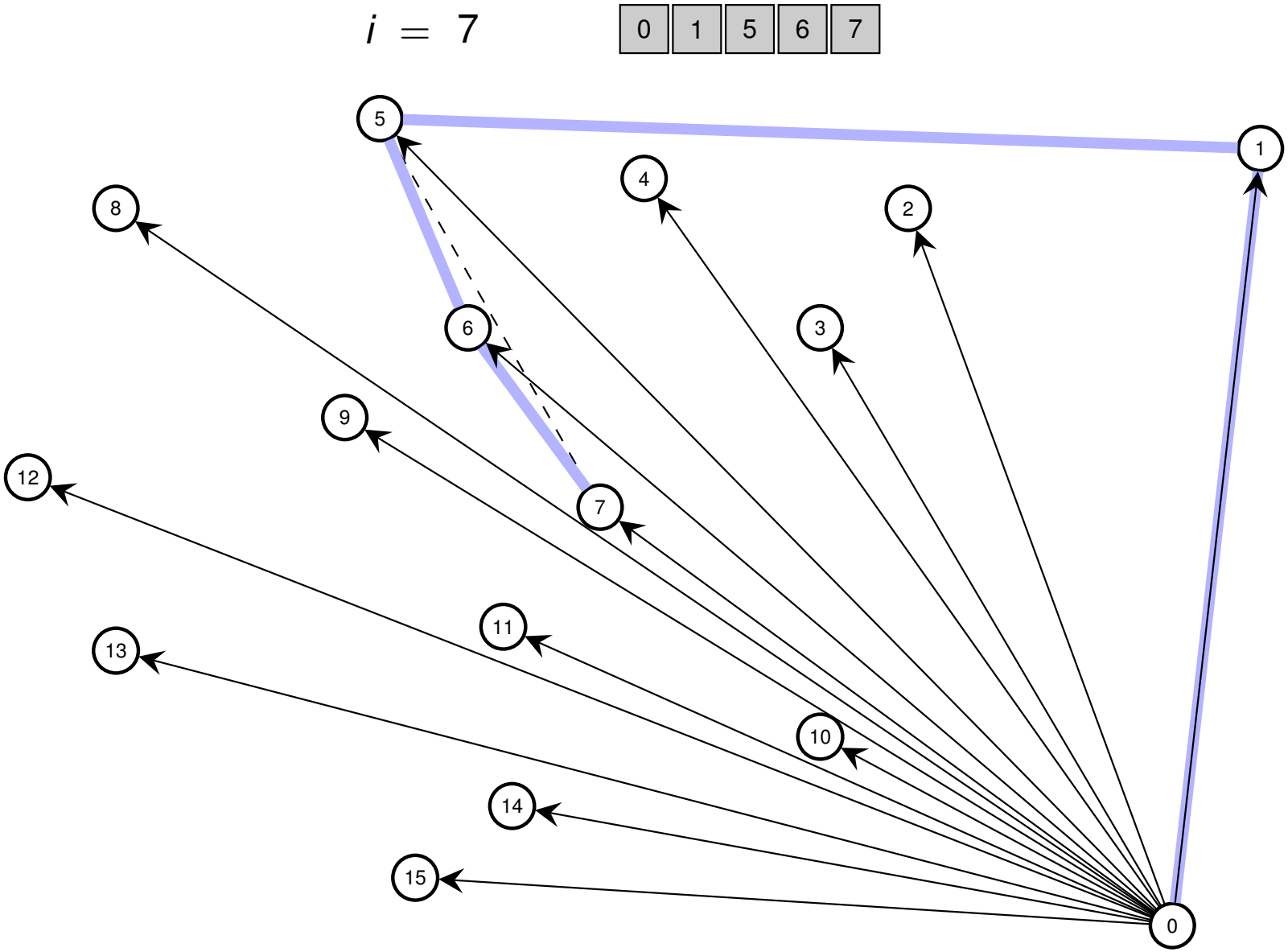

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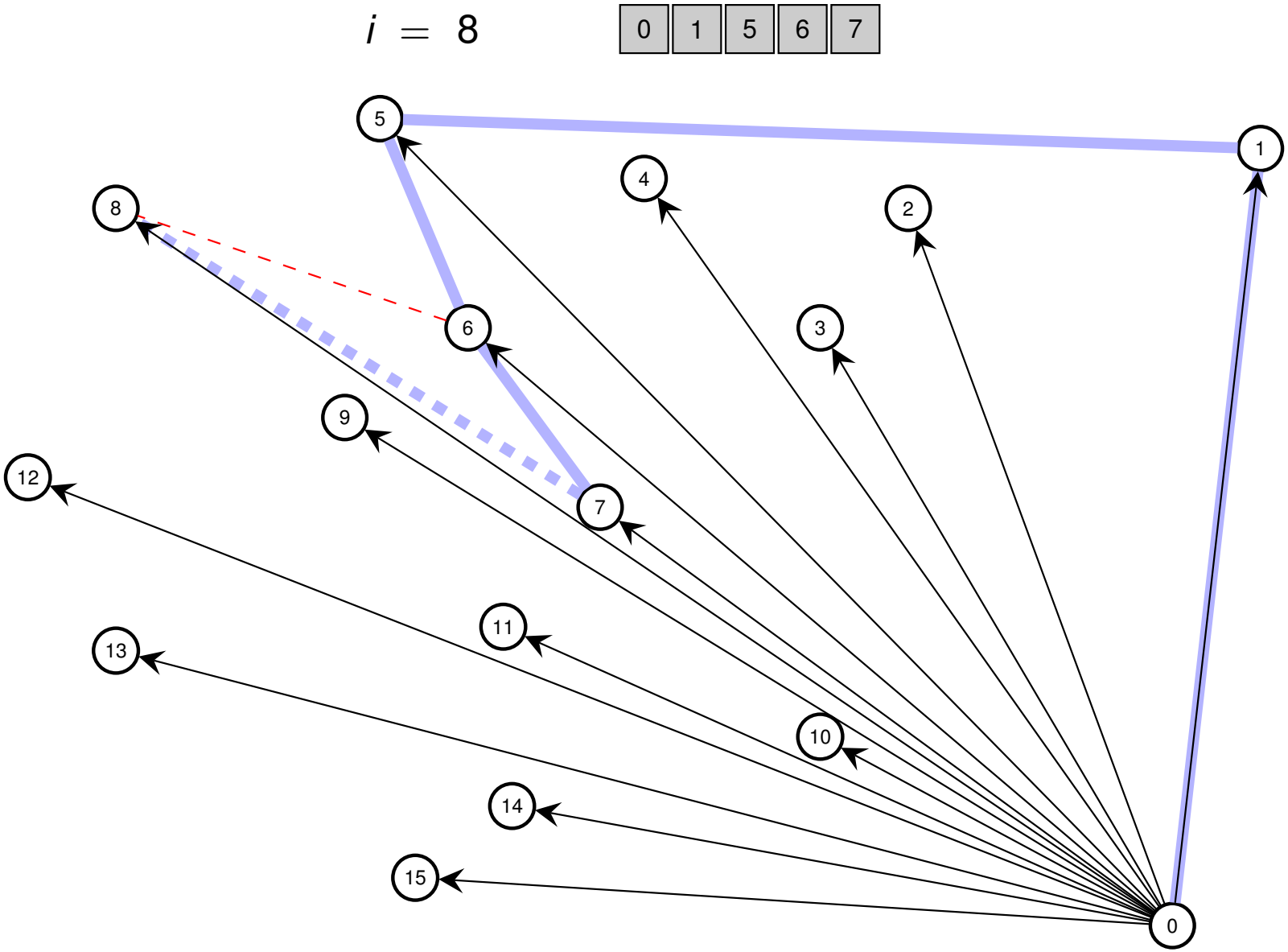

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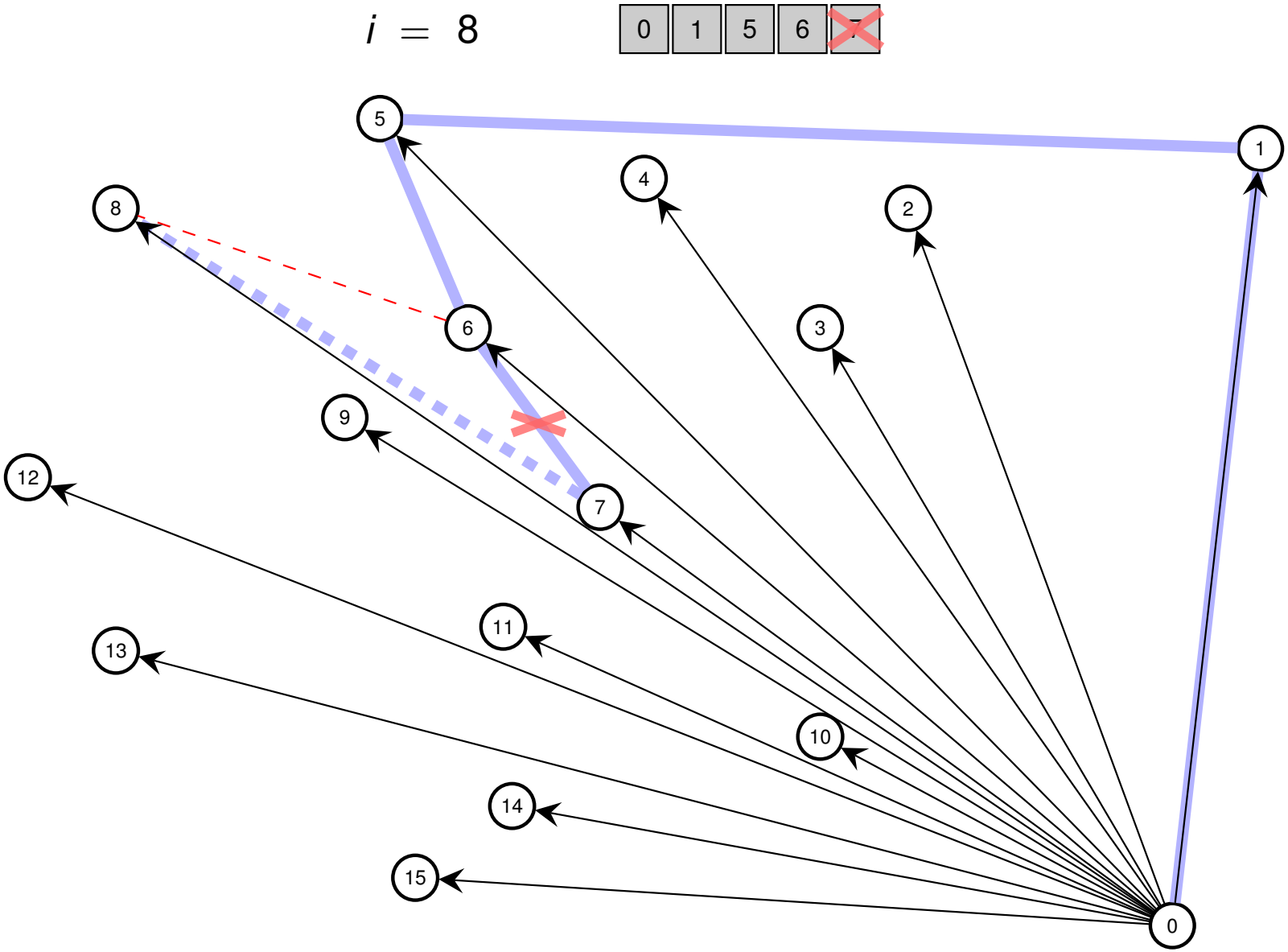

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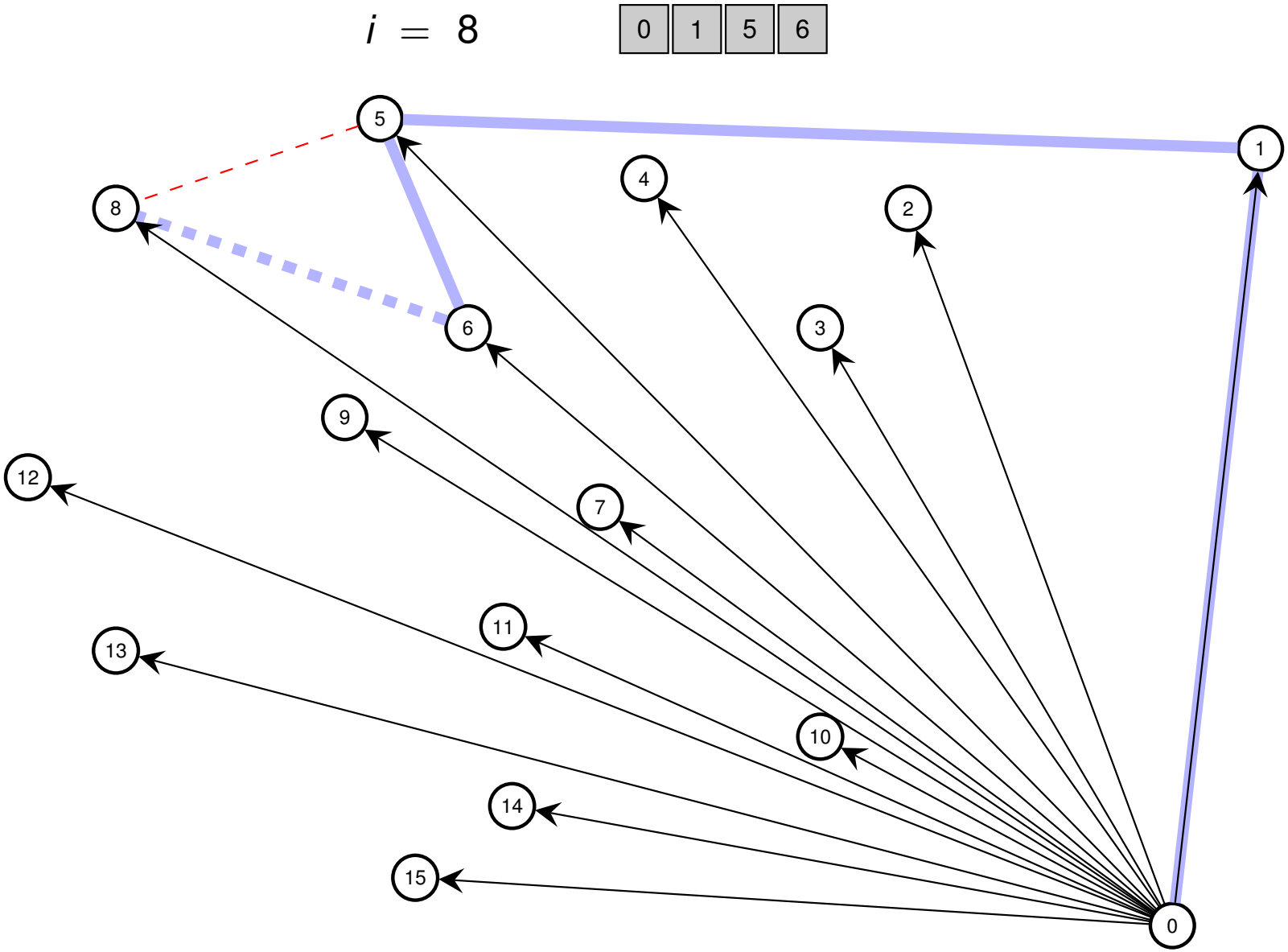

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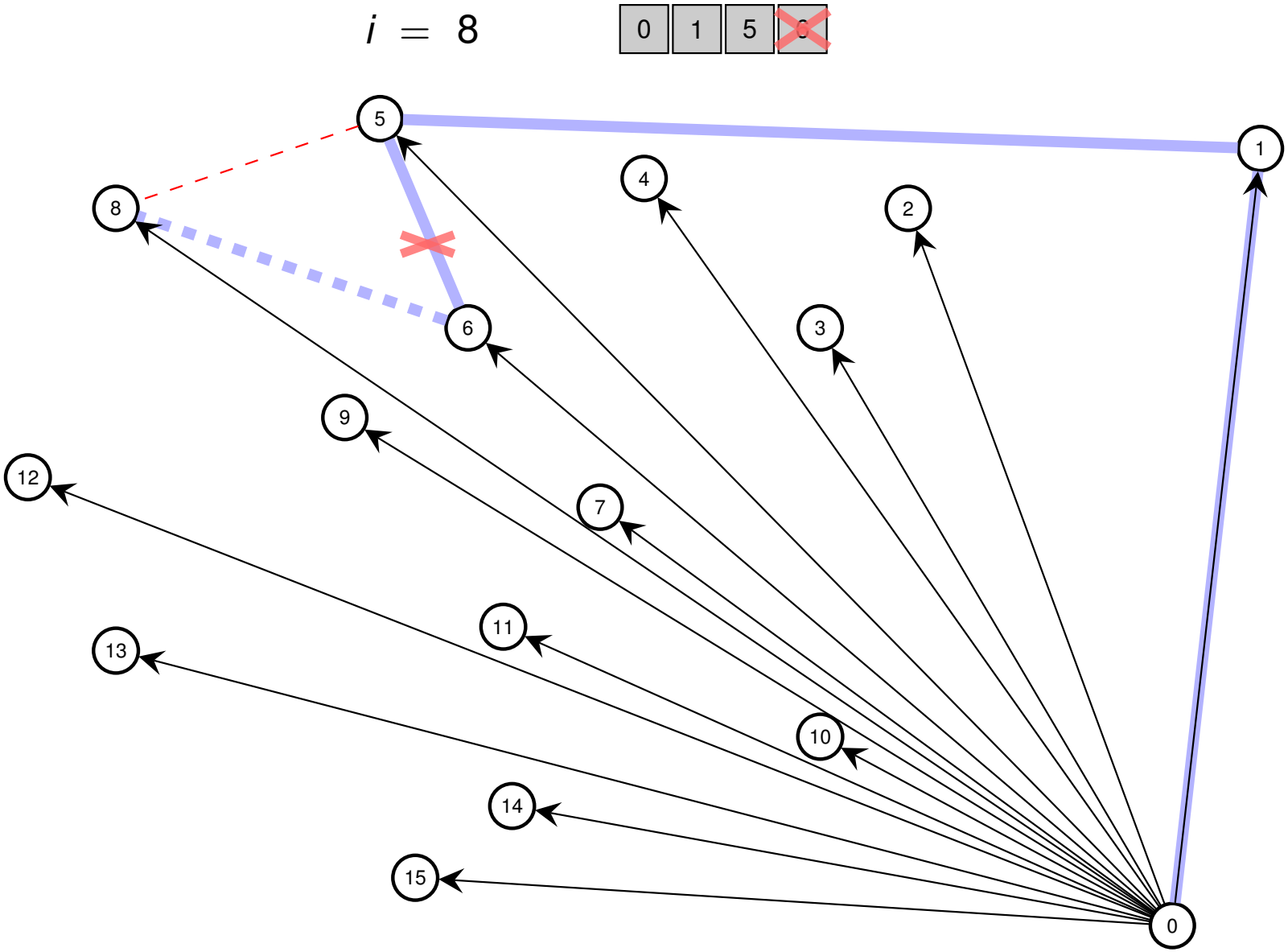

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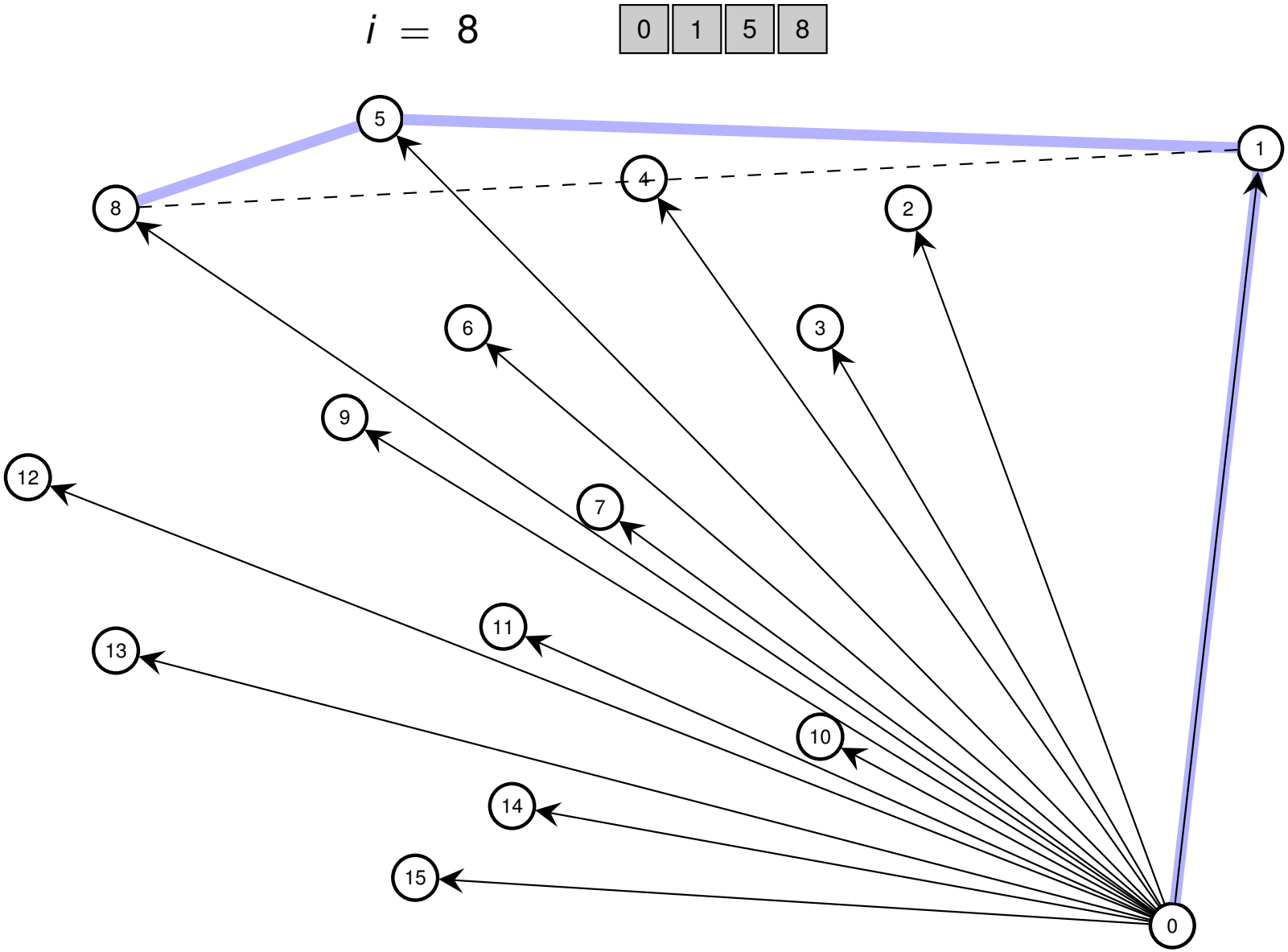

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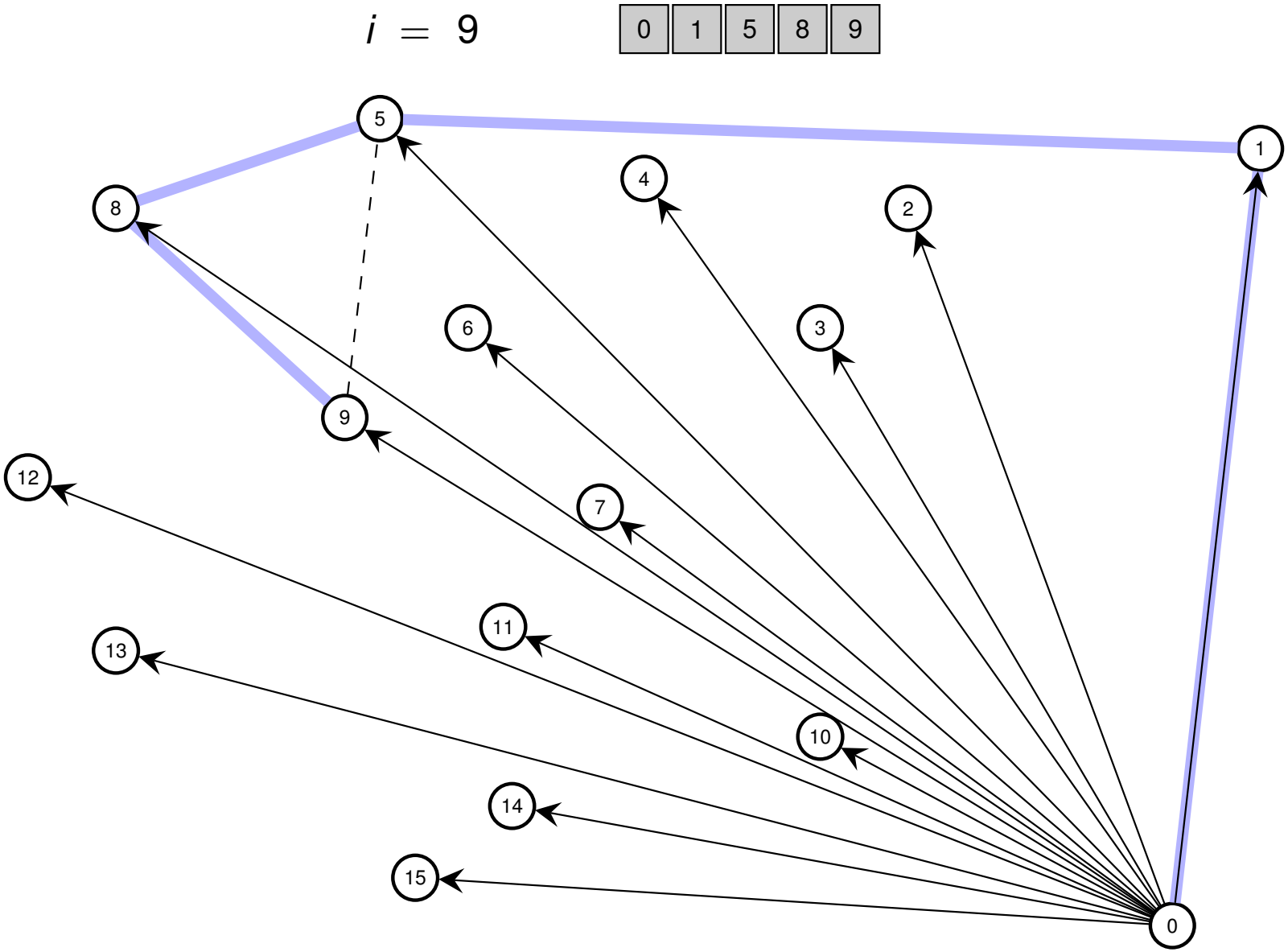

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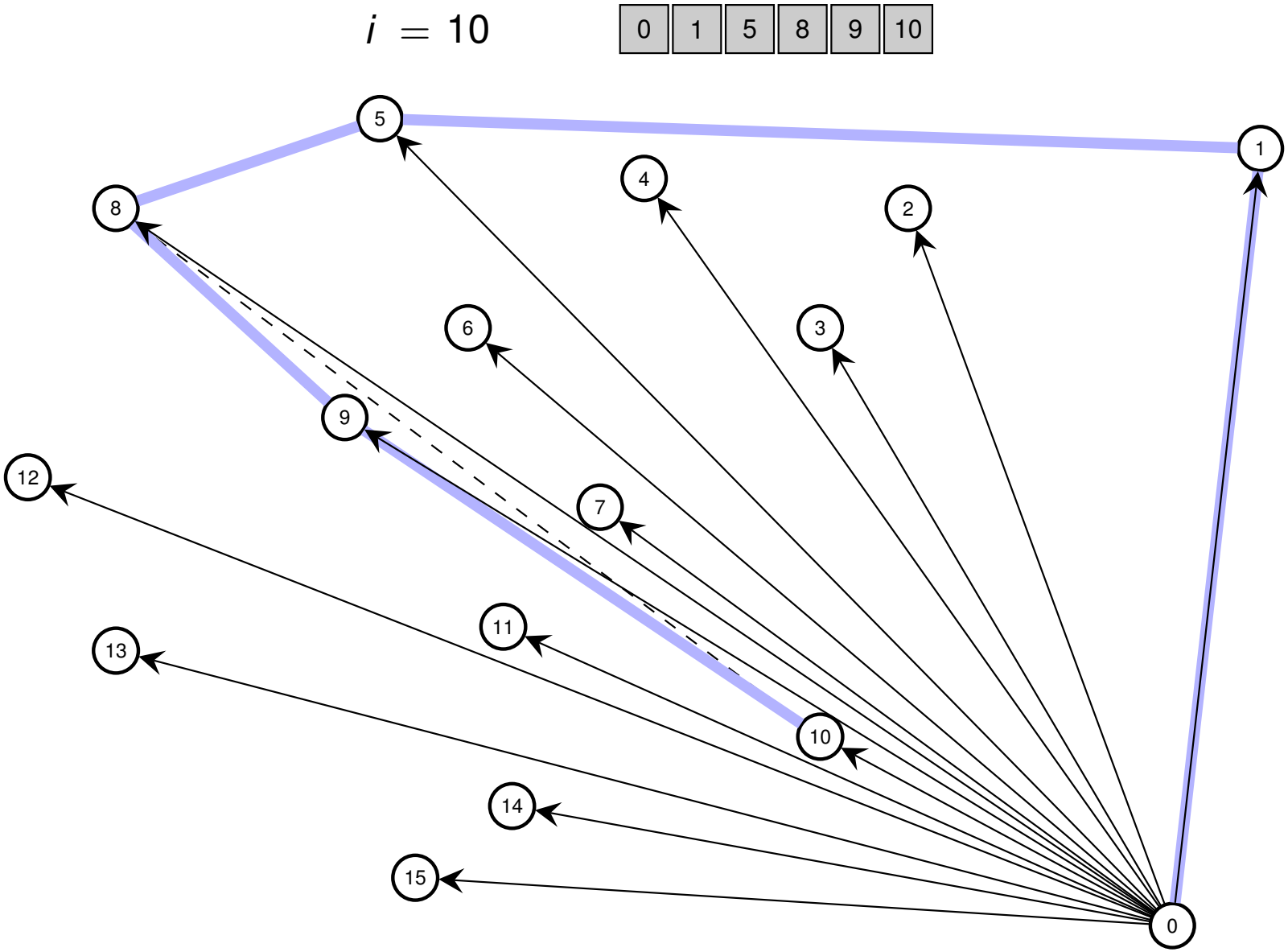

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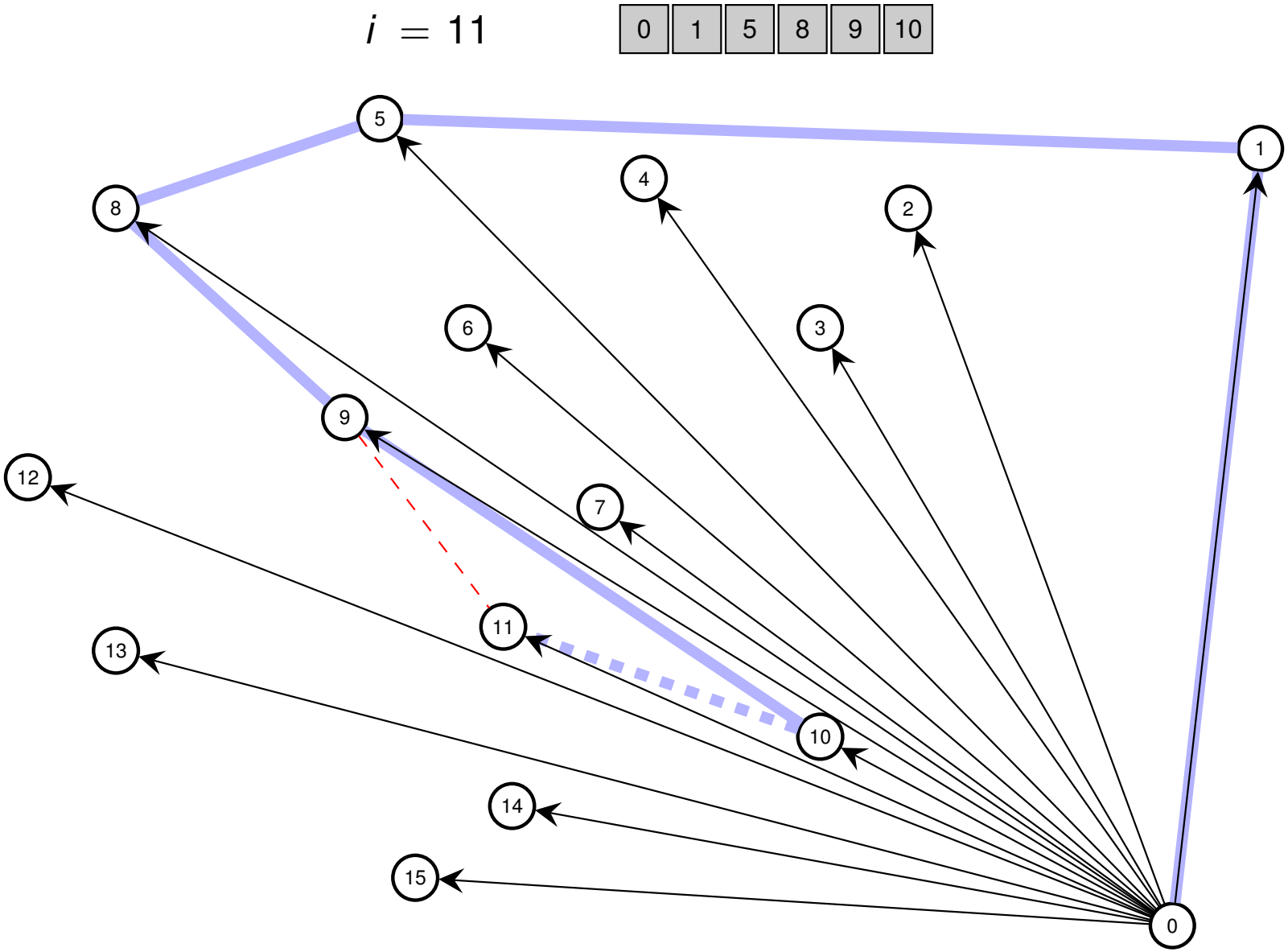

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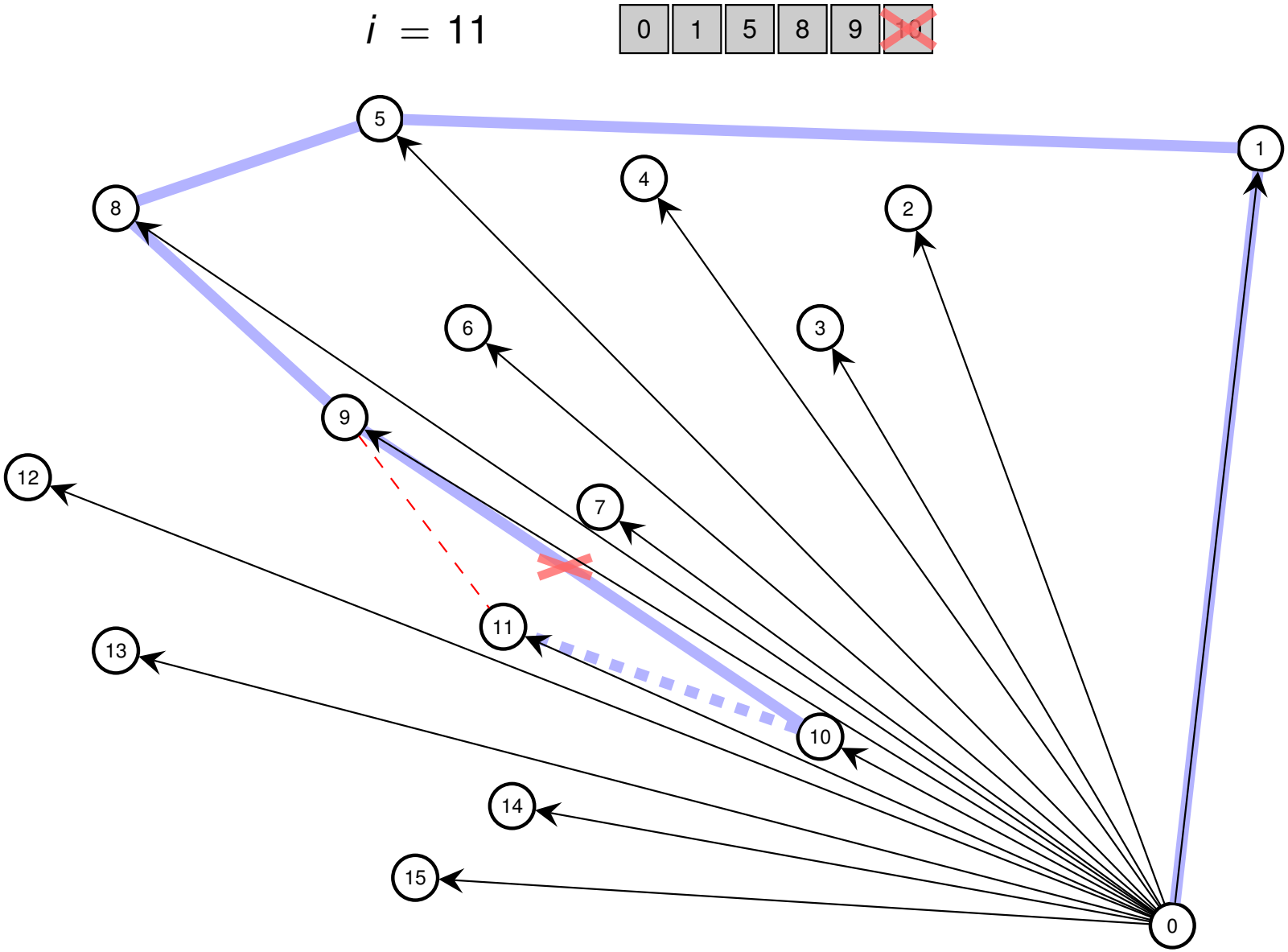

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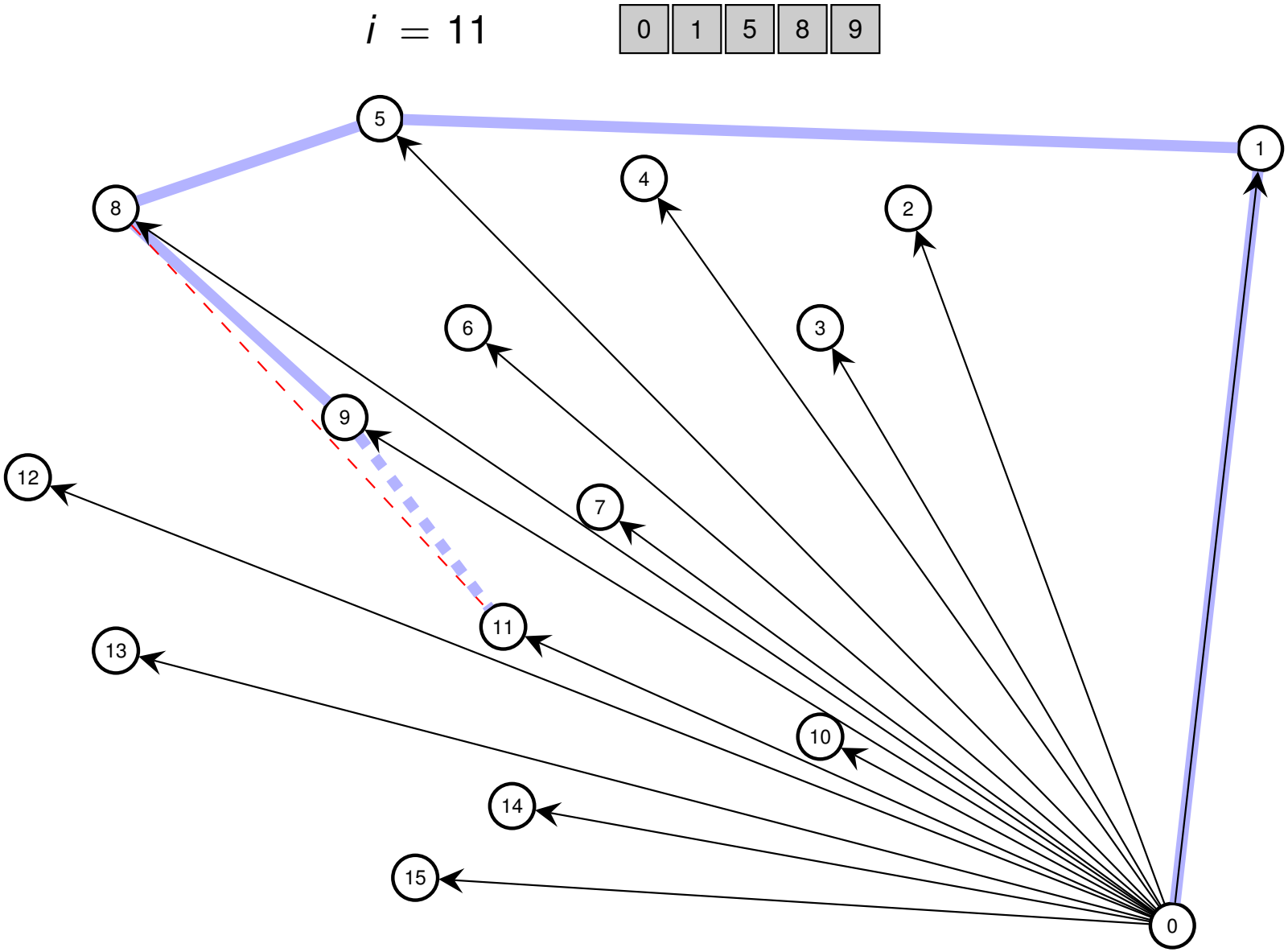

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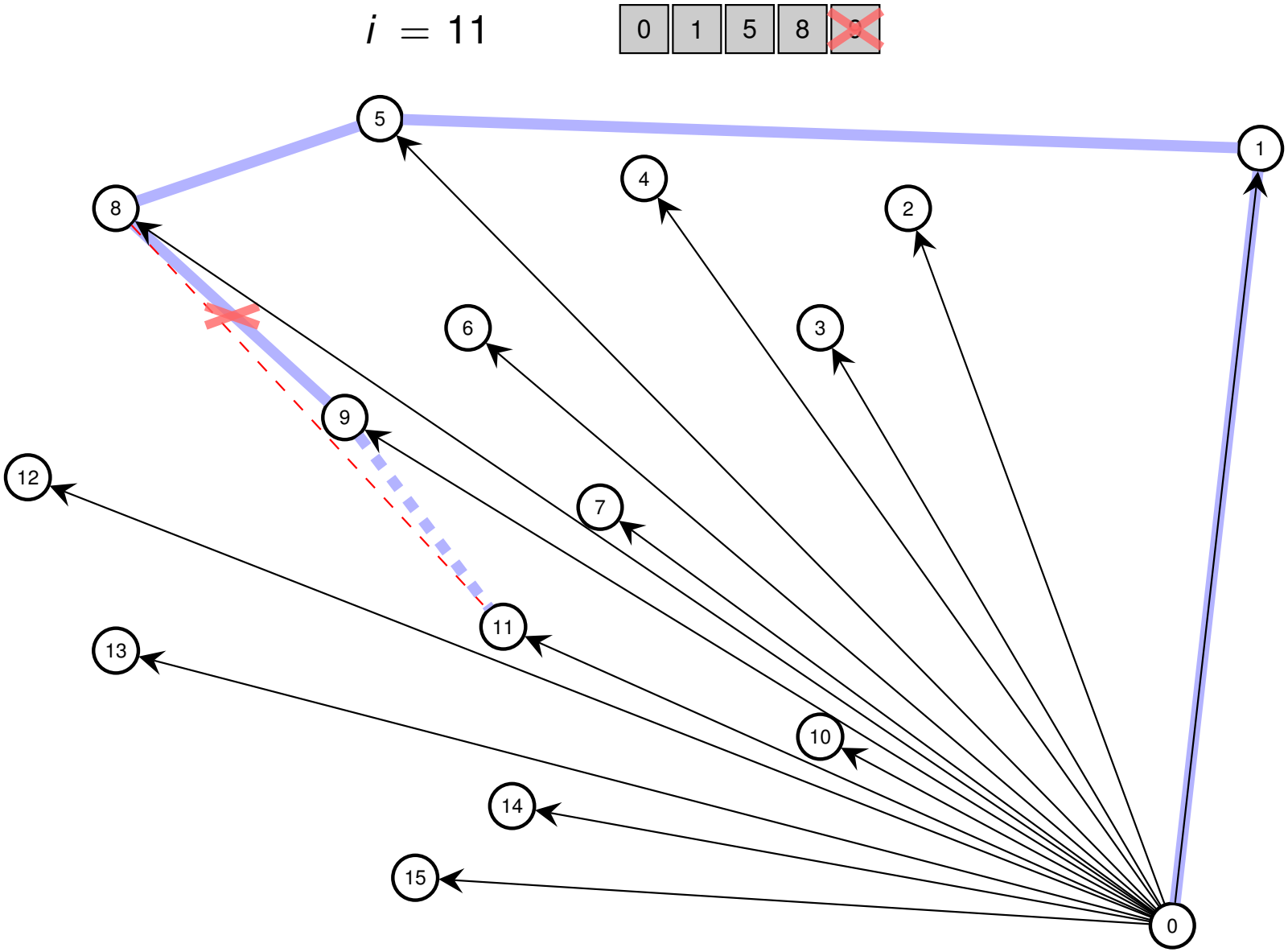

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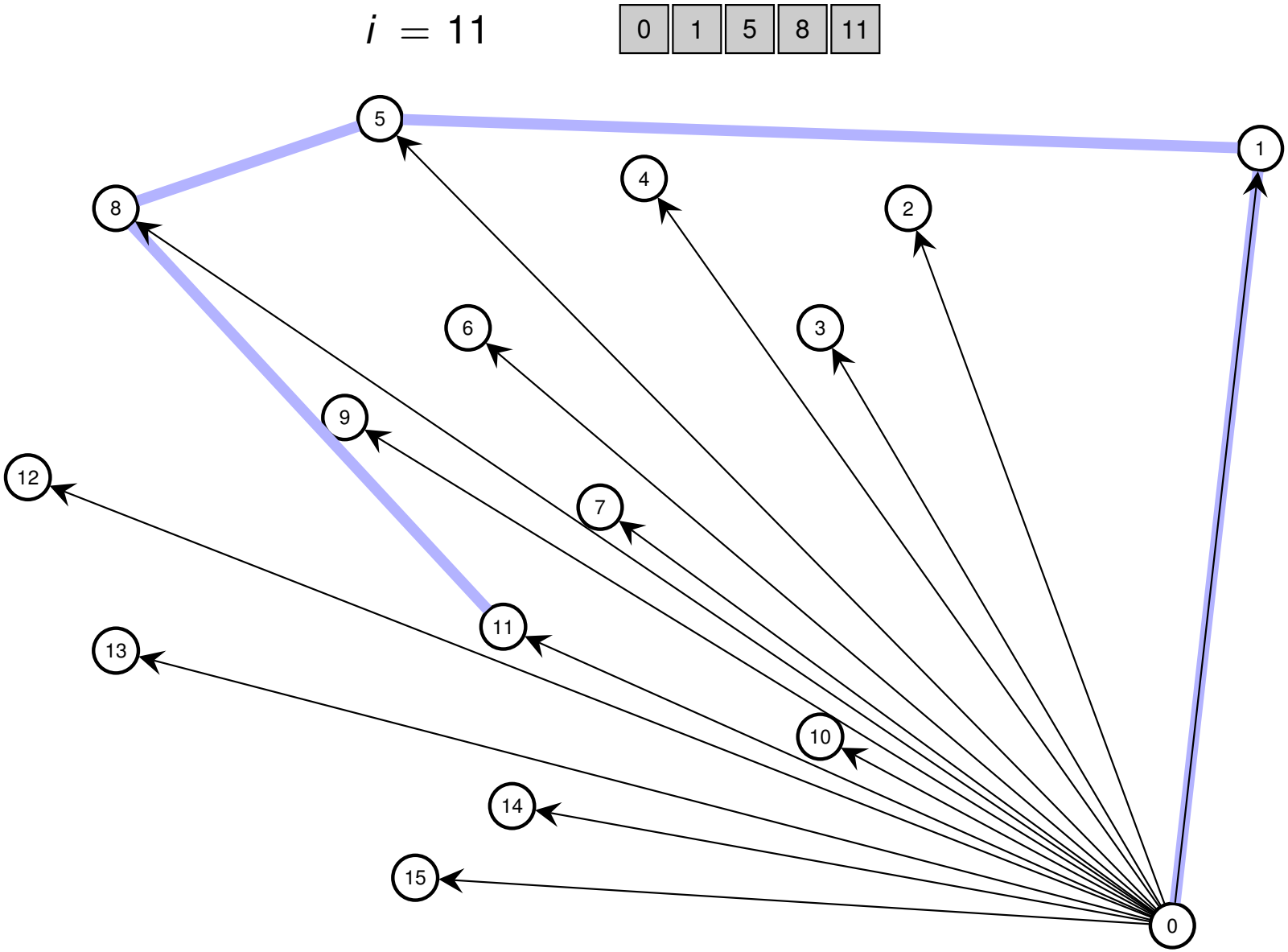

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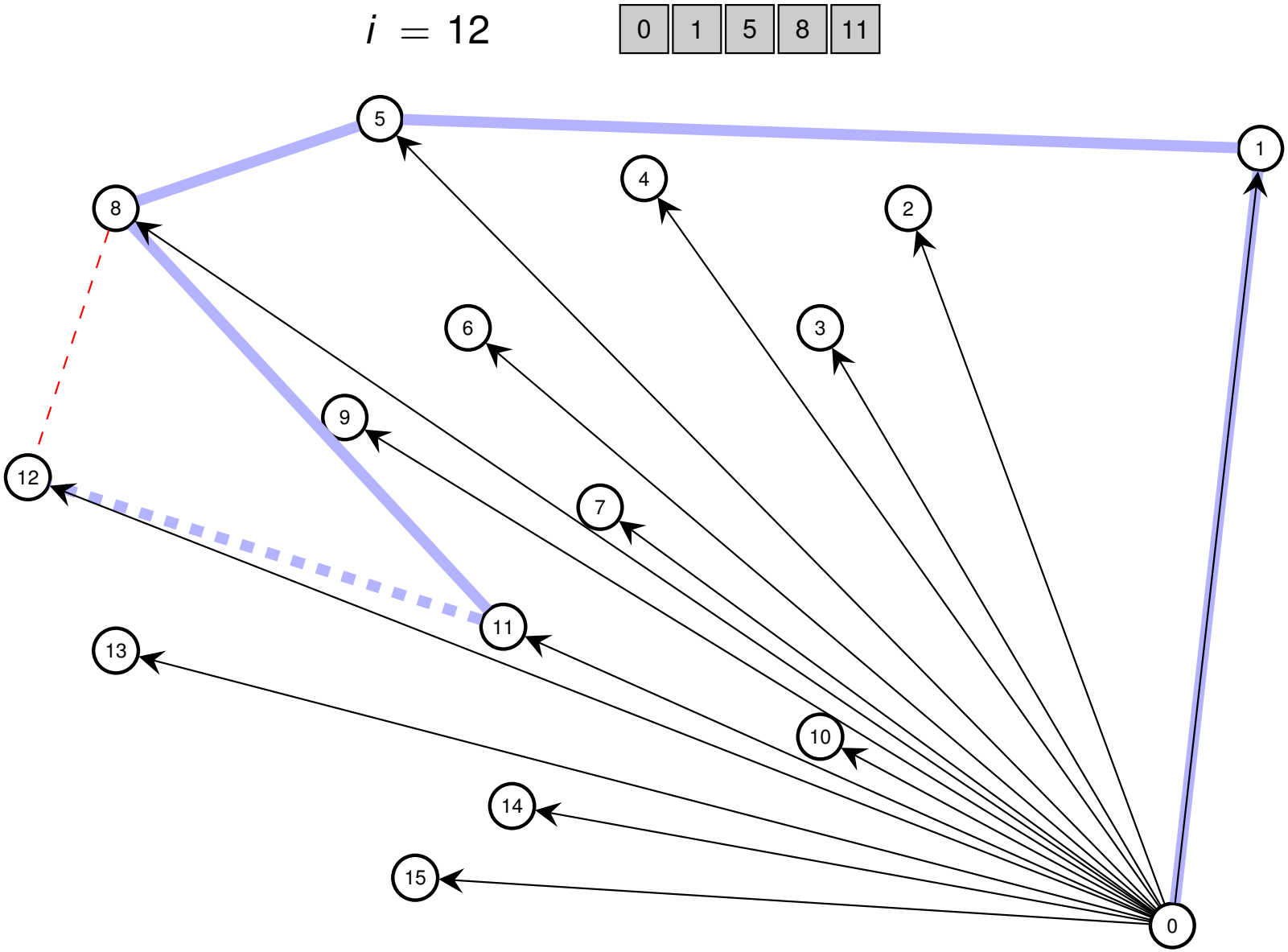

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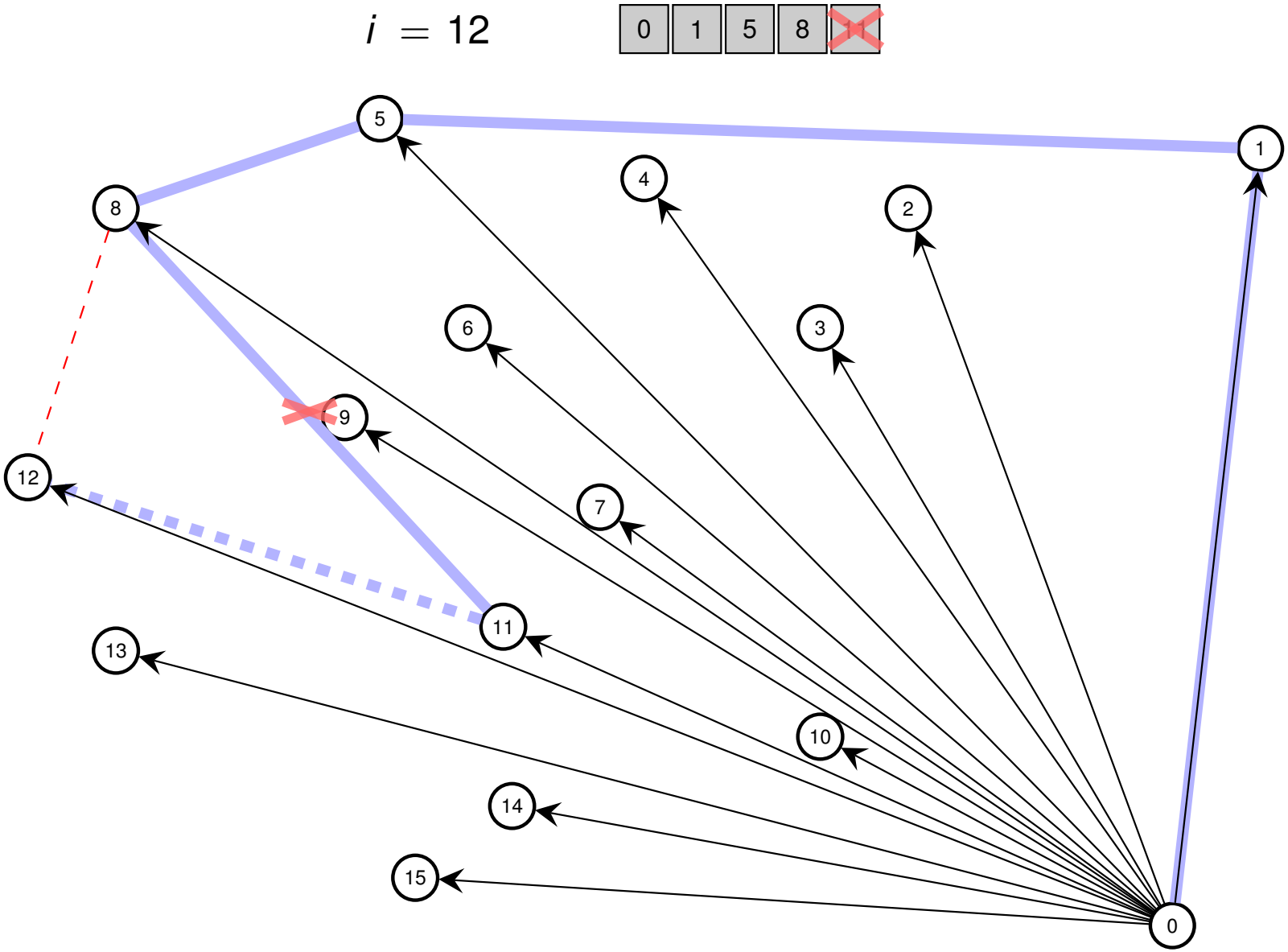

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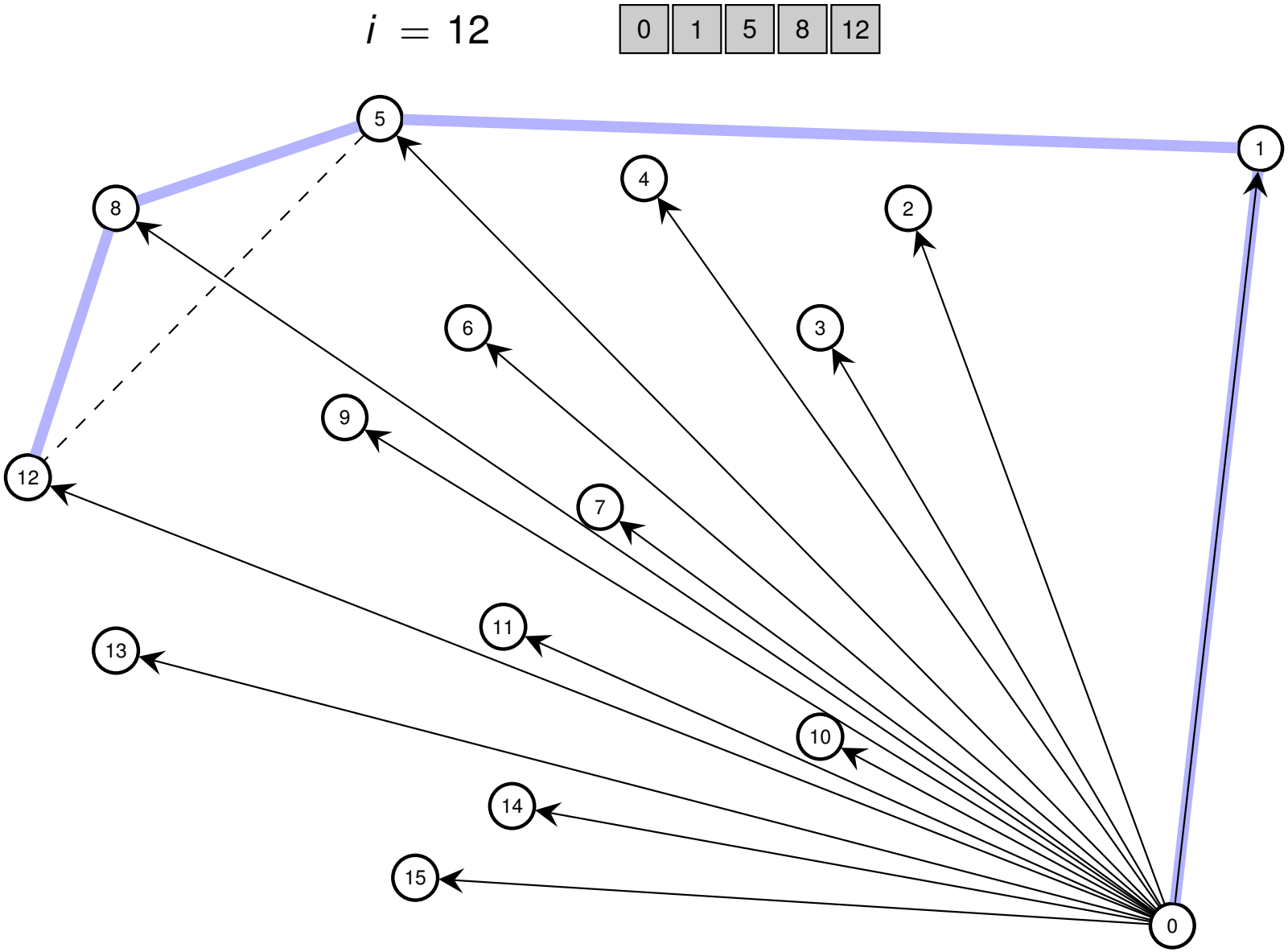

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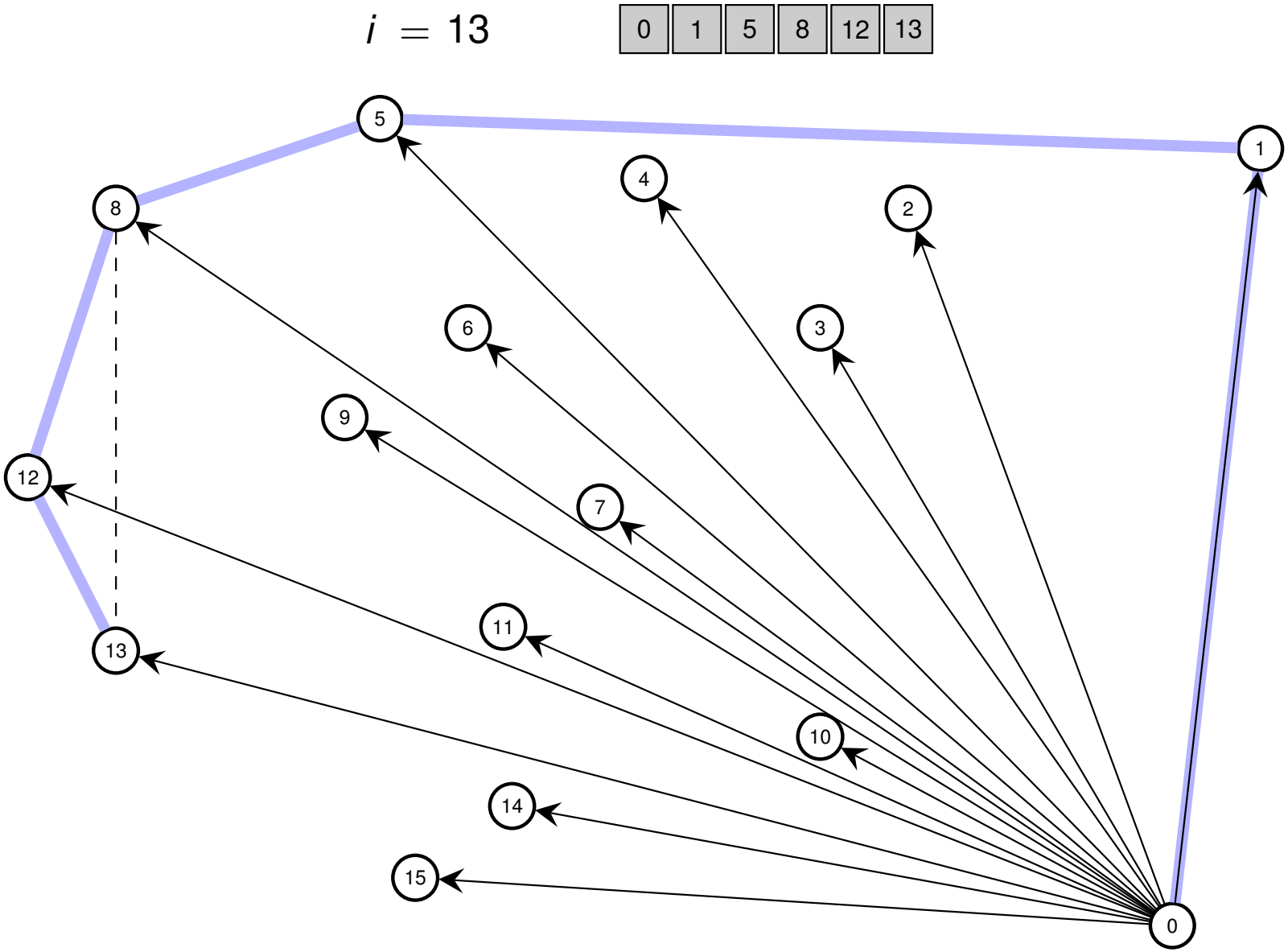

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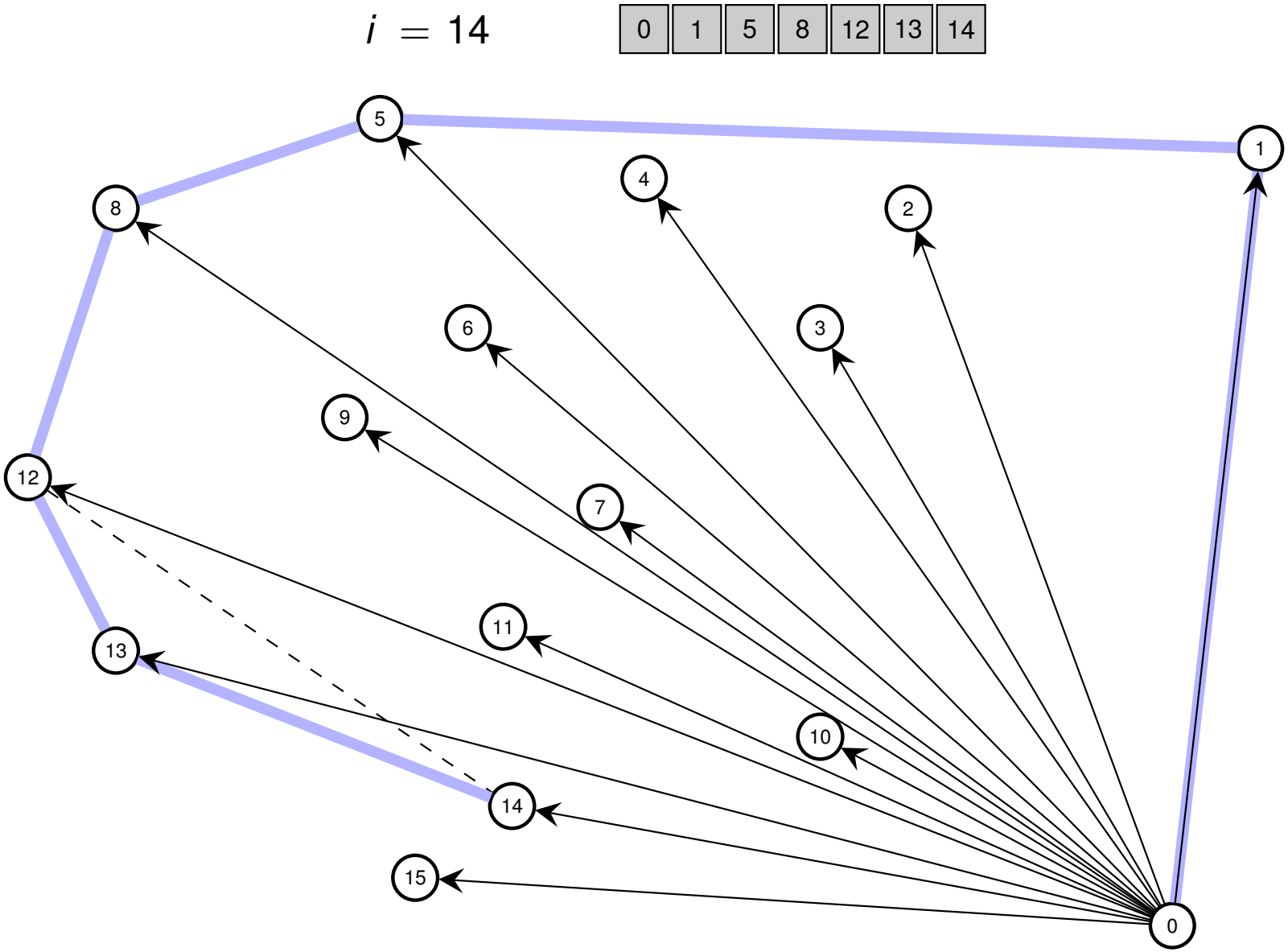

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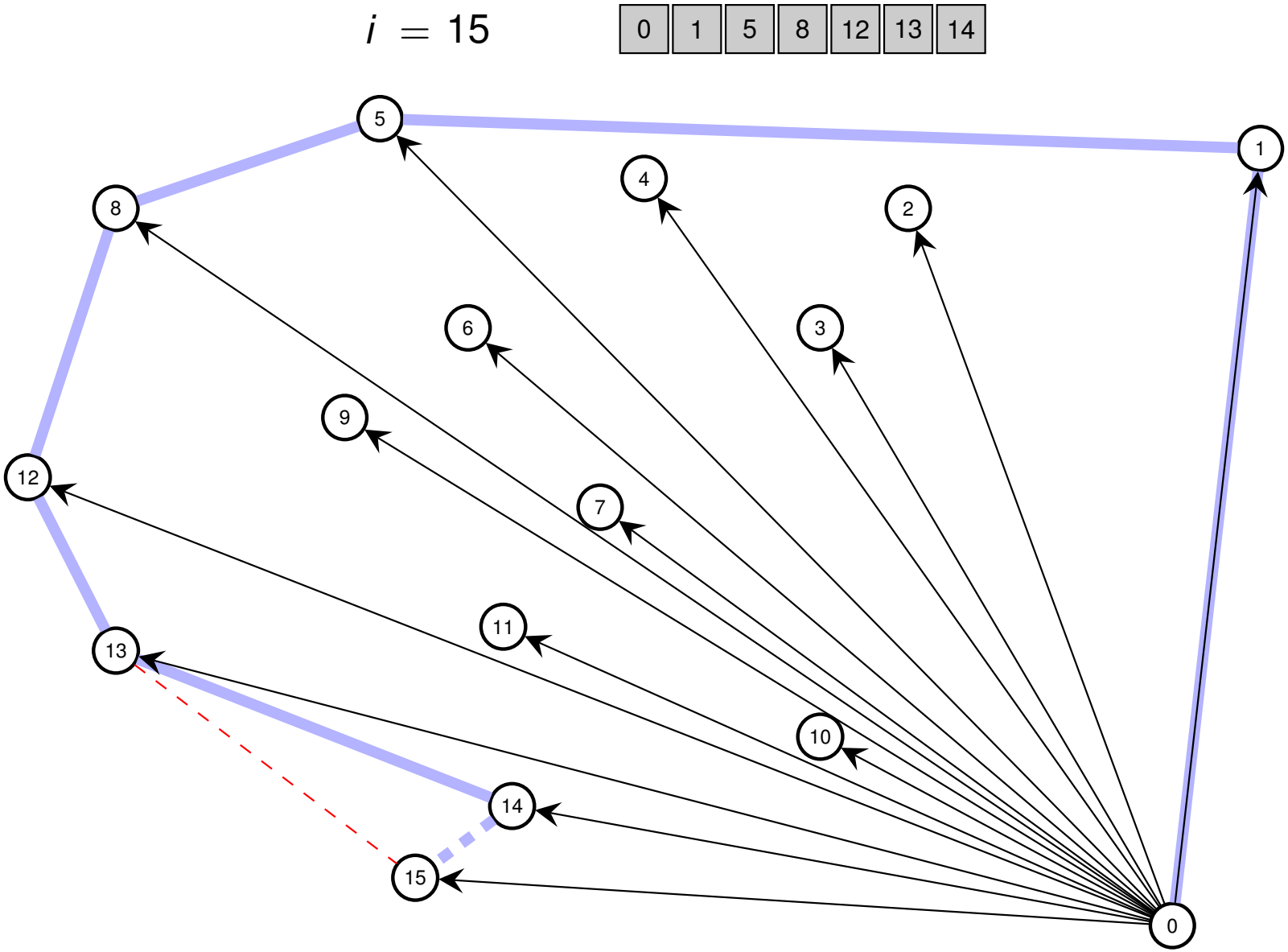

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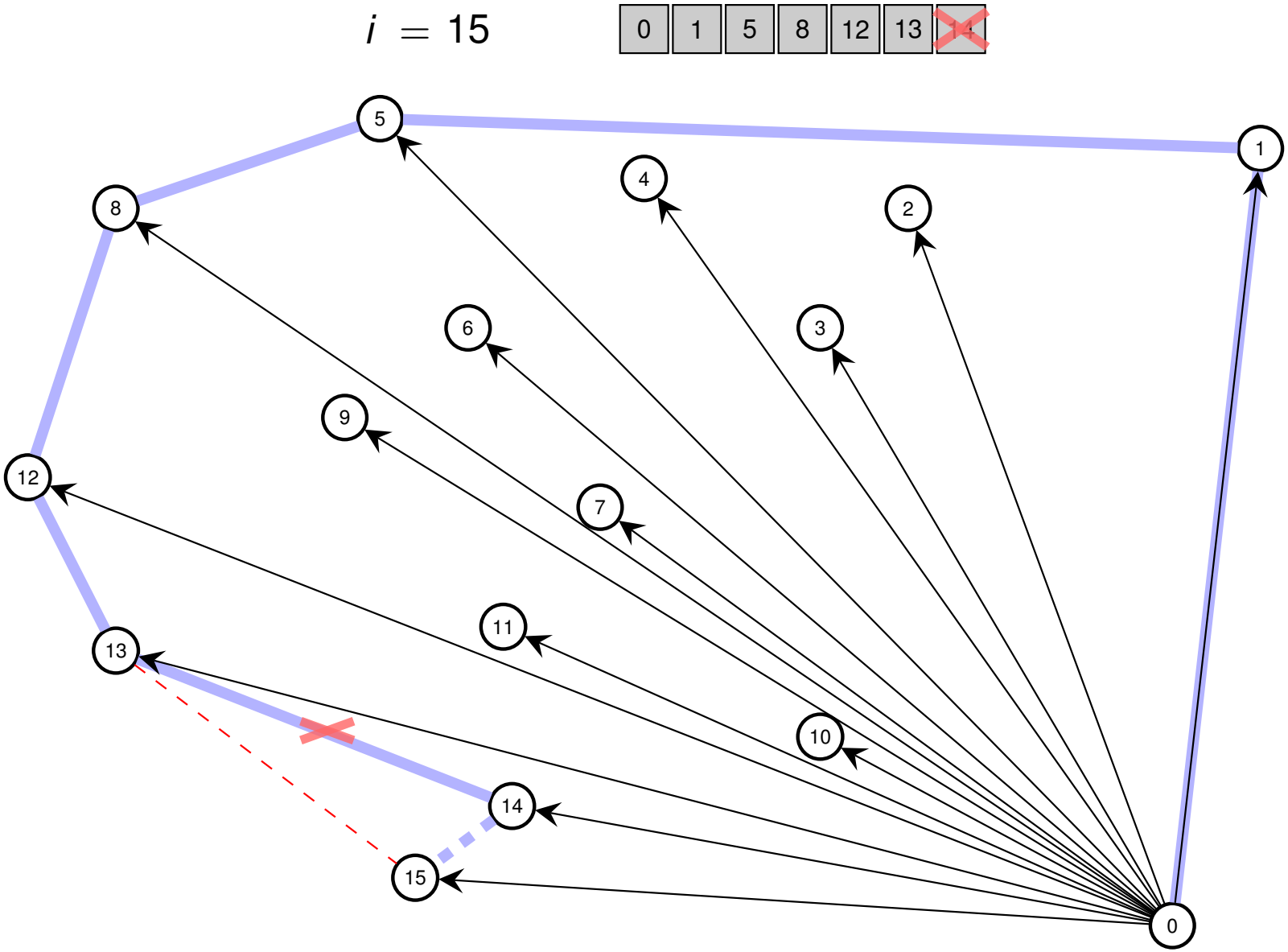

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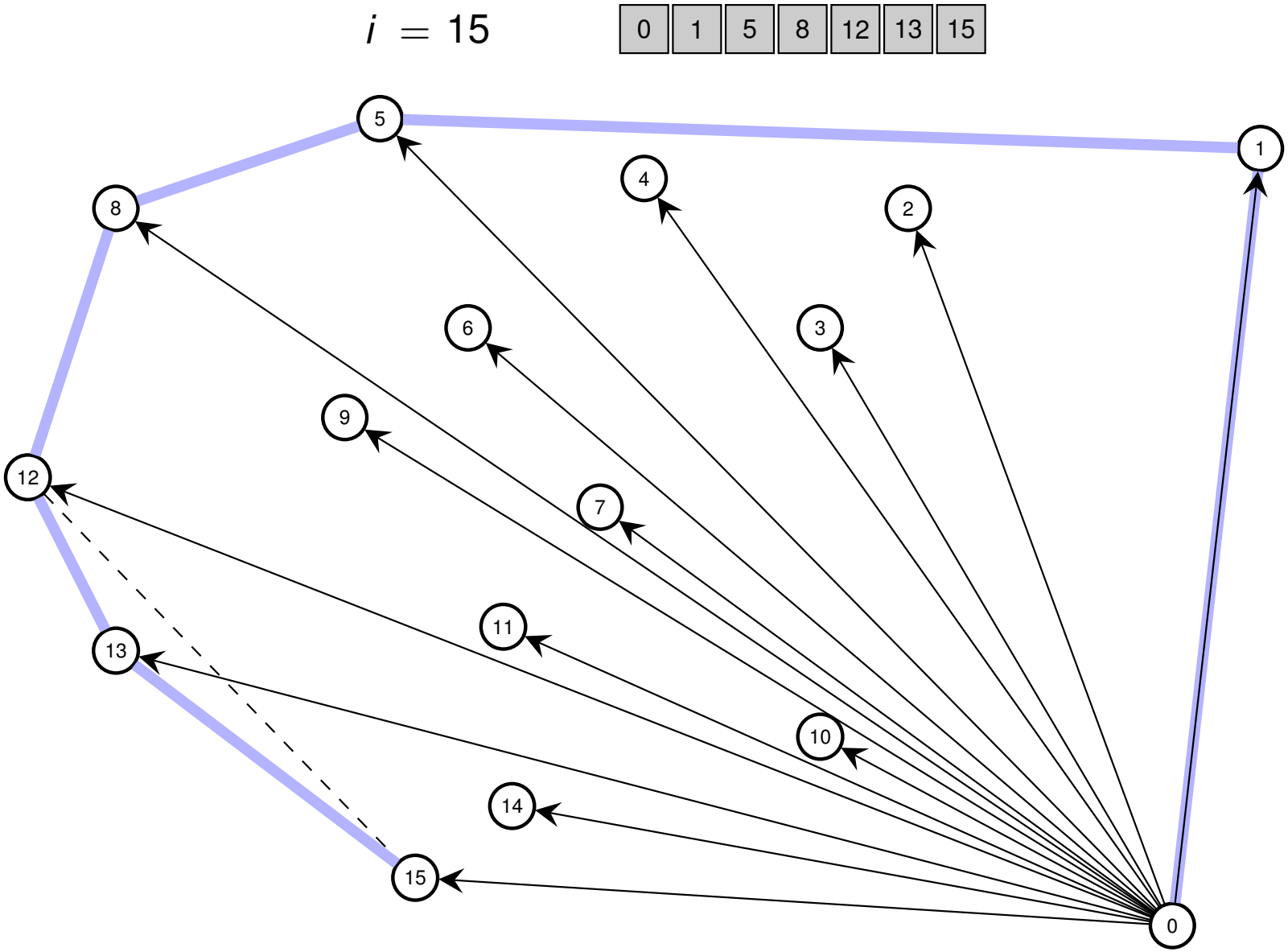
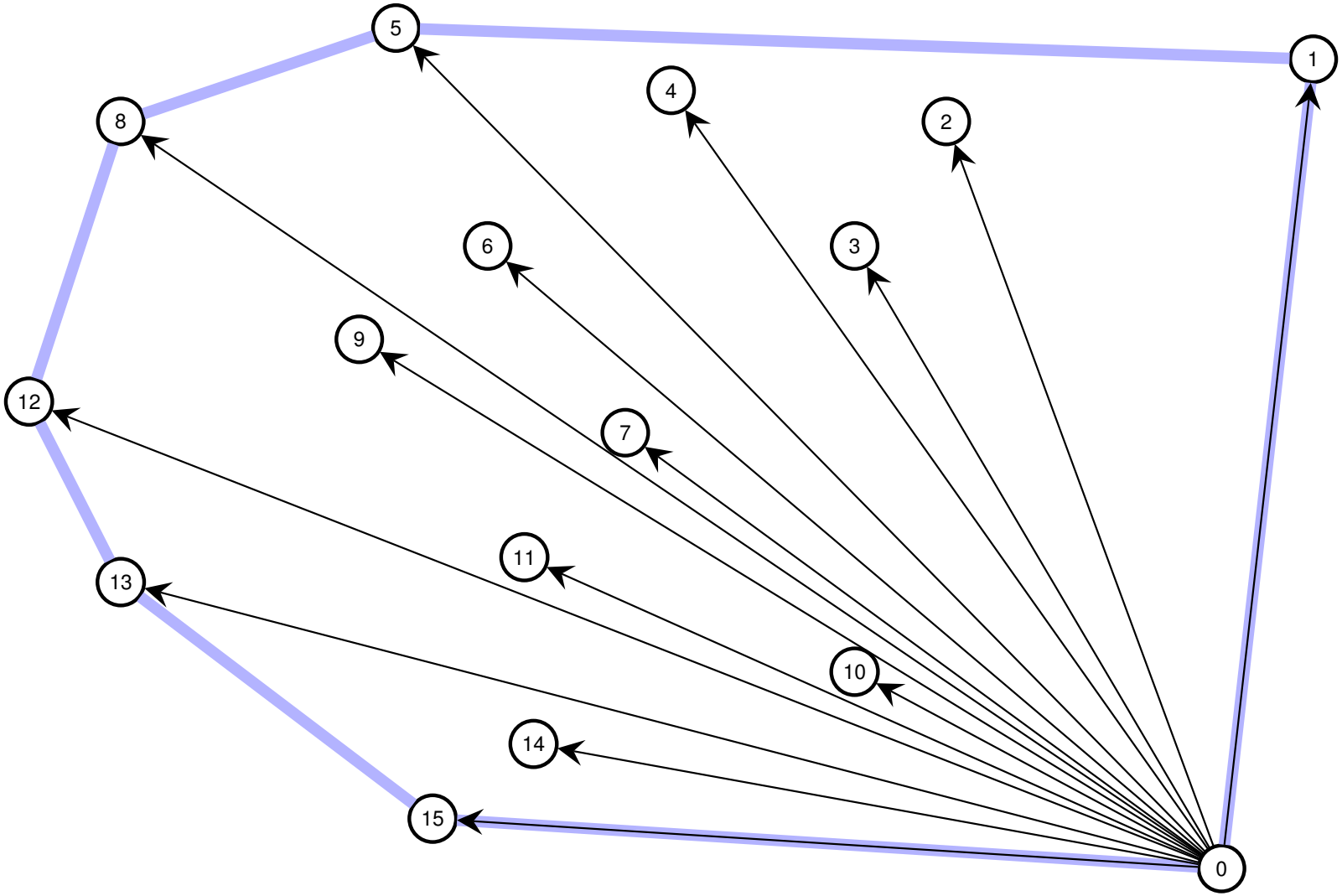


Complete Run of Graham's Scan

$i = 15$ ✓

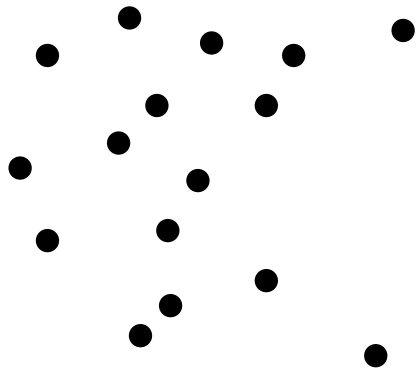
0	1	5	8	12	13	15



Jarvis' March (Gift wrapping)

Intuition

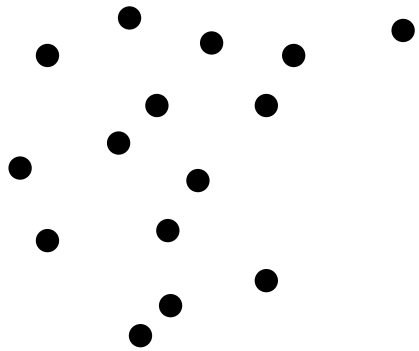
- Wrapping taut paper around the points



Jarvis' March (Gift wrapping)

Intuition

- Wrapping taut paper around the points
  1. Tape end of paper at lowest point



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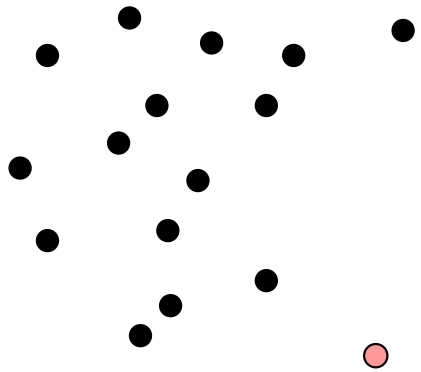

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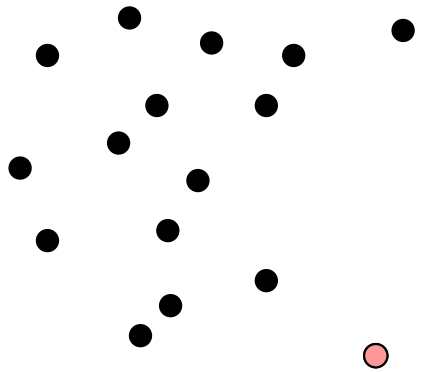

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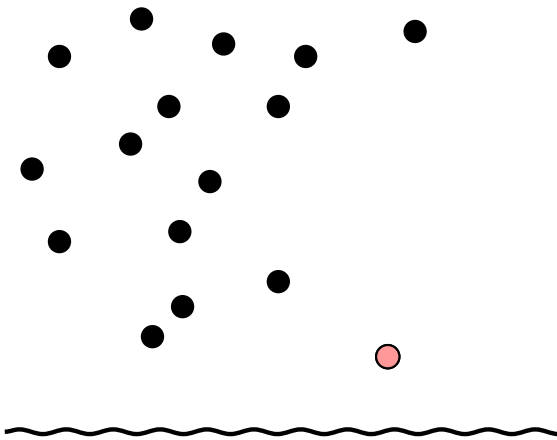

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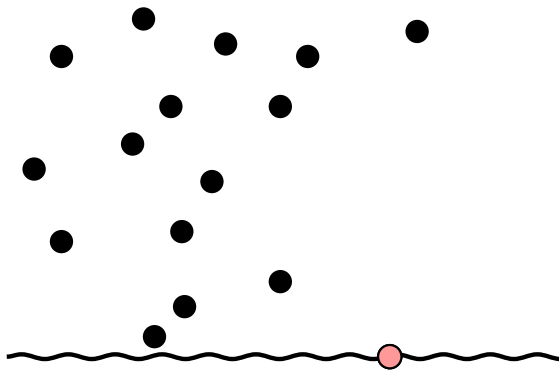
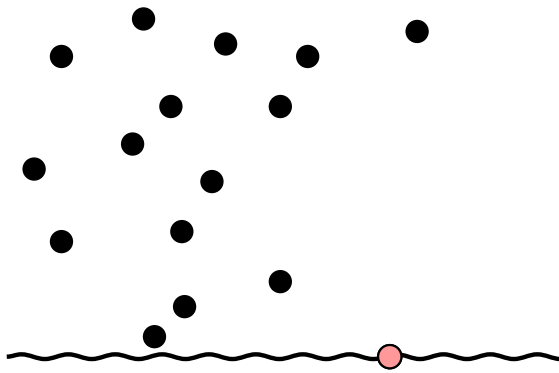


Jarvis' March (Gift wrapping)

Intuition

- Wrapping taut paper around the points
  1. Tape end of paper at lowest point
  2. Pull paper to the right until it touches a point



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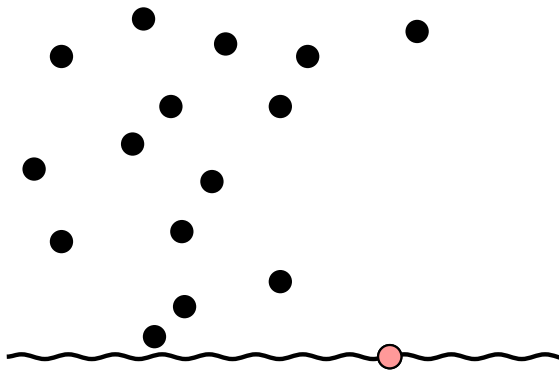

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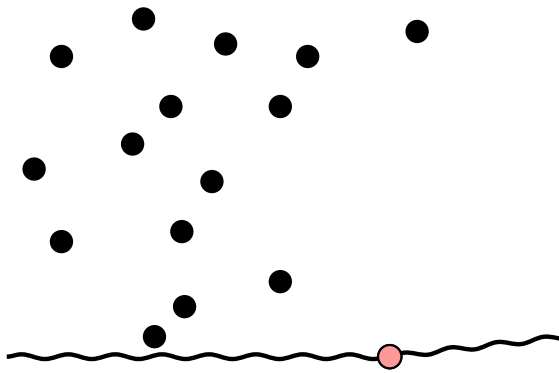

Jarvis' March (Gift wrapping)

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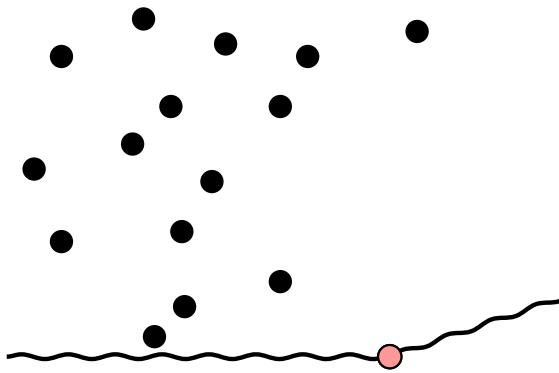

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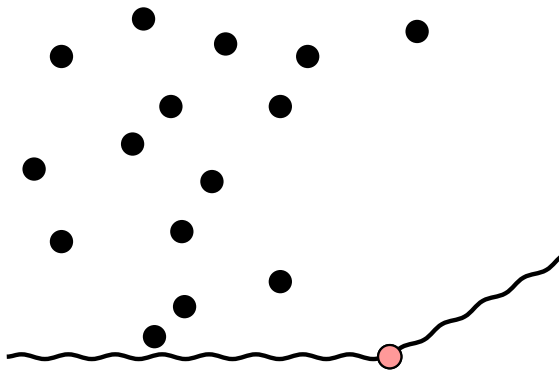

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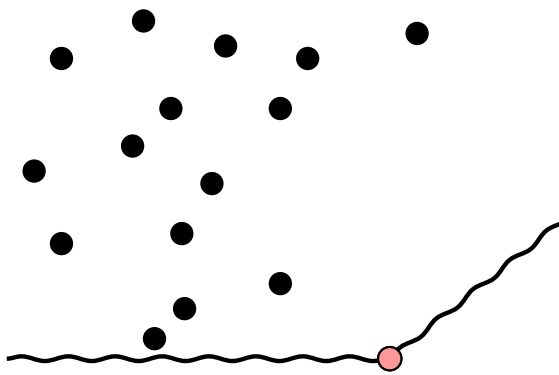

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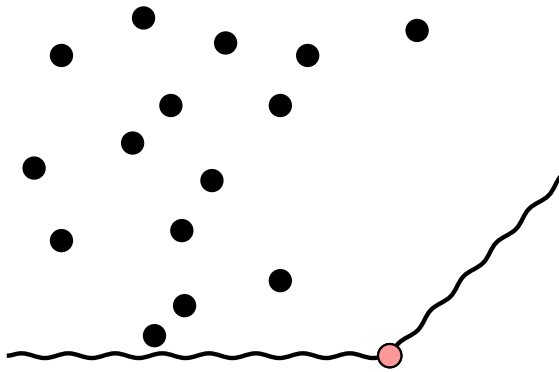

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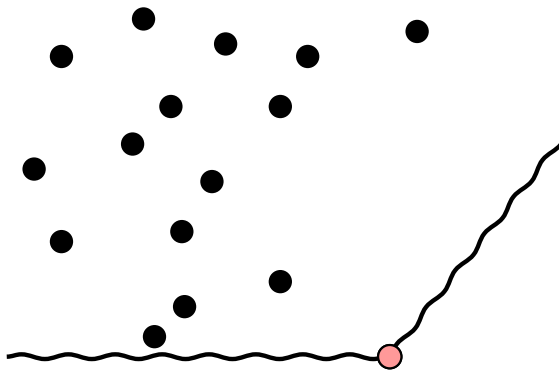

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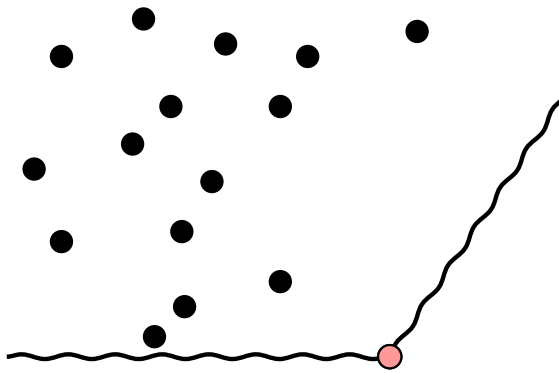

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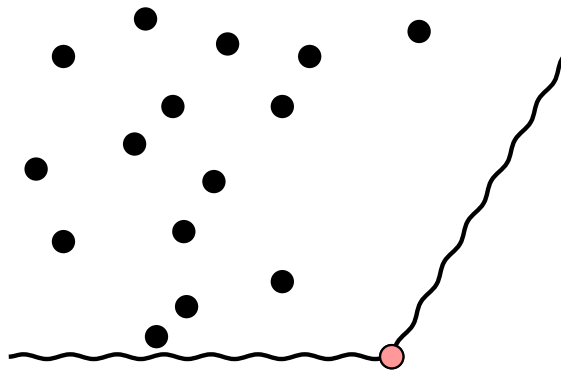

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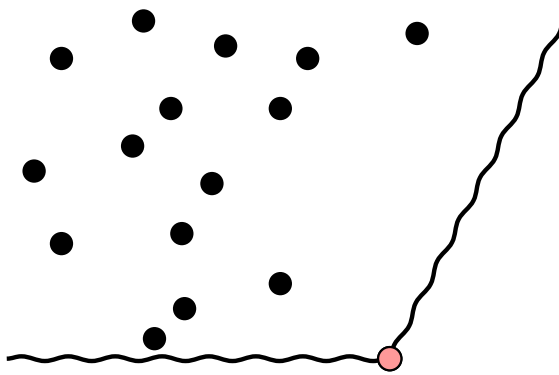

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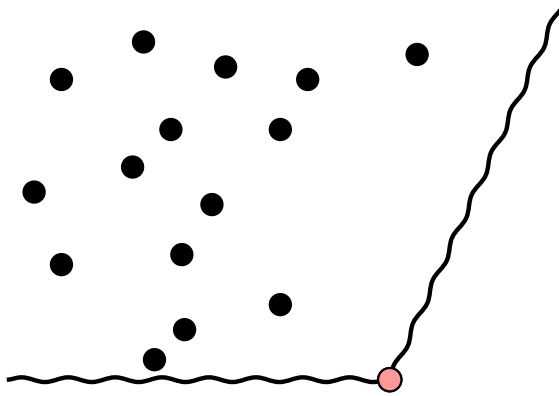

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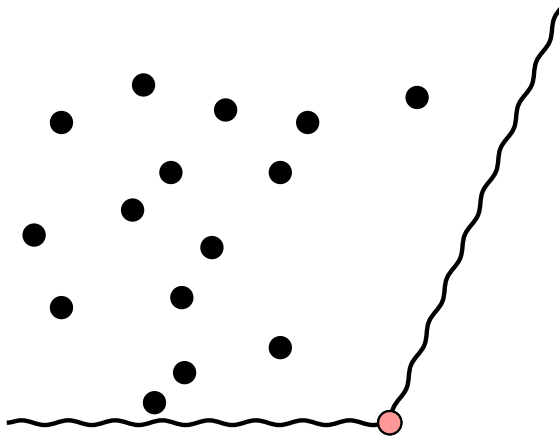

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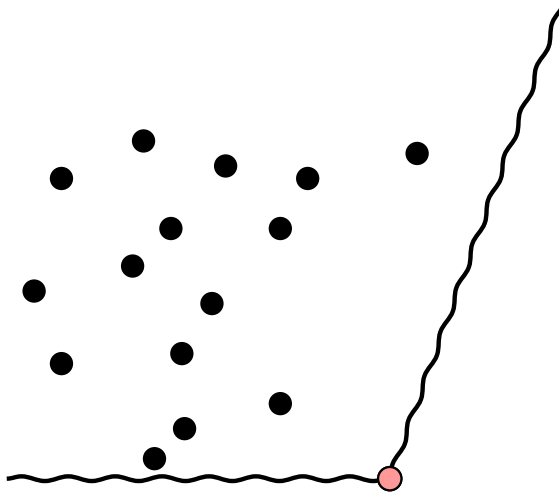

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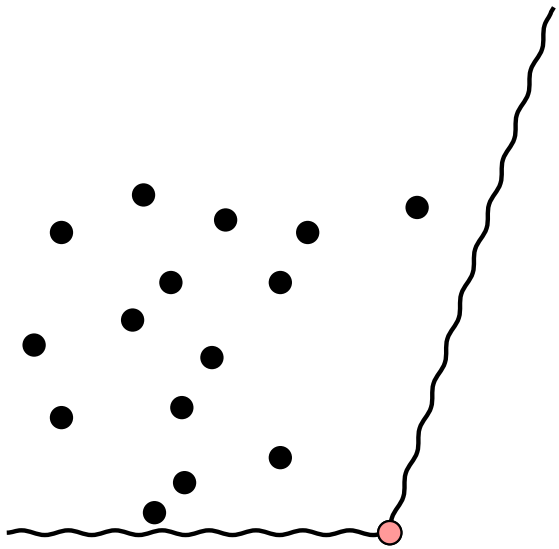

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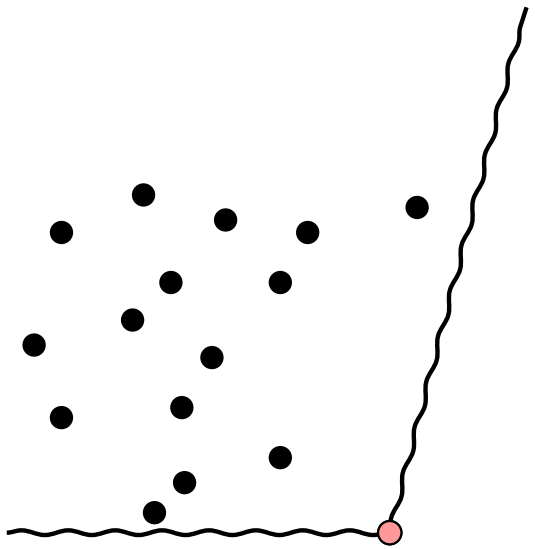

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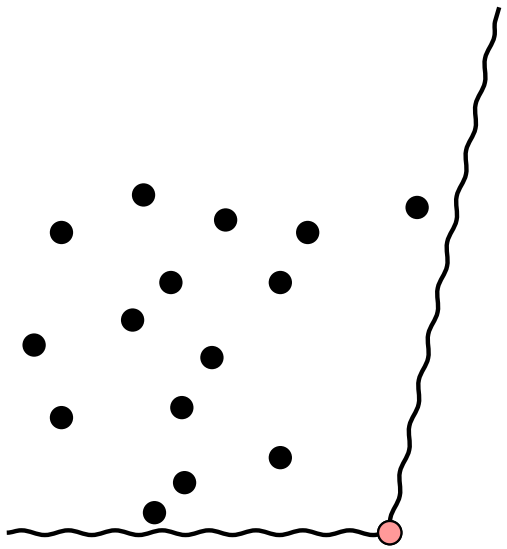

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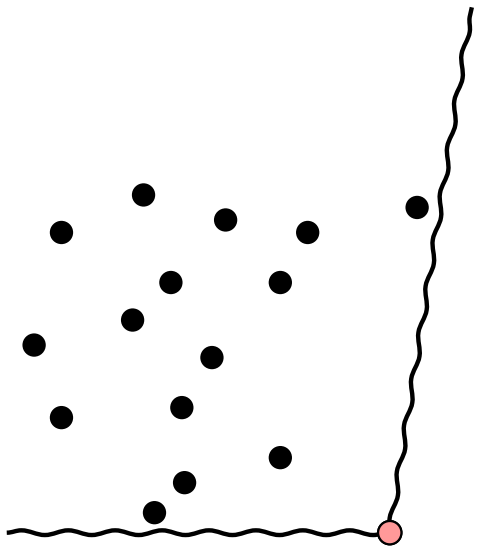

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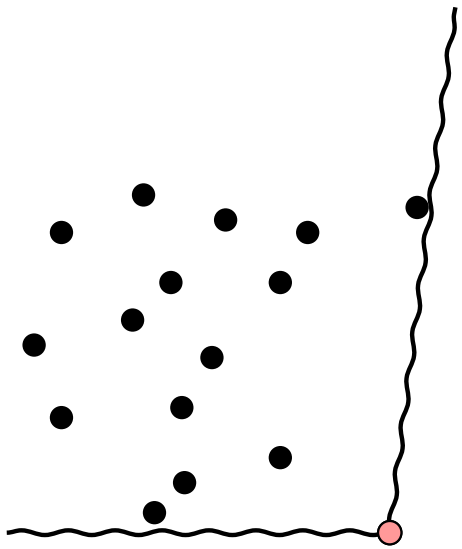

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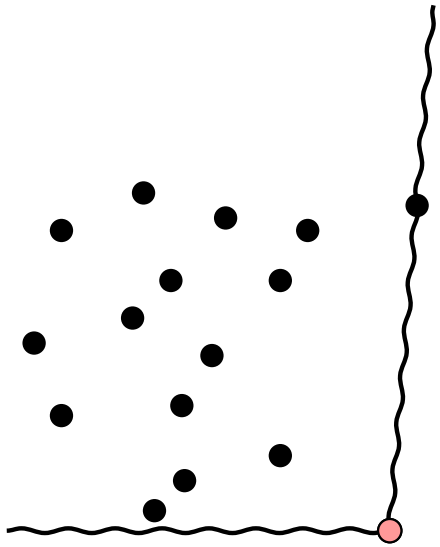

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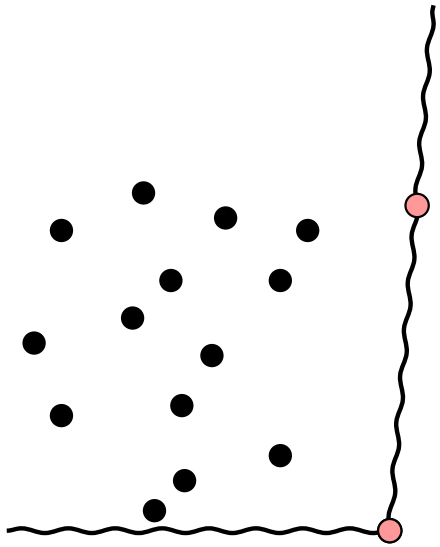

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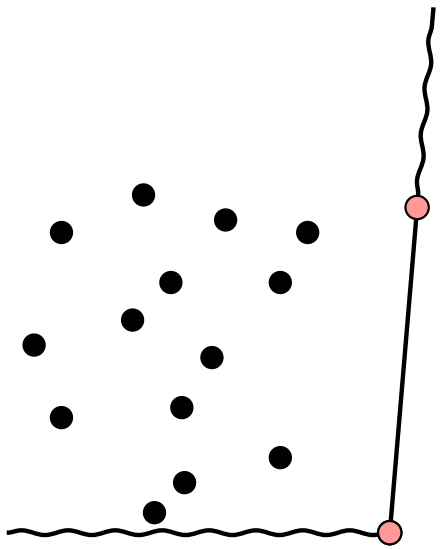

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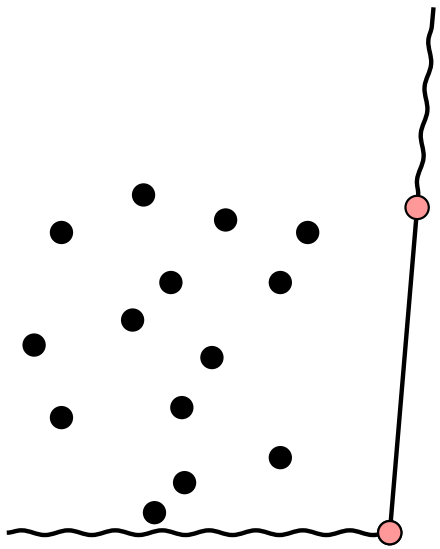

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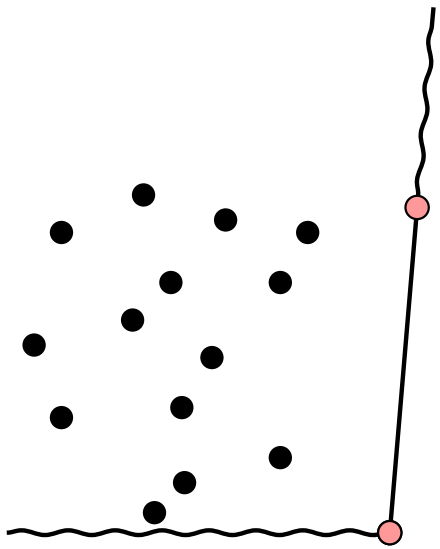

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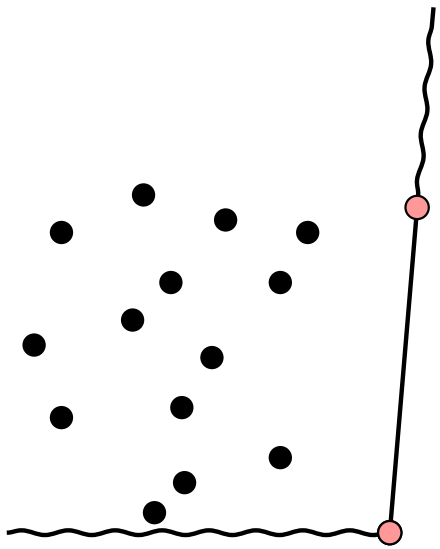

Algorithm

1. Let p_0 be the lowest point
2. Next point the one with smallest angle w.r.t. p_0

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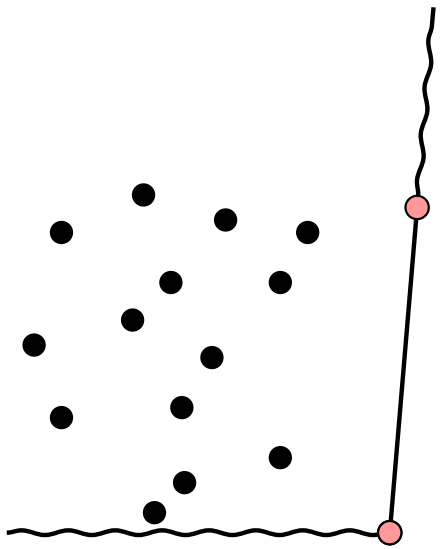

Algorithm

1. Let p_0 be the lowest point
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3. Continue until highest point p_k

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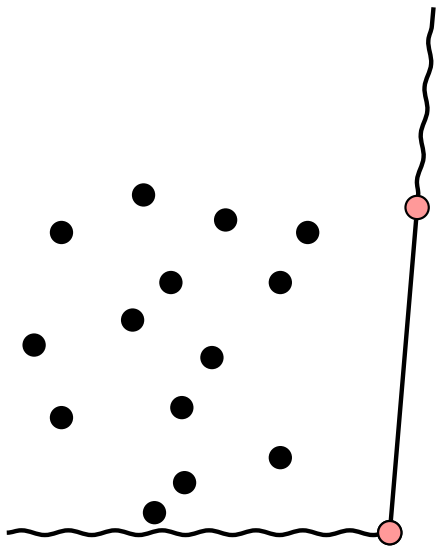

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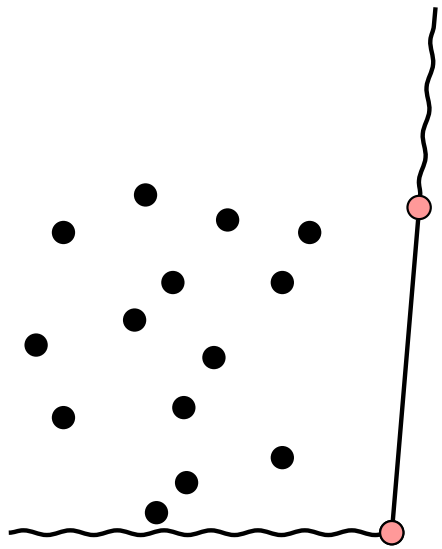

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Here, we rotate the coordinate system by 180°!

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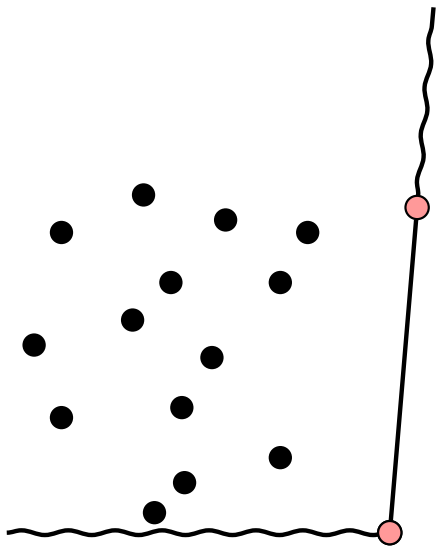

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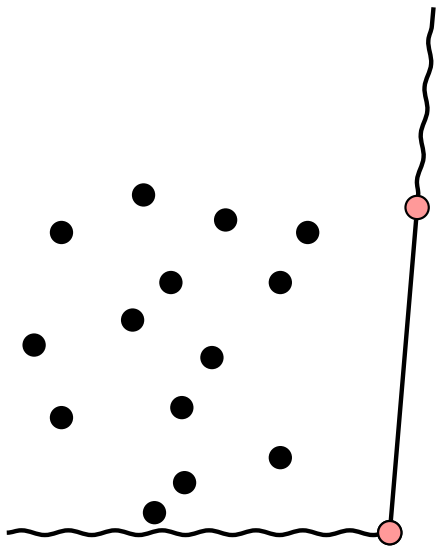

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Runtime: $O(n \cdot h)$, where h is the number of points on the convex hull

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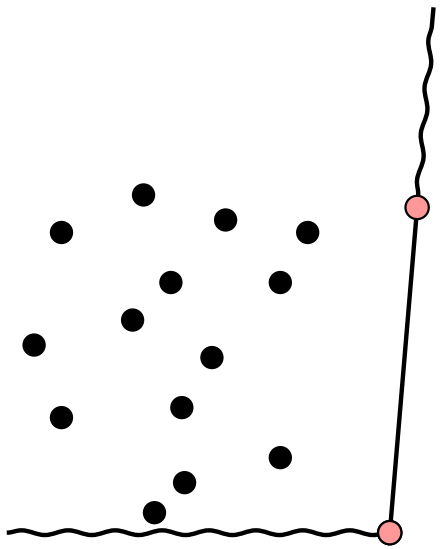

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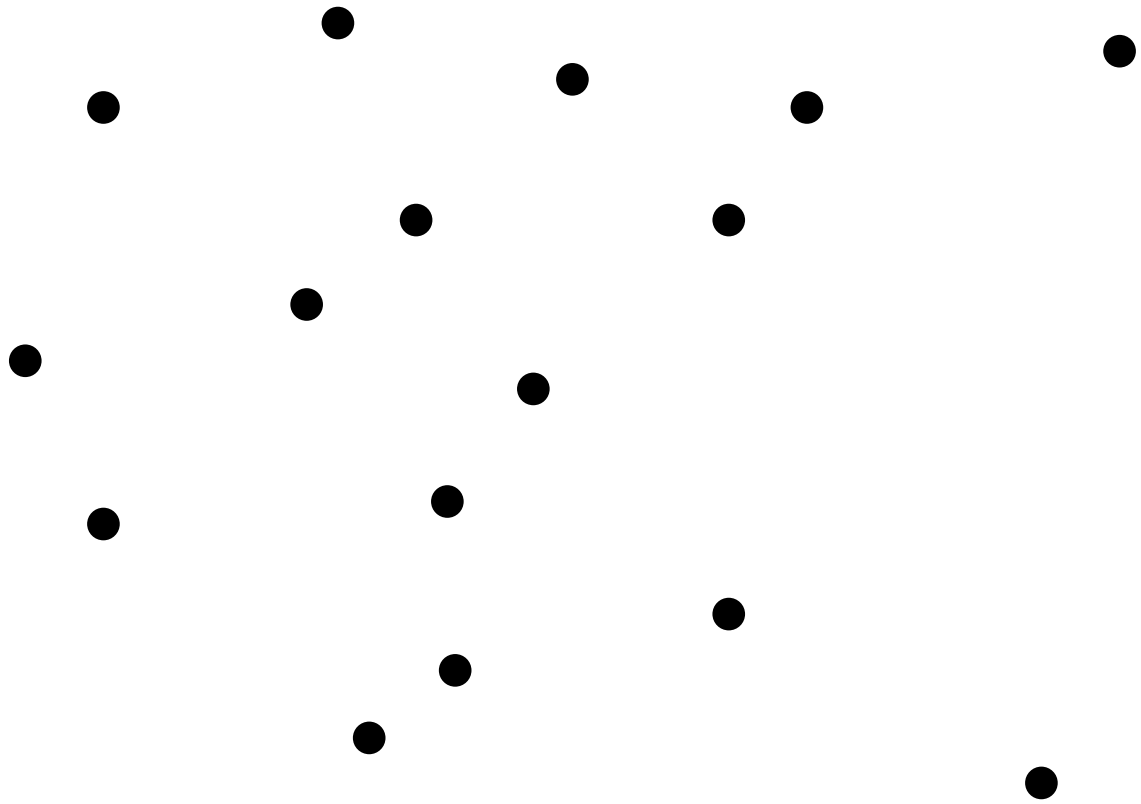
Output sensitive algorithm!

Execution of Jarvis' March



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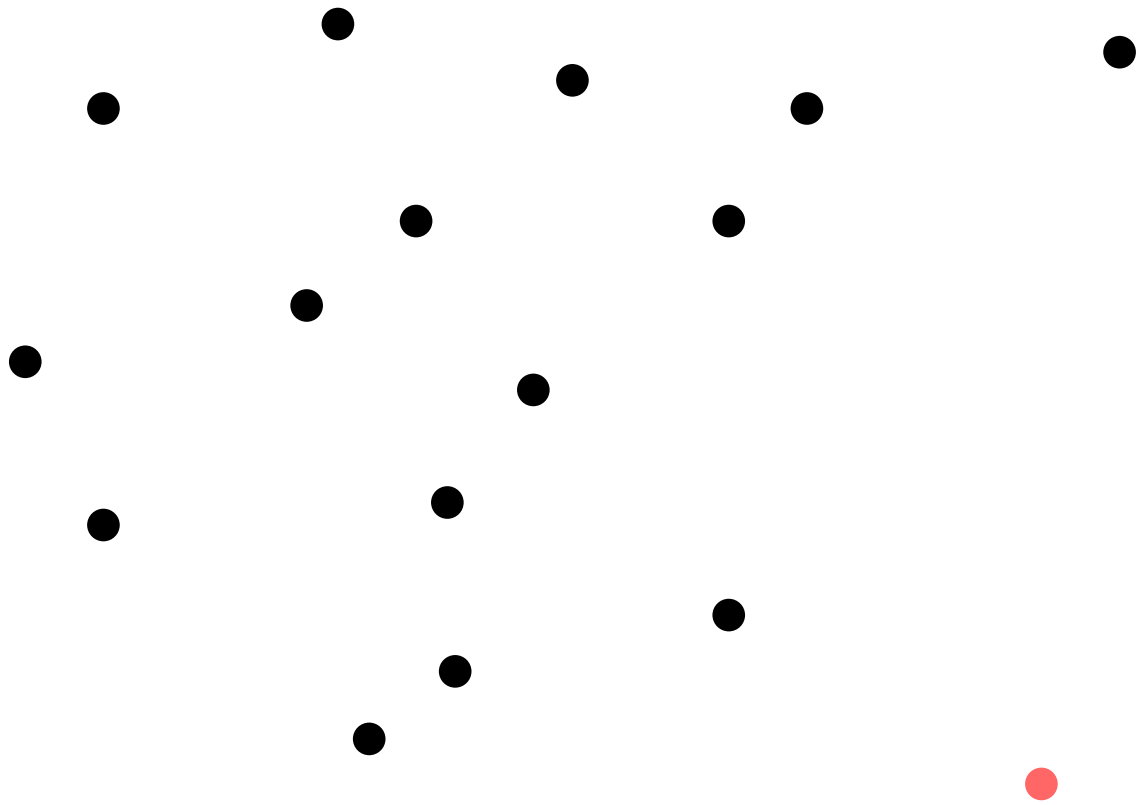

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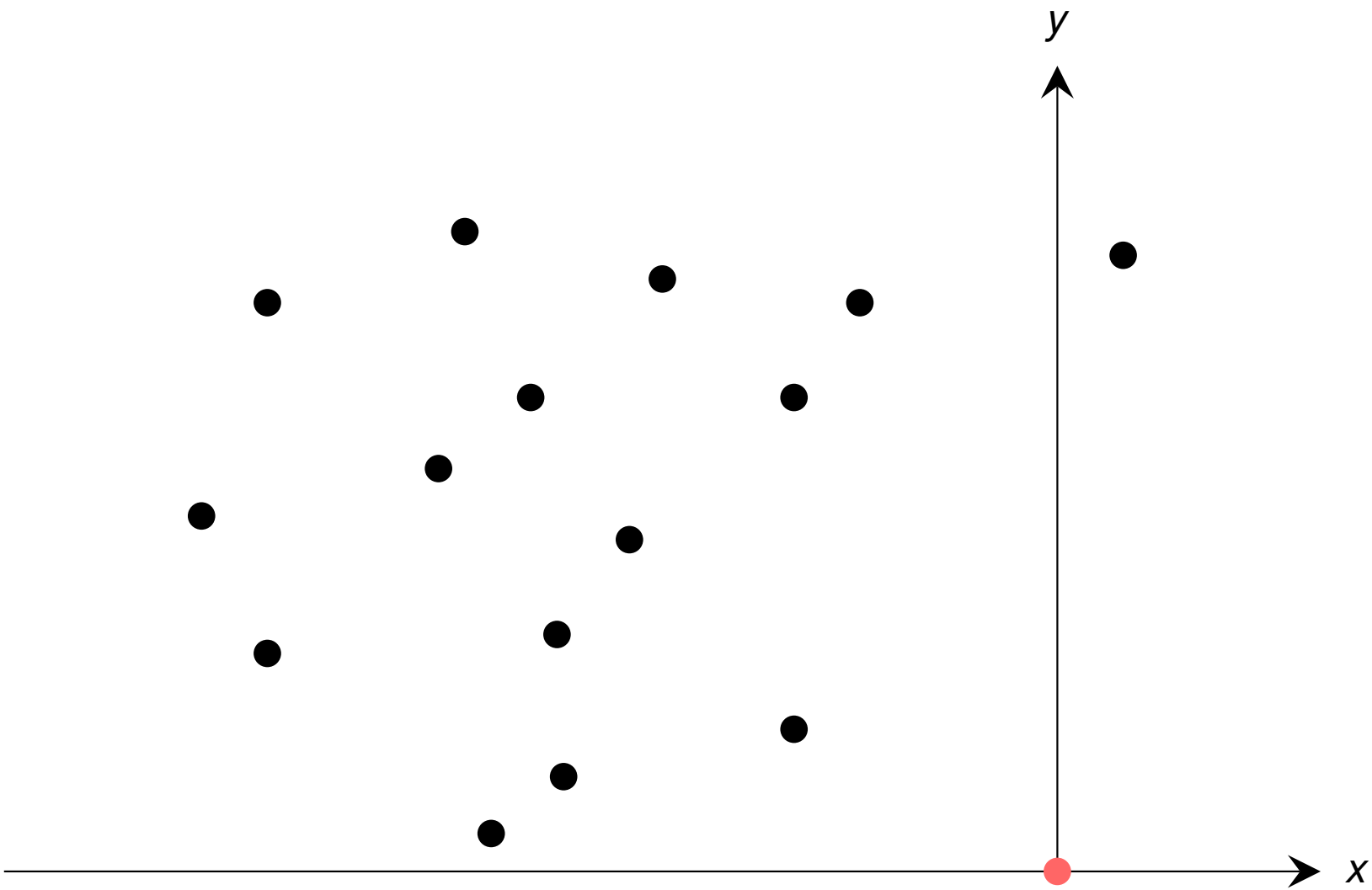

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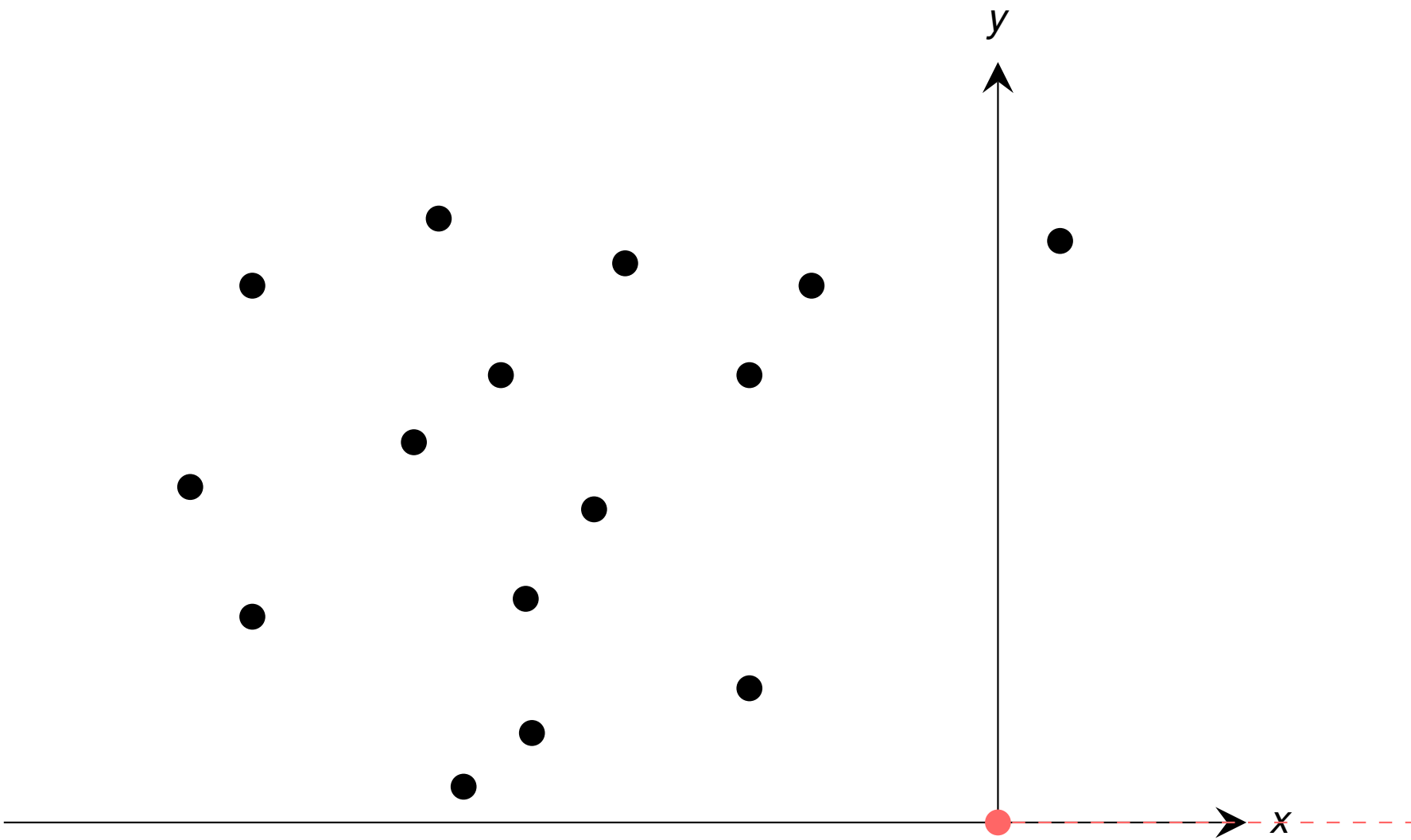

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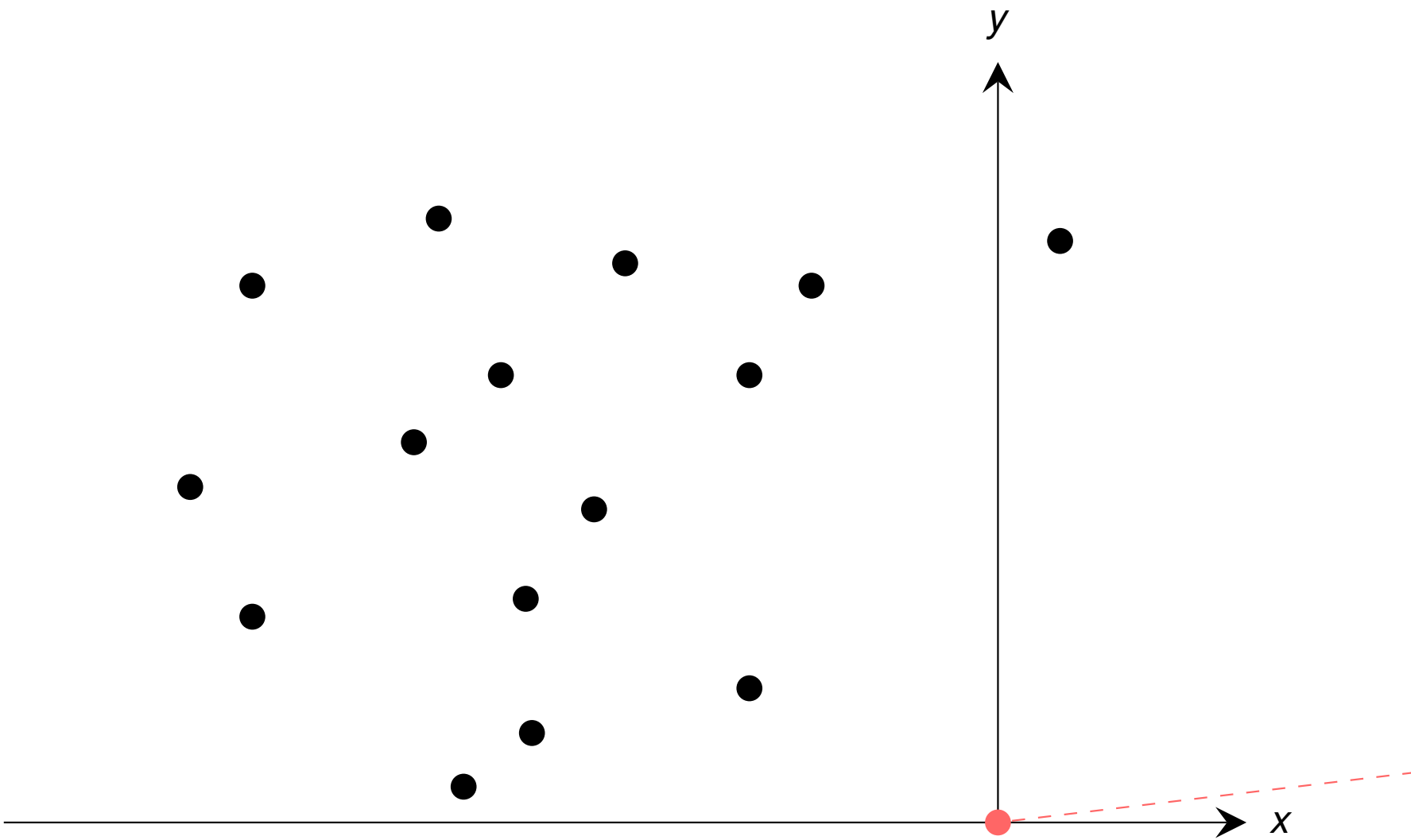

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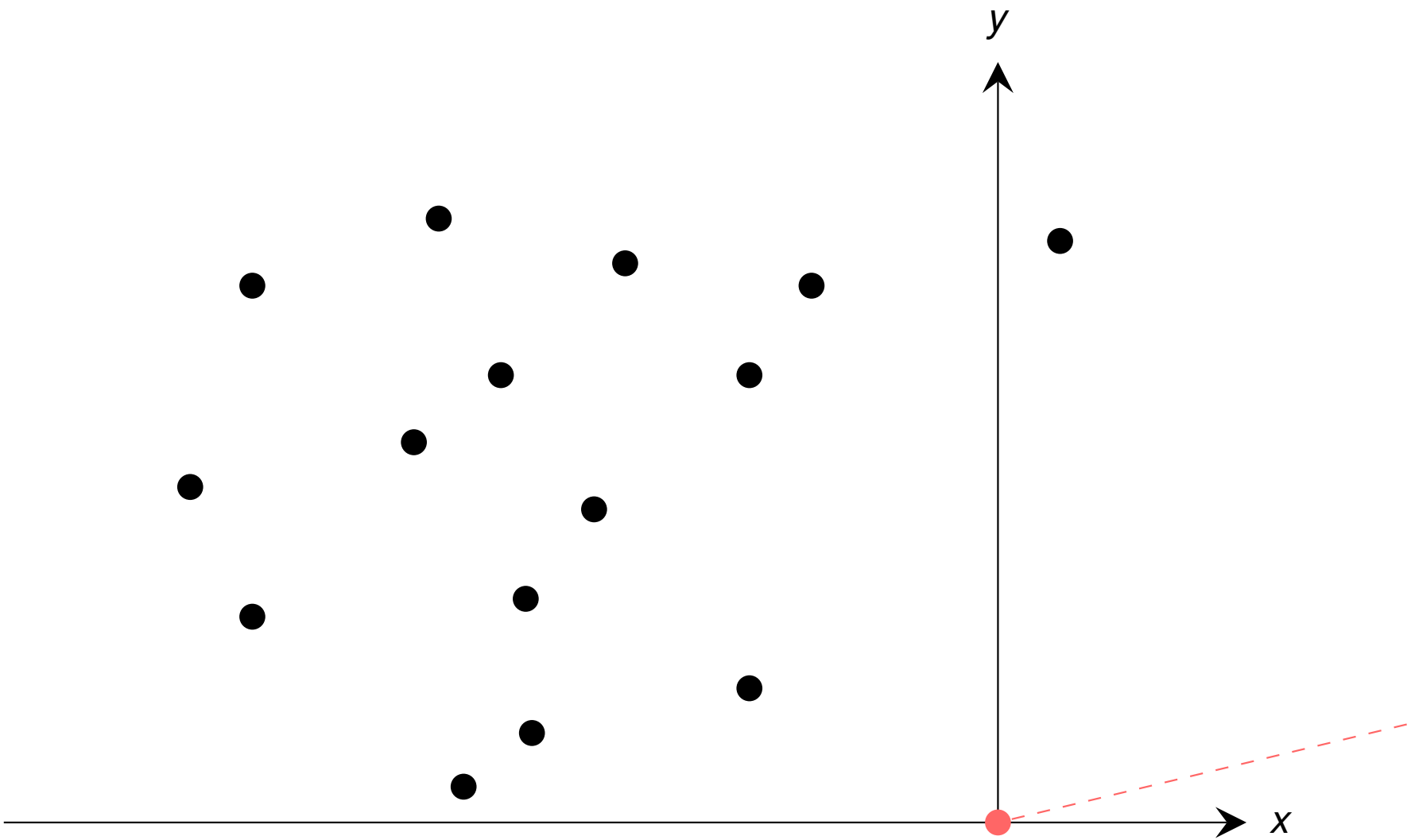

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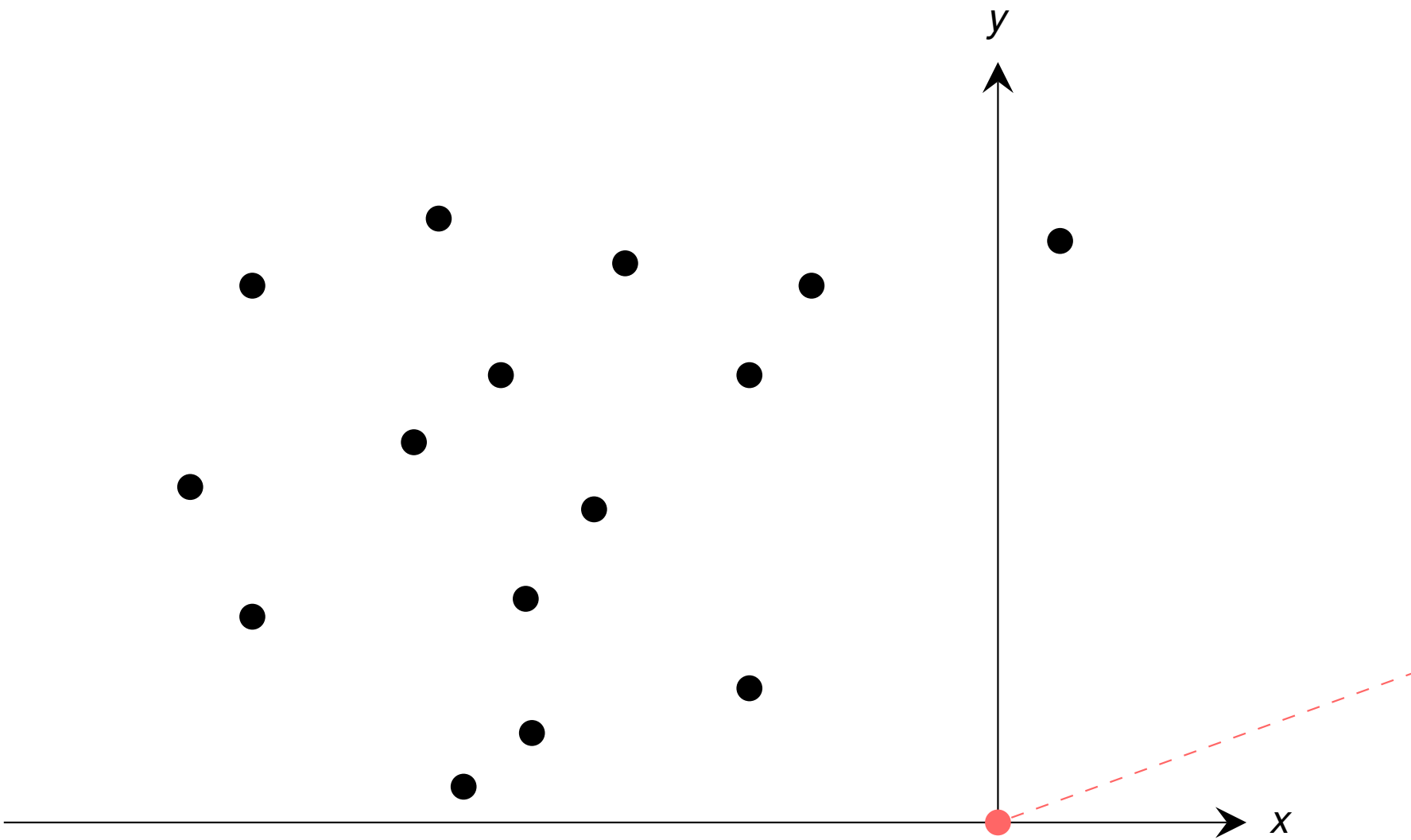

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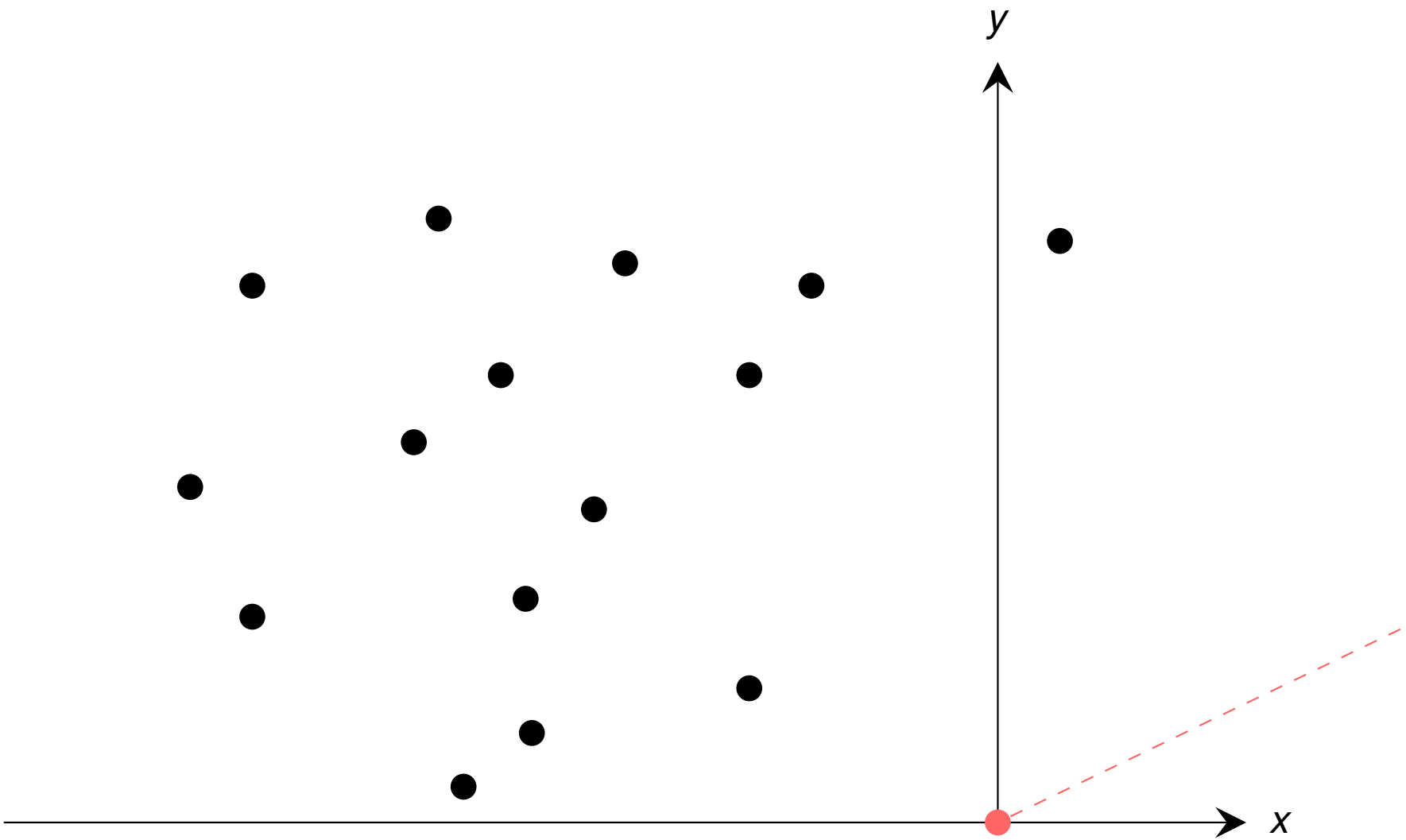

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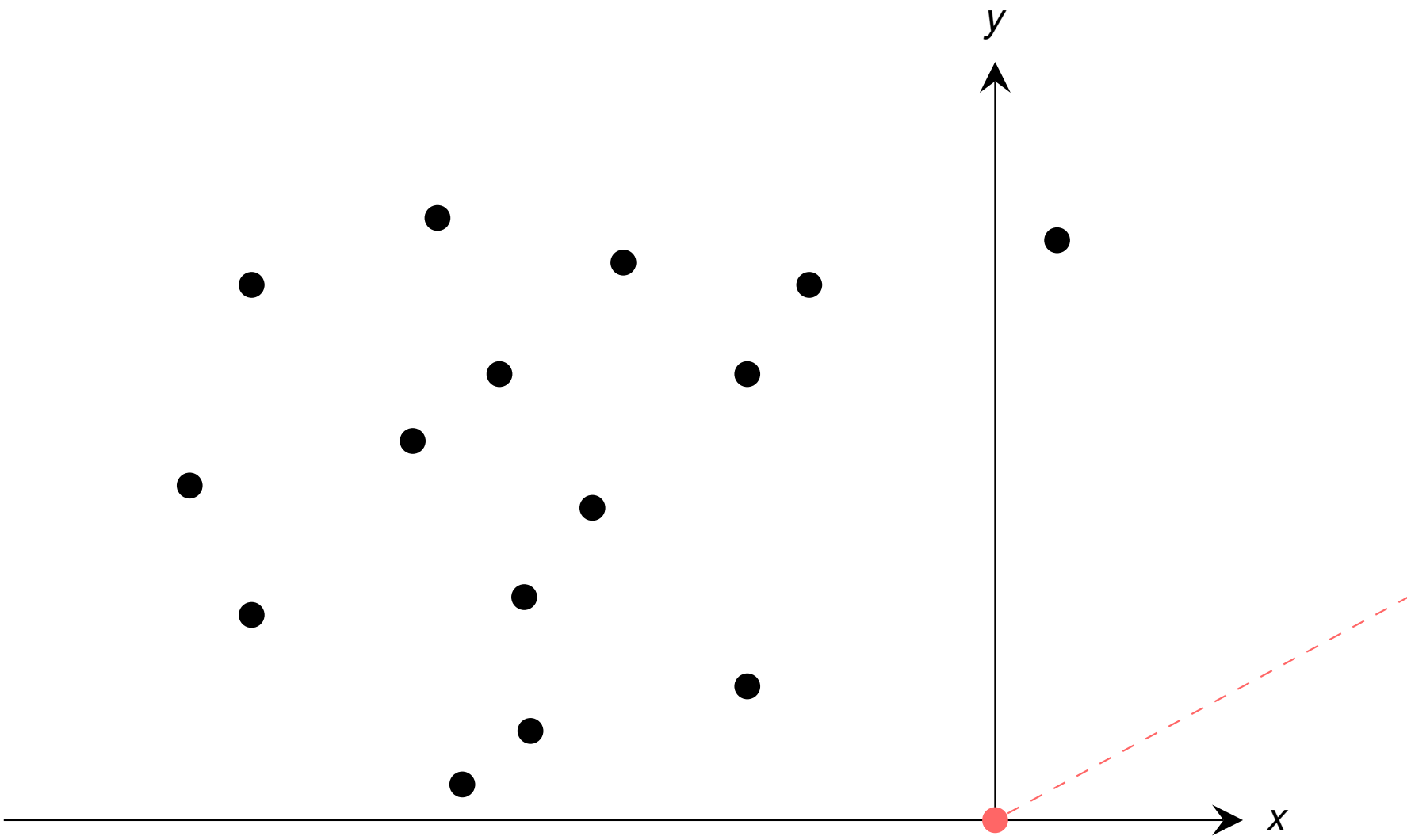

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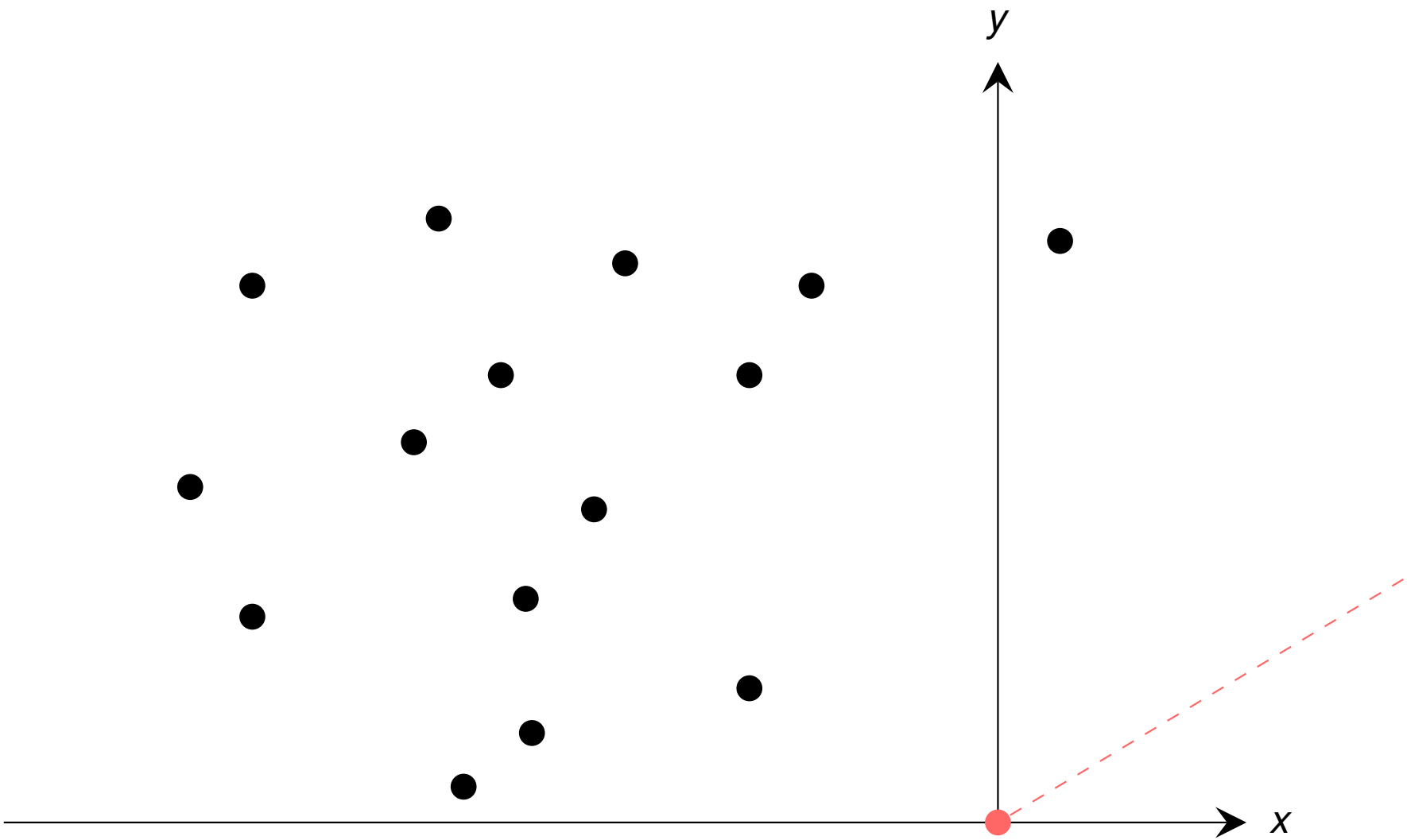

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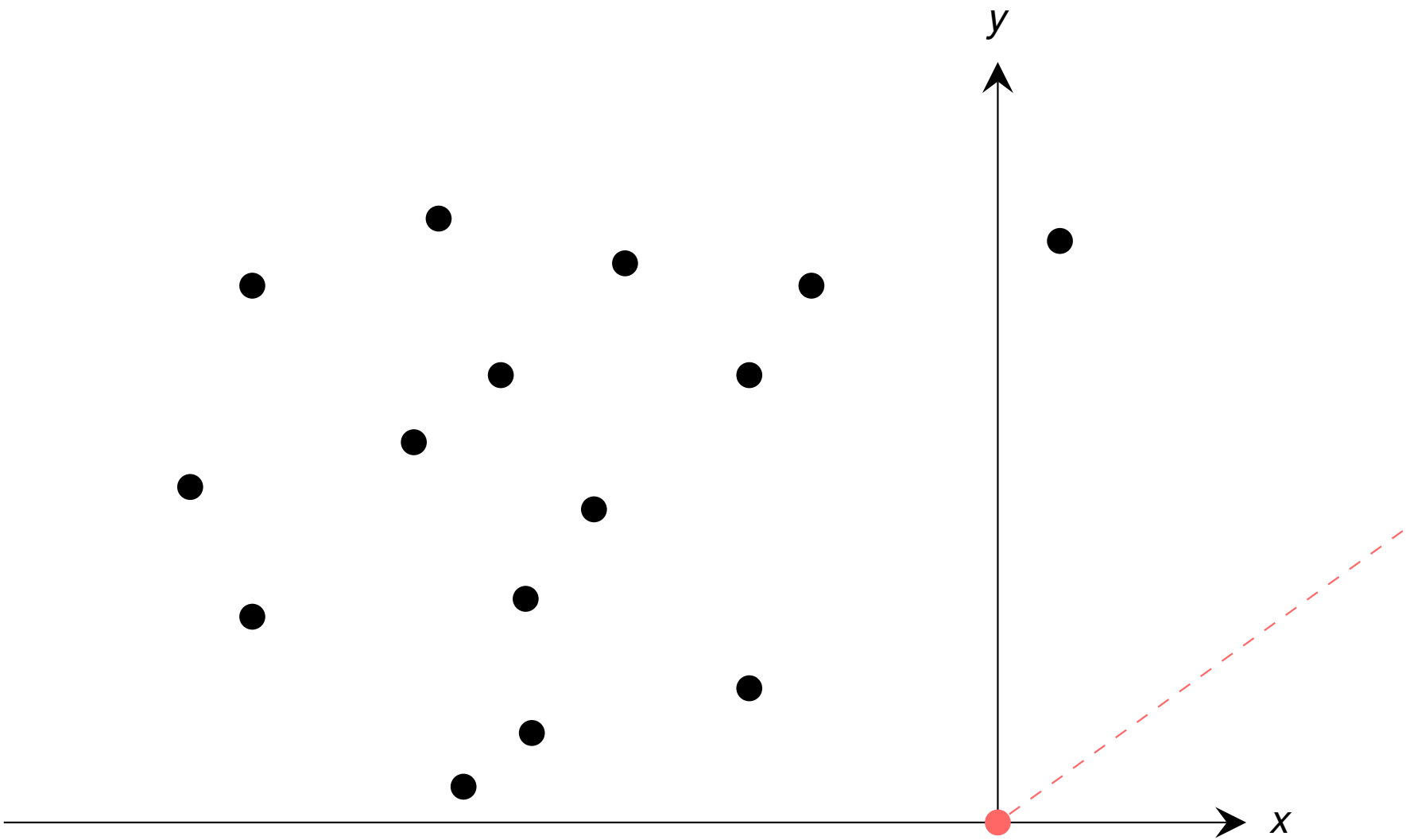

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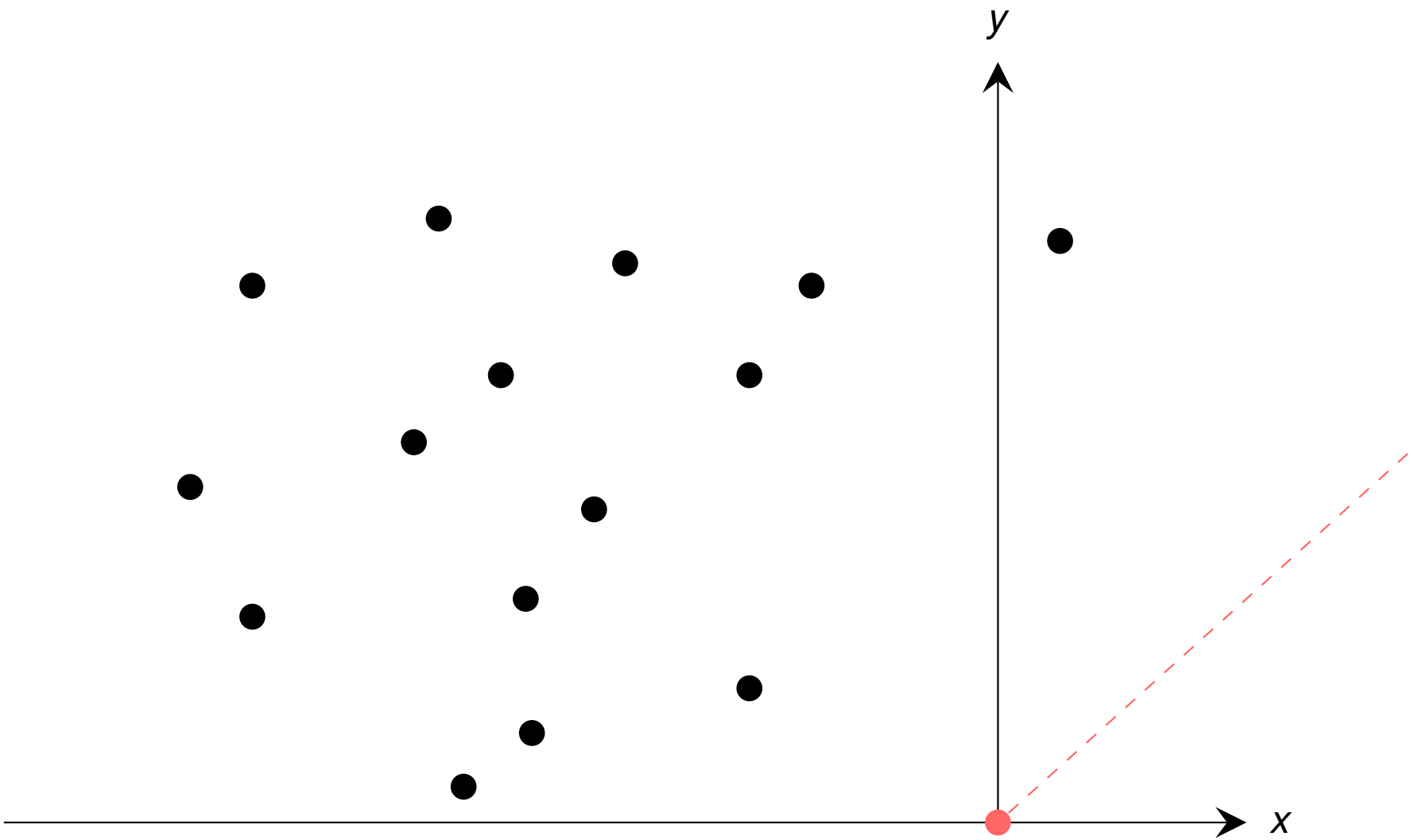

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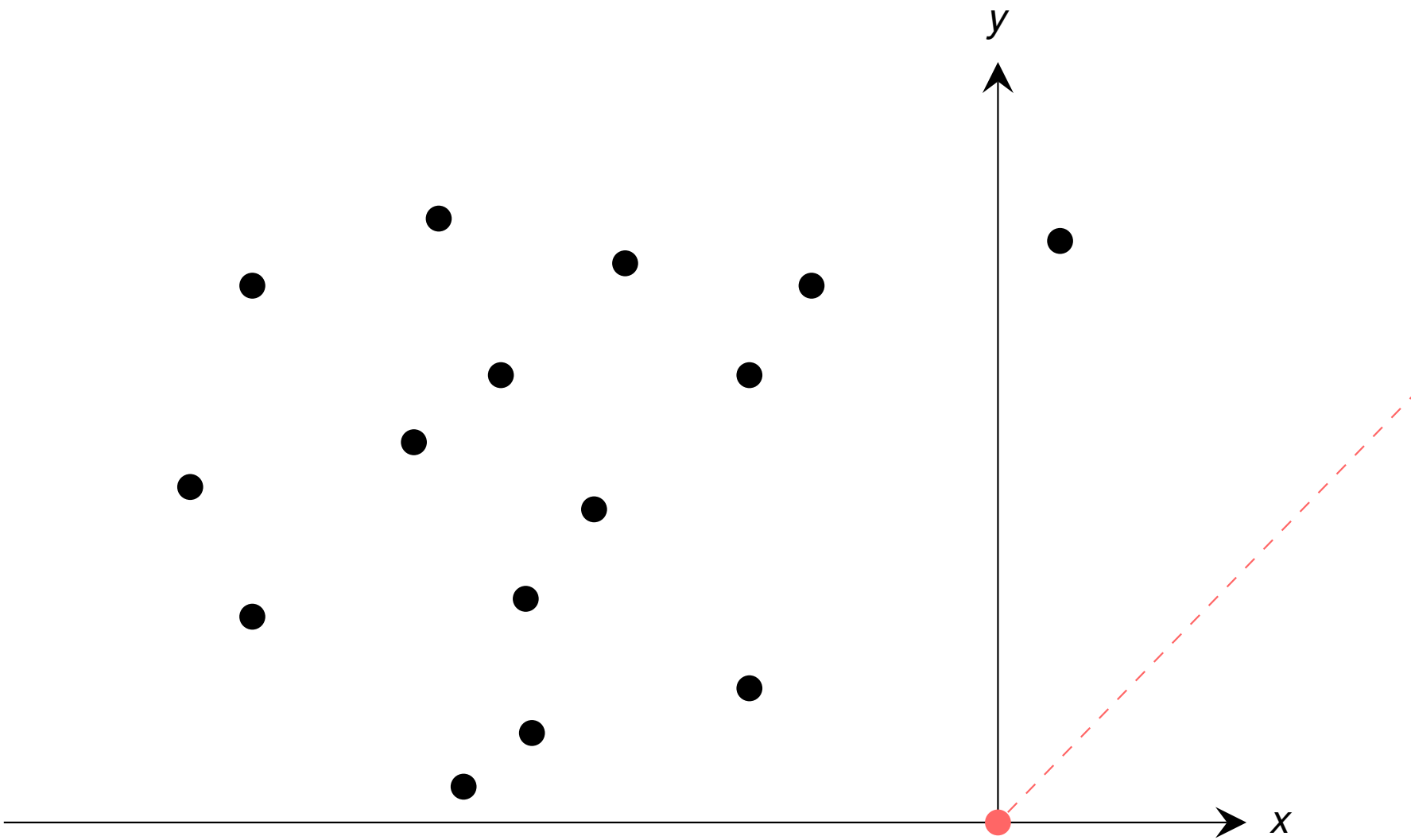

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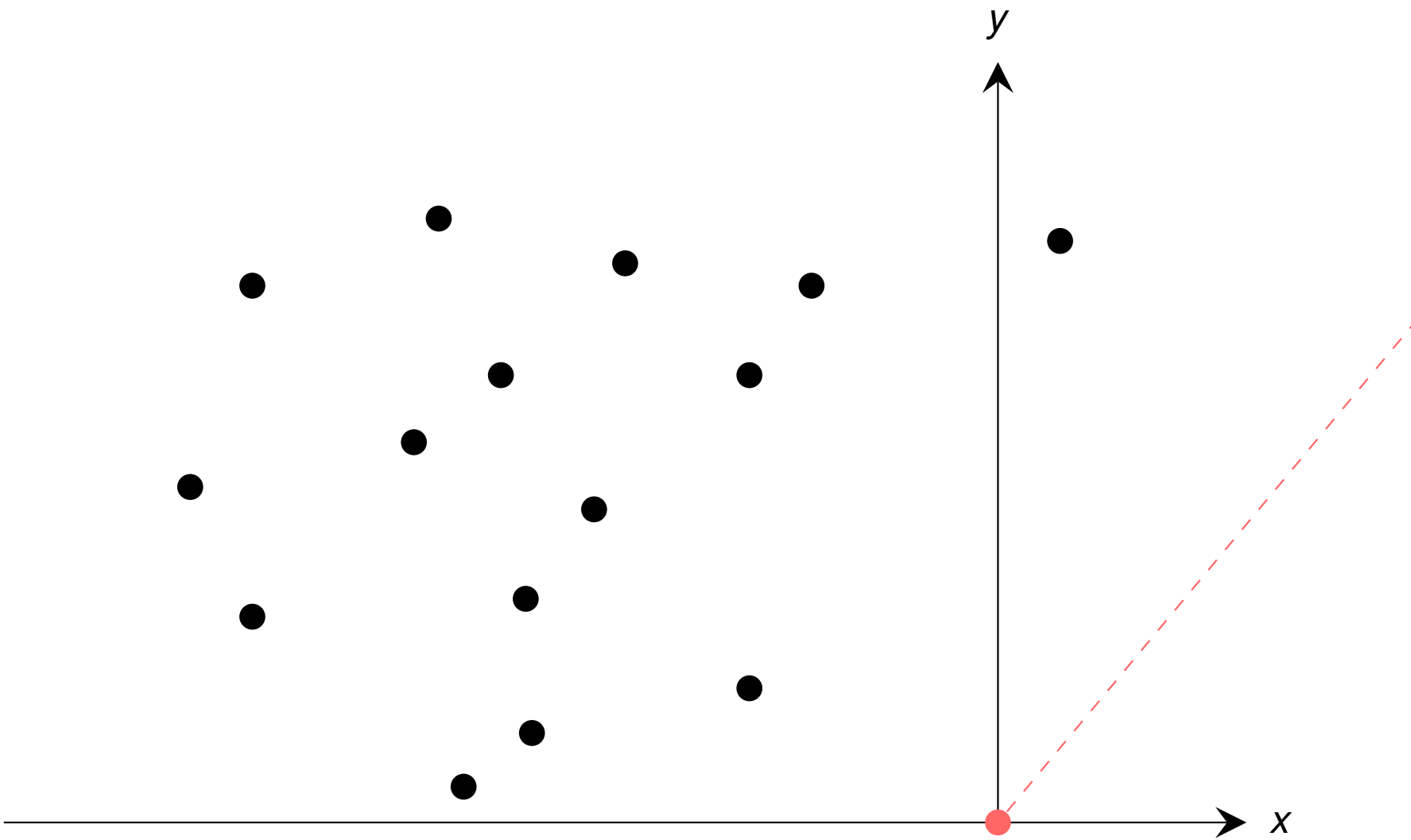

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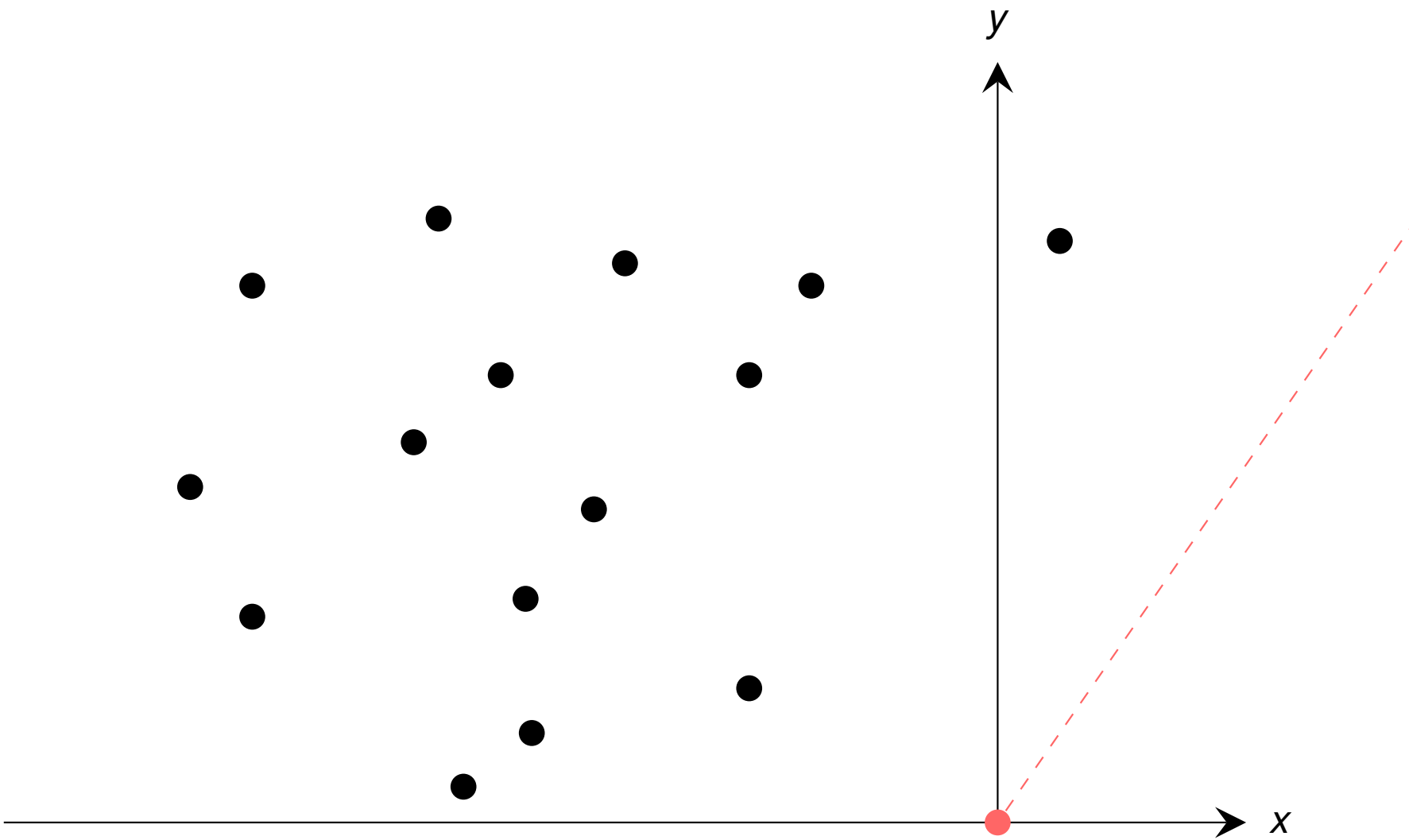
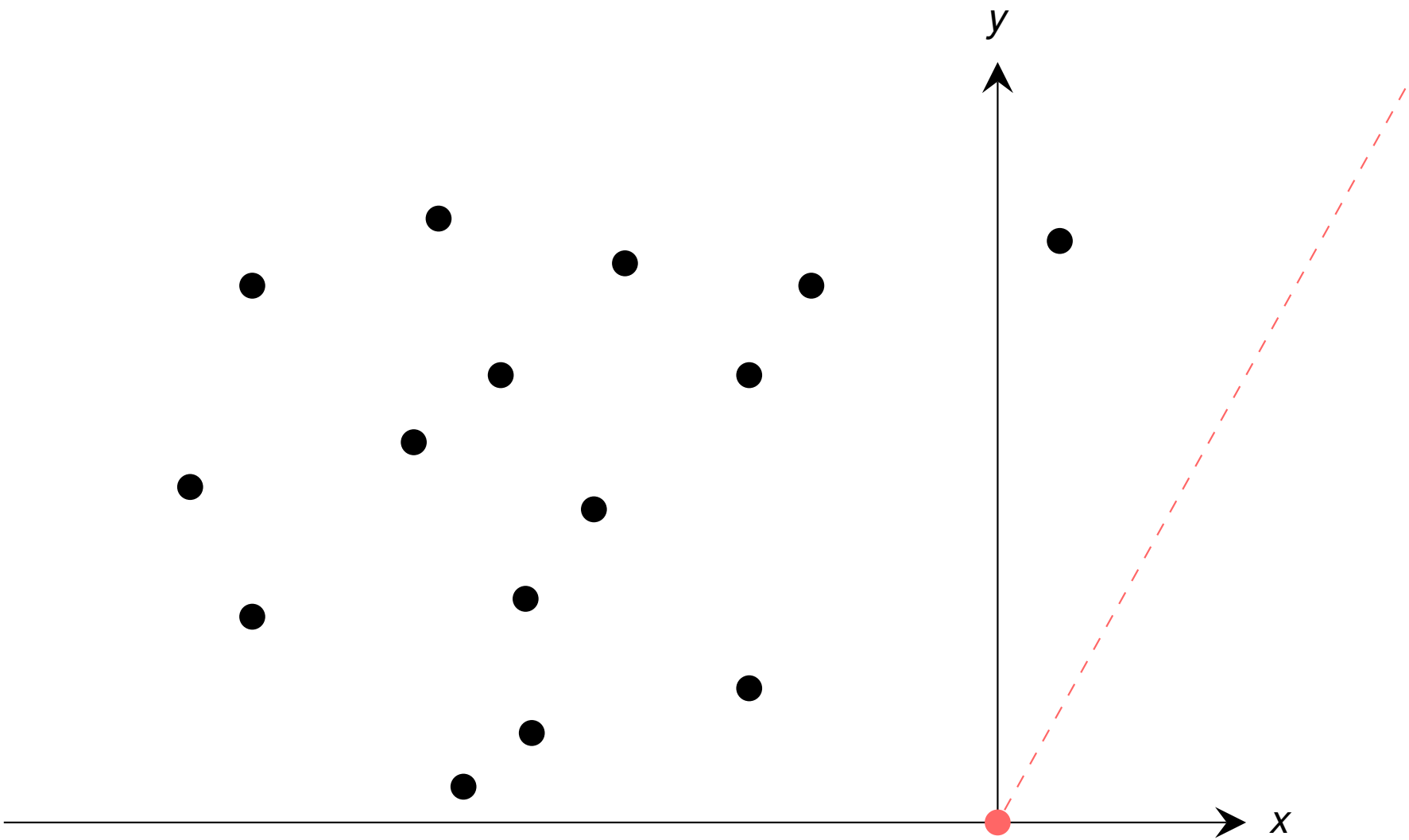


Execution of Jarvis' March



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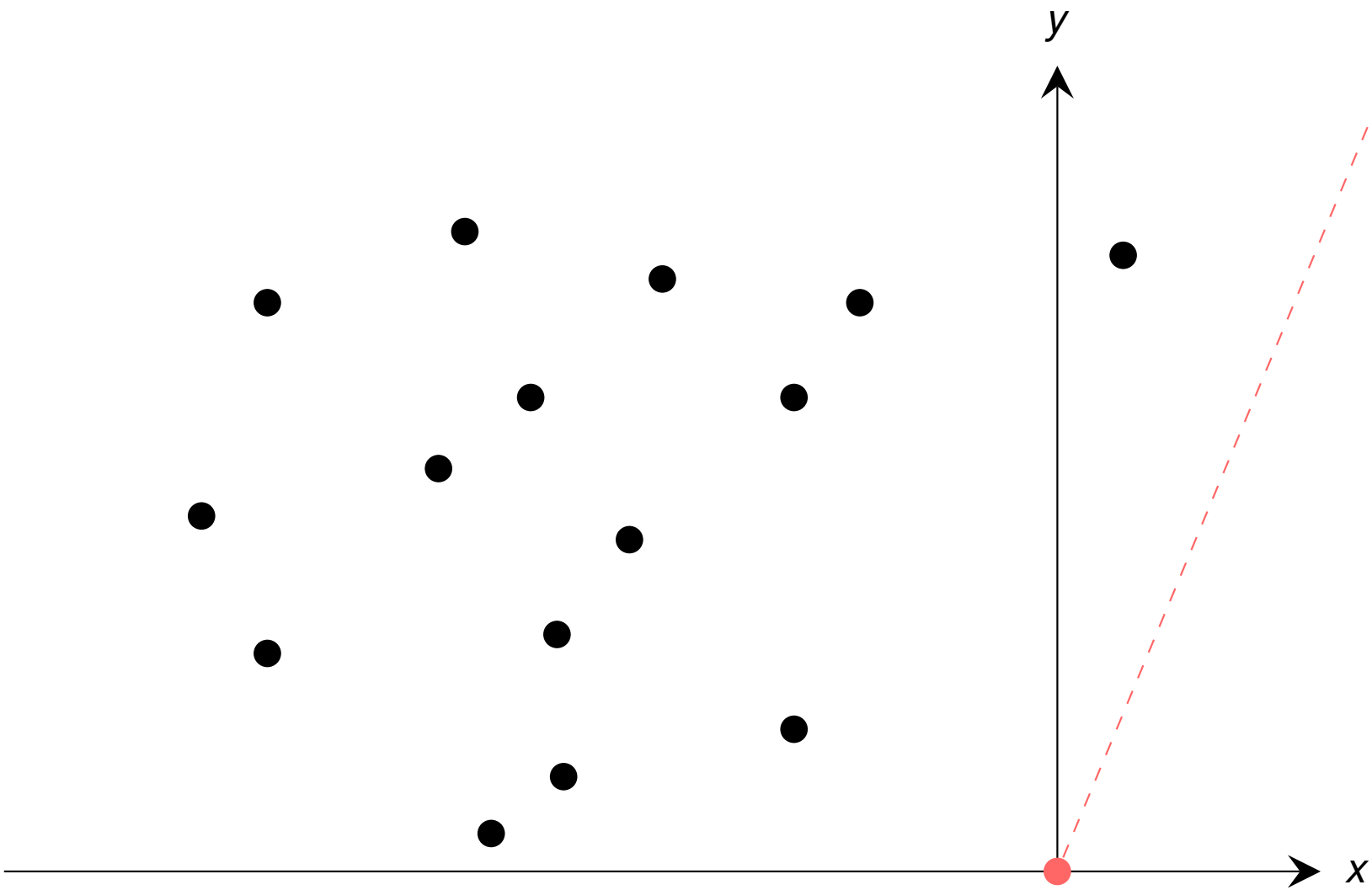

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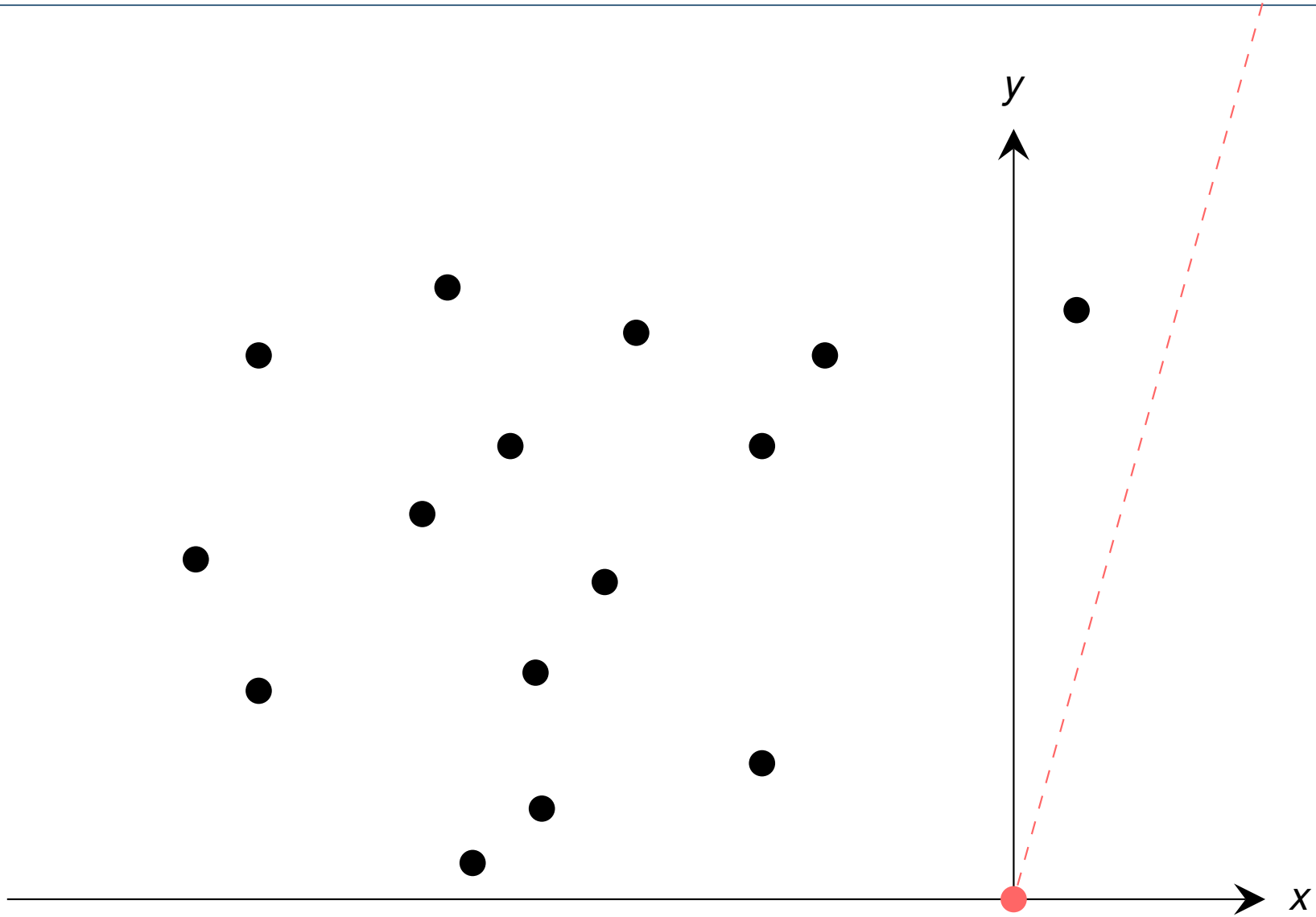

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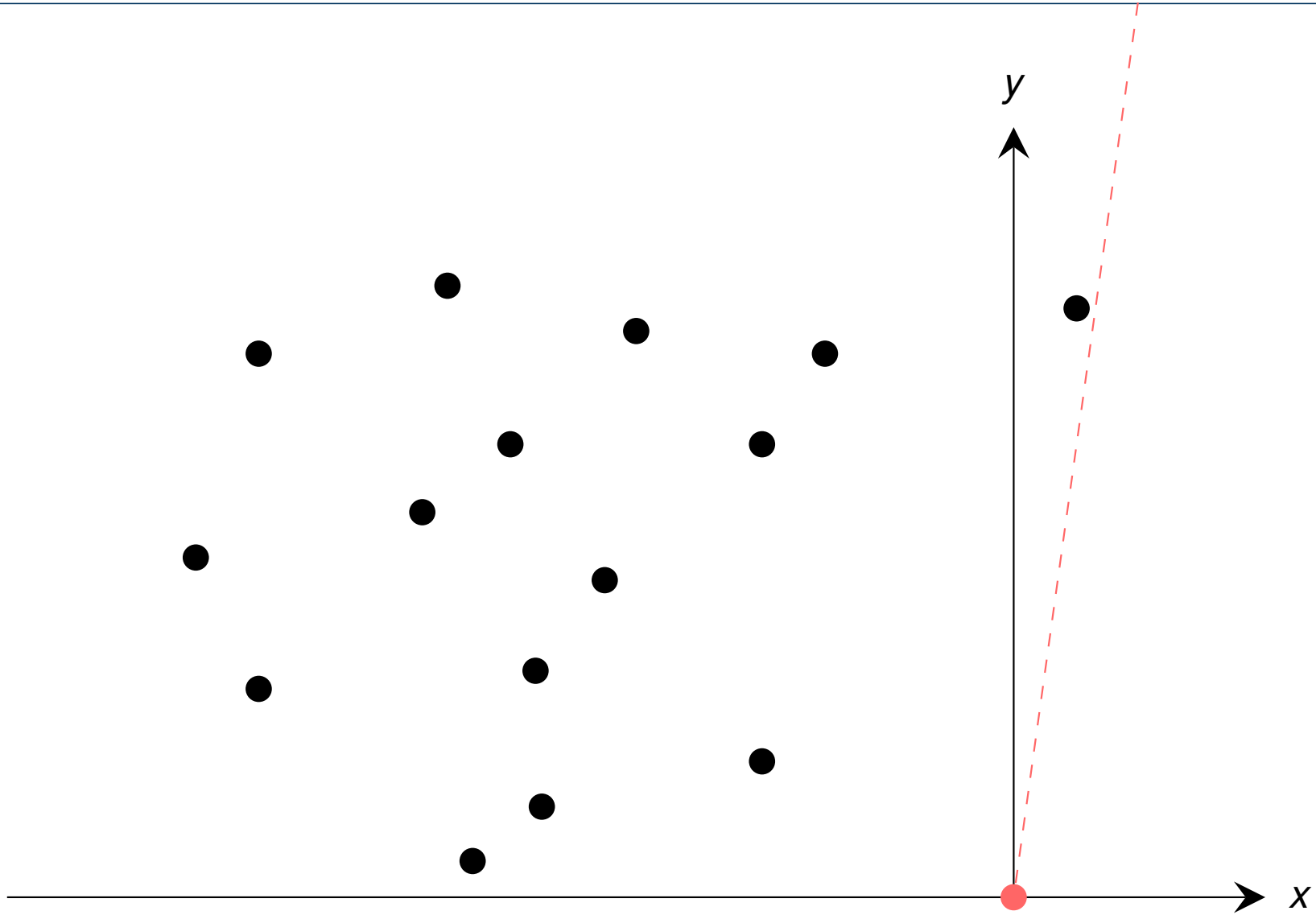

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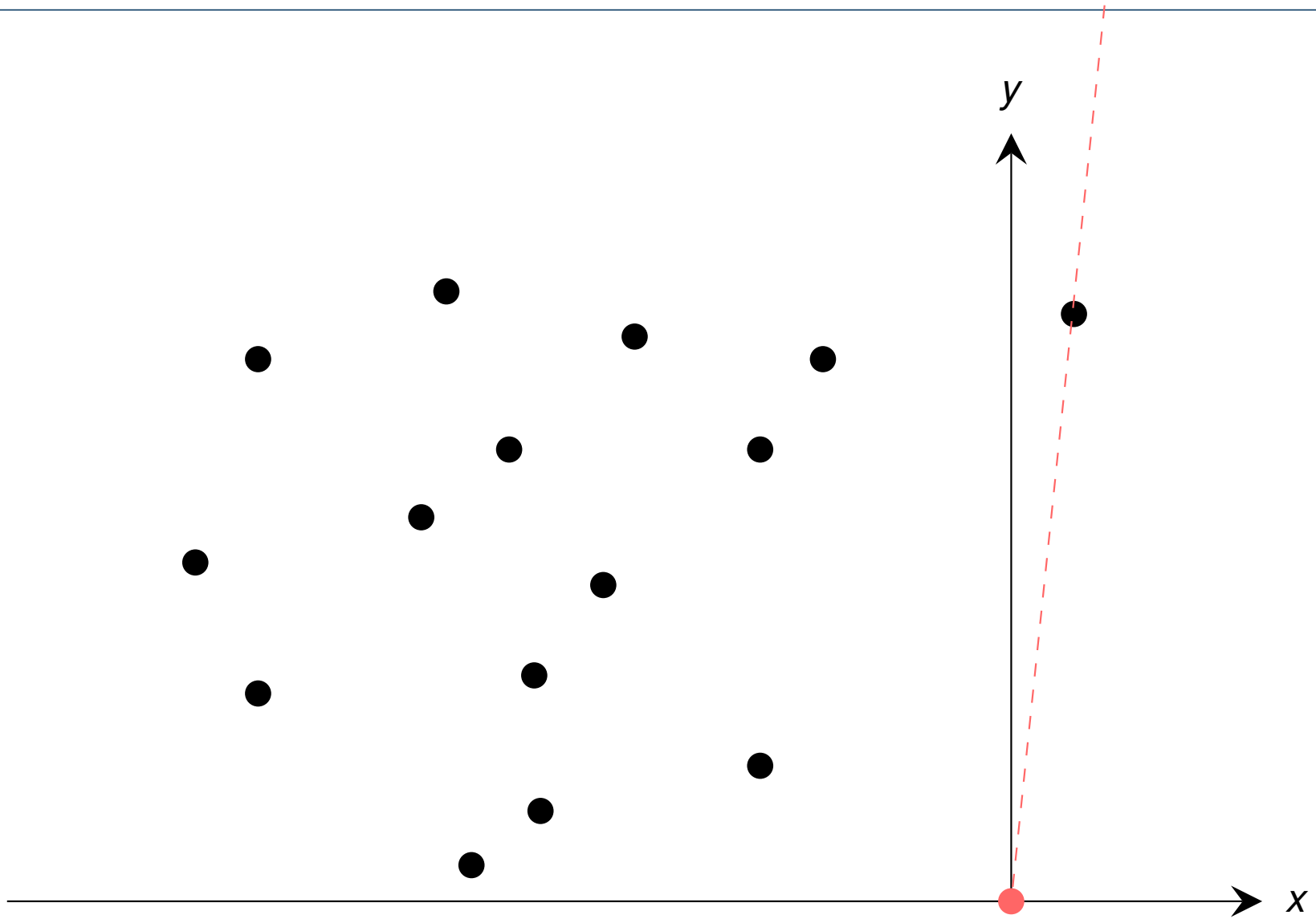

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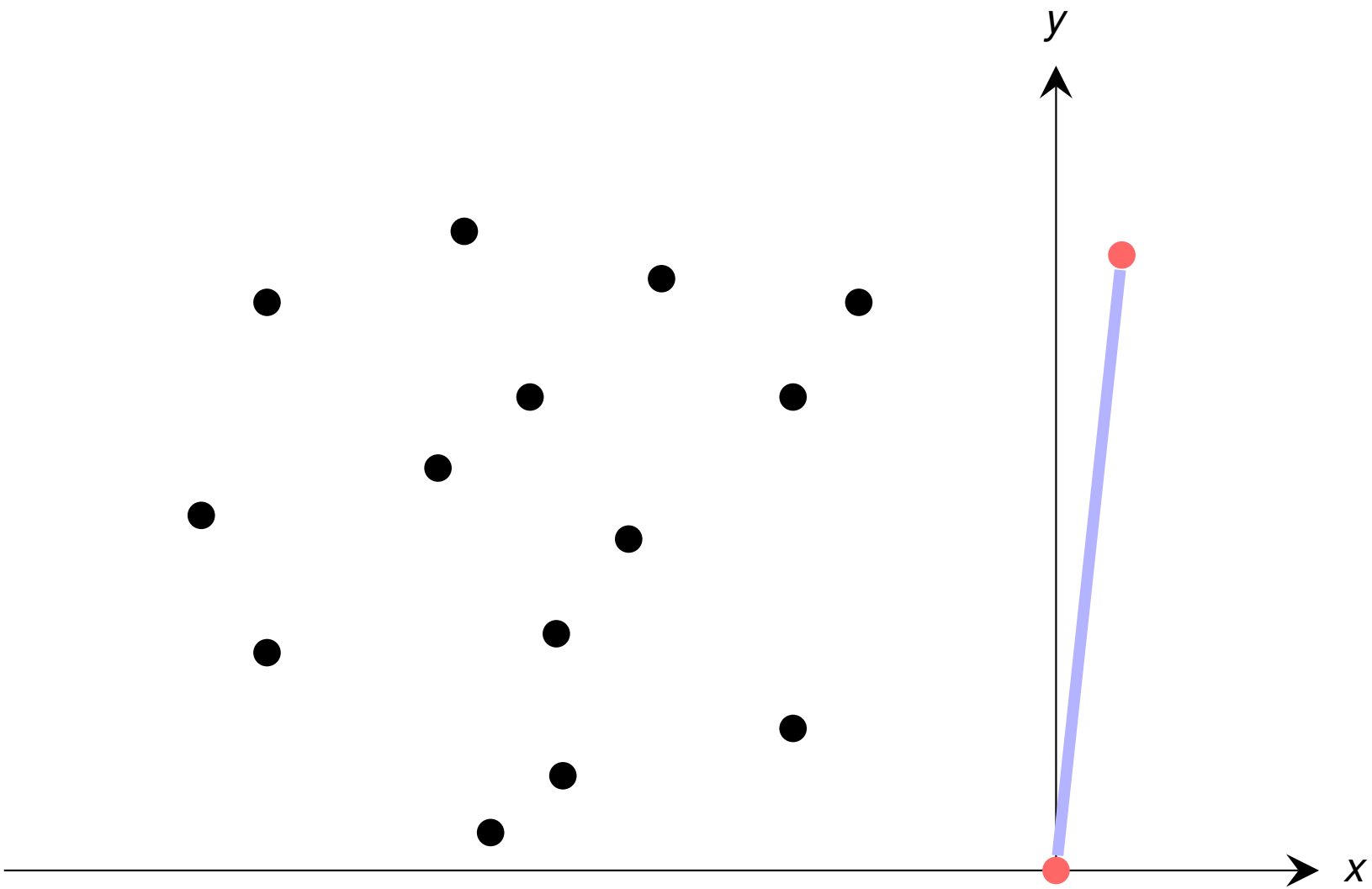

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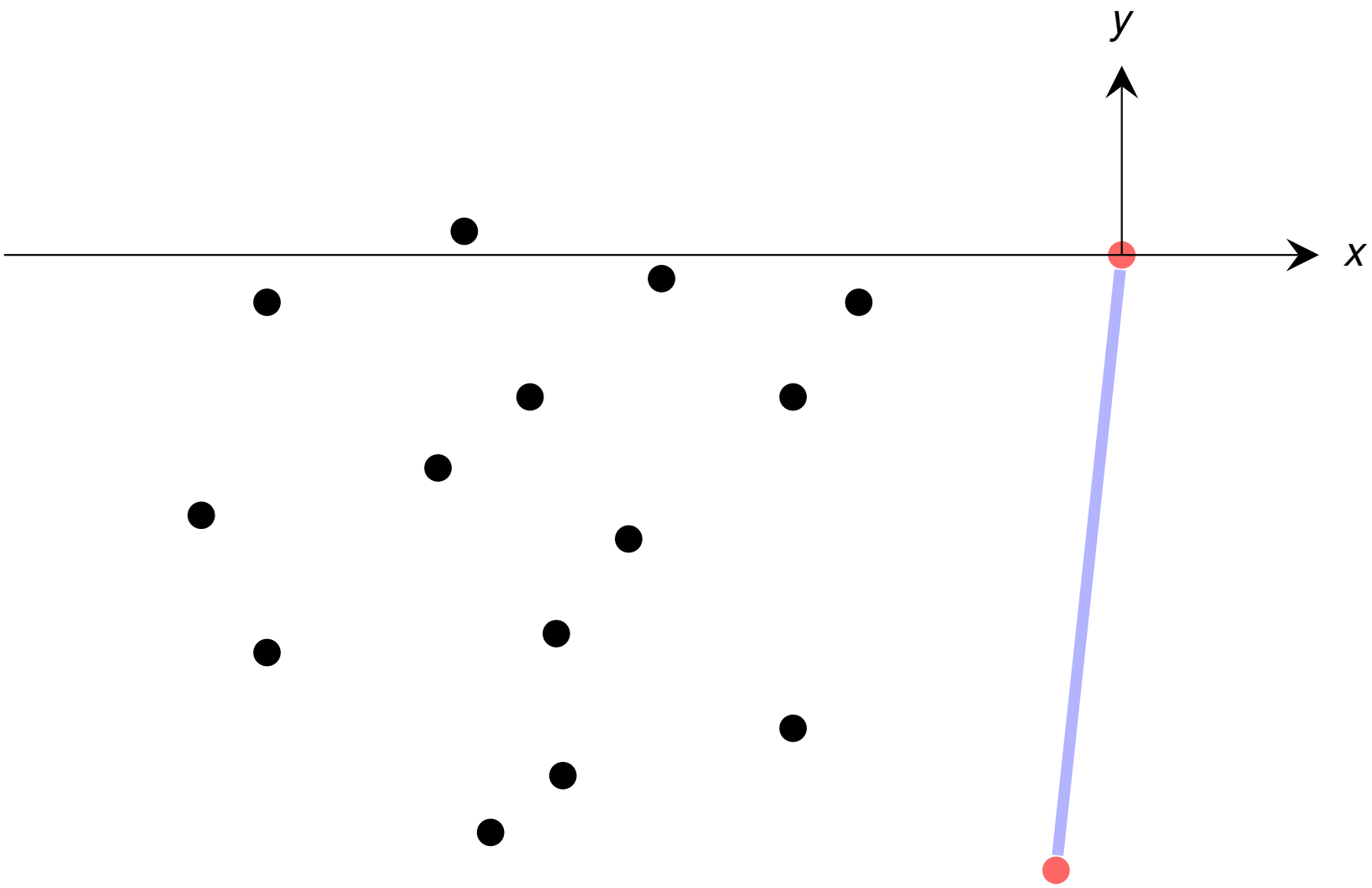

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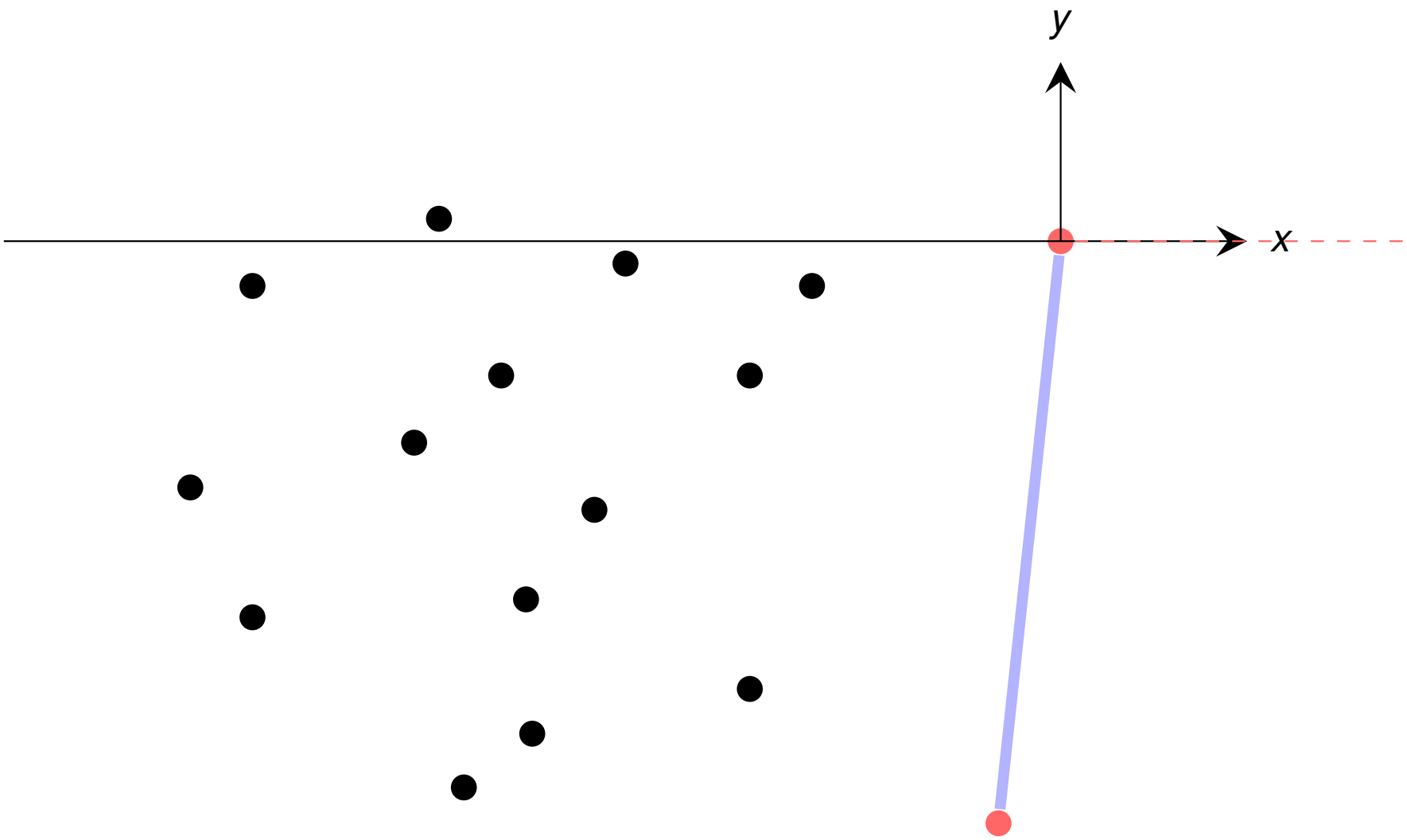

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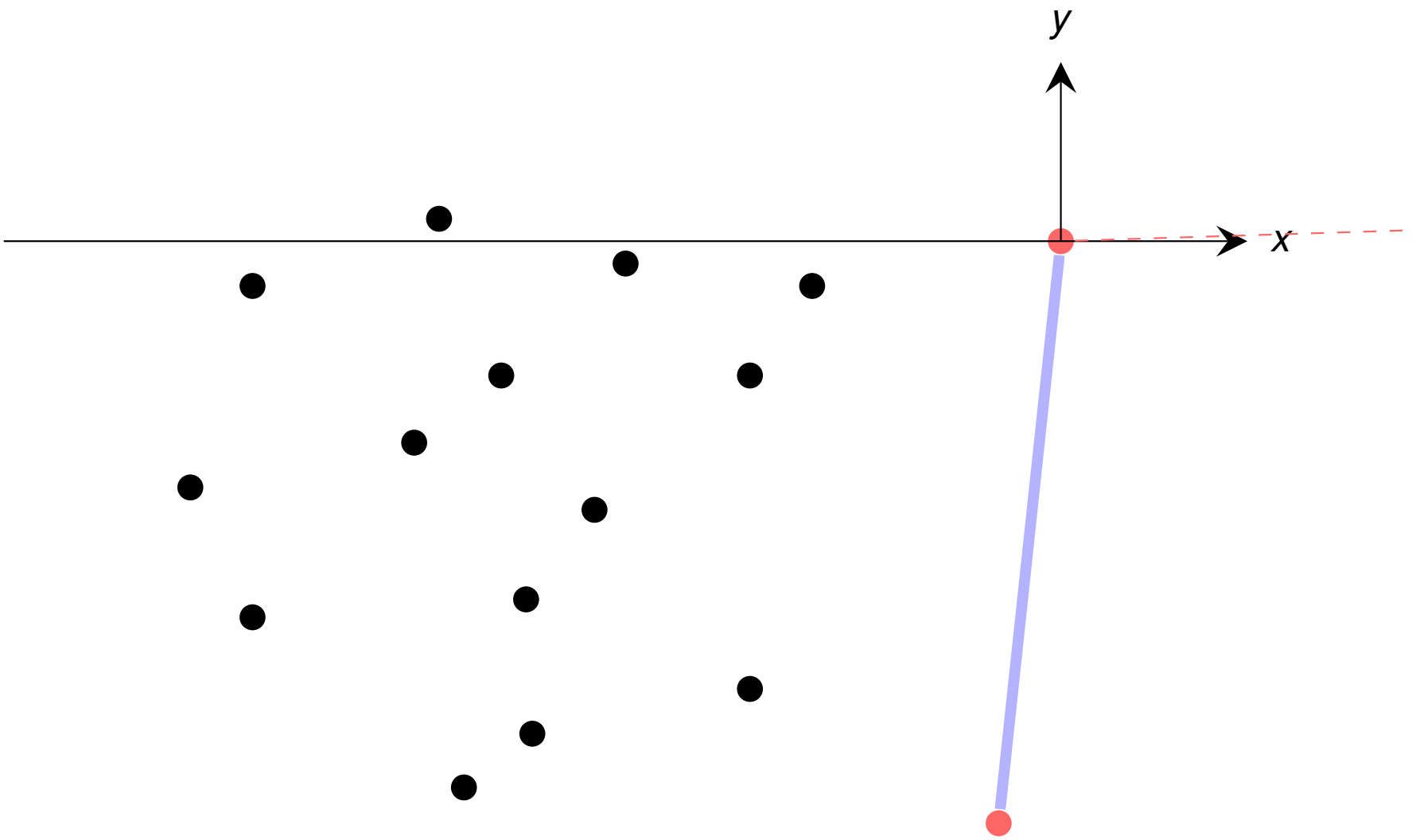

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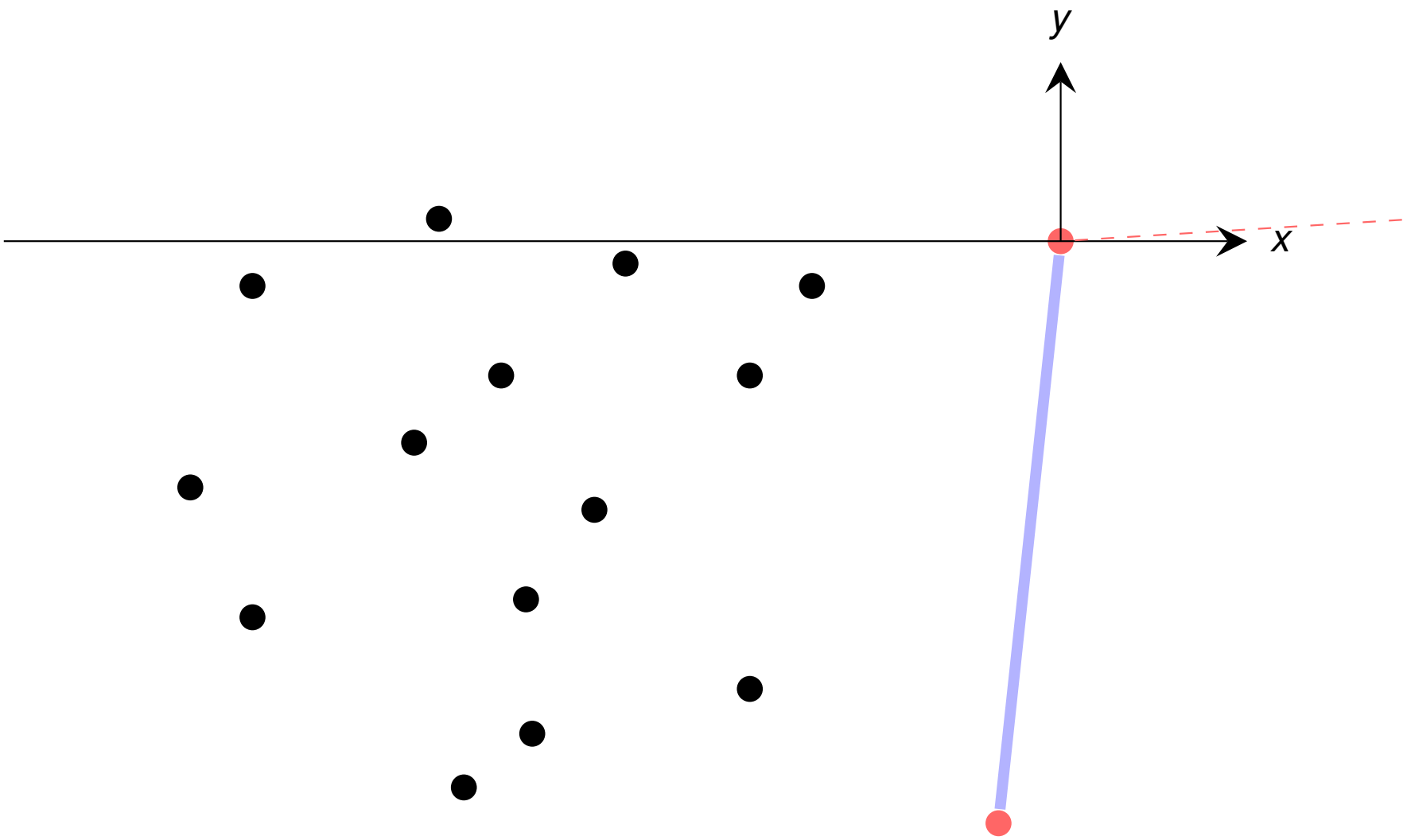

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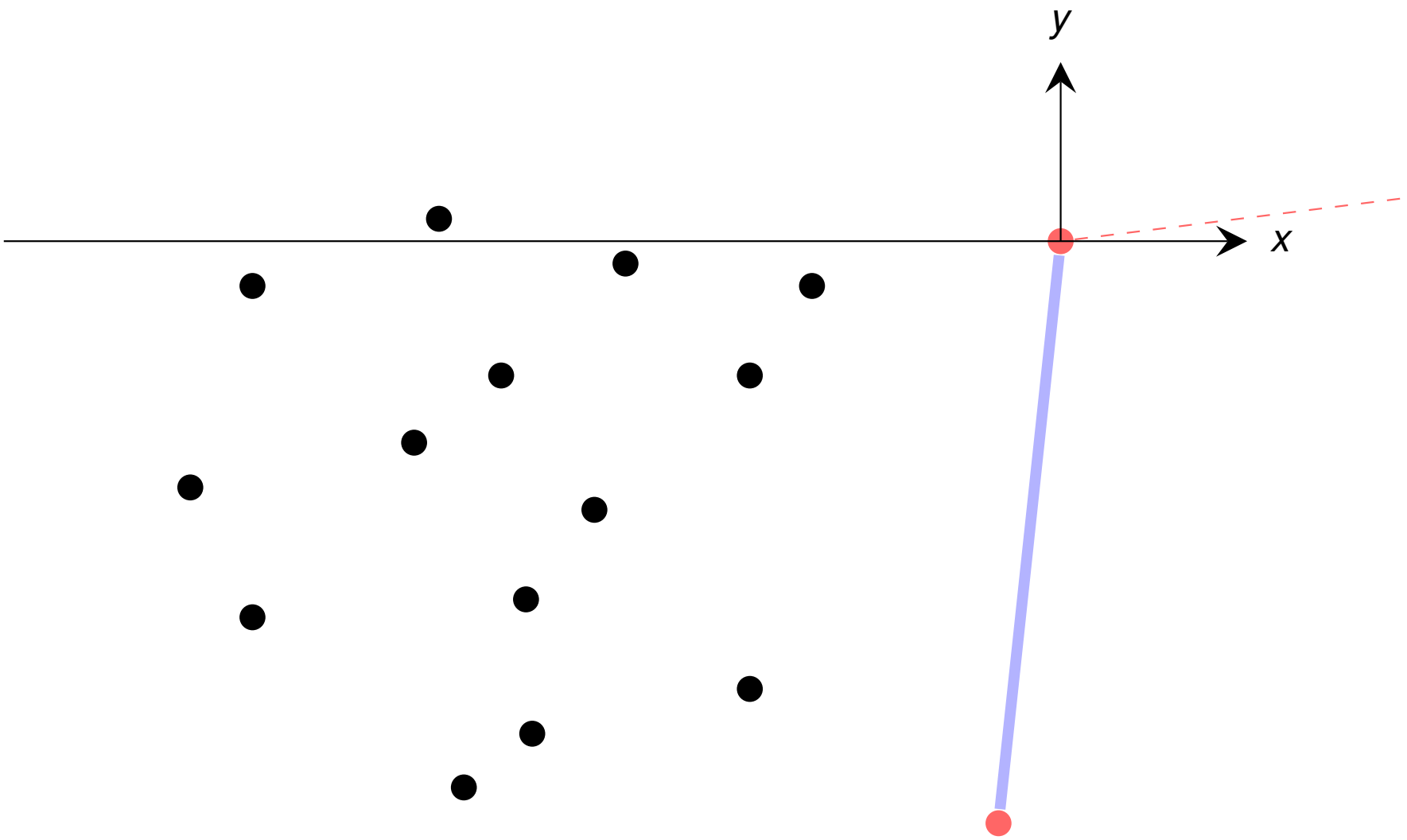

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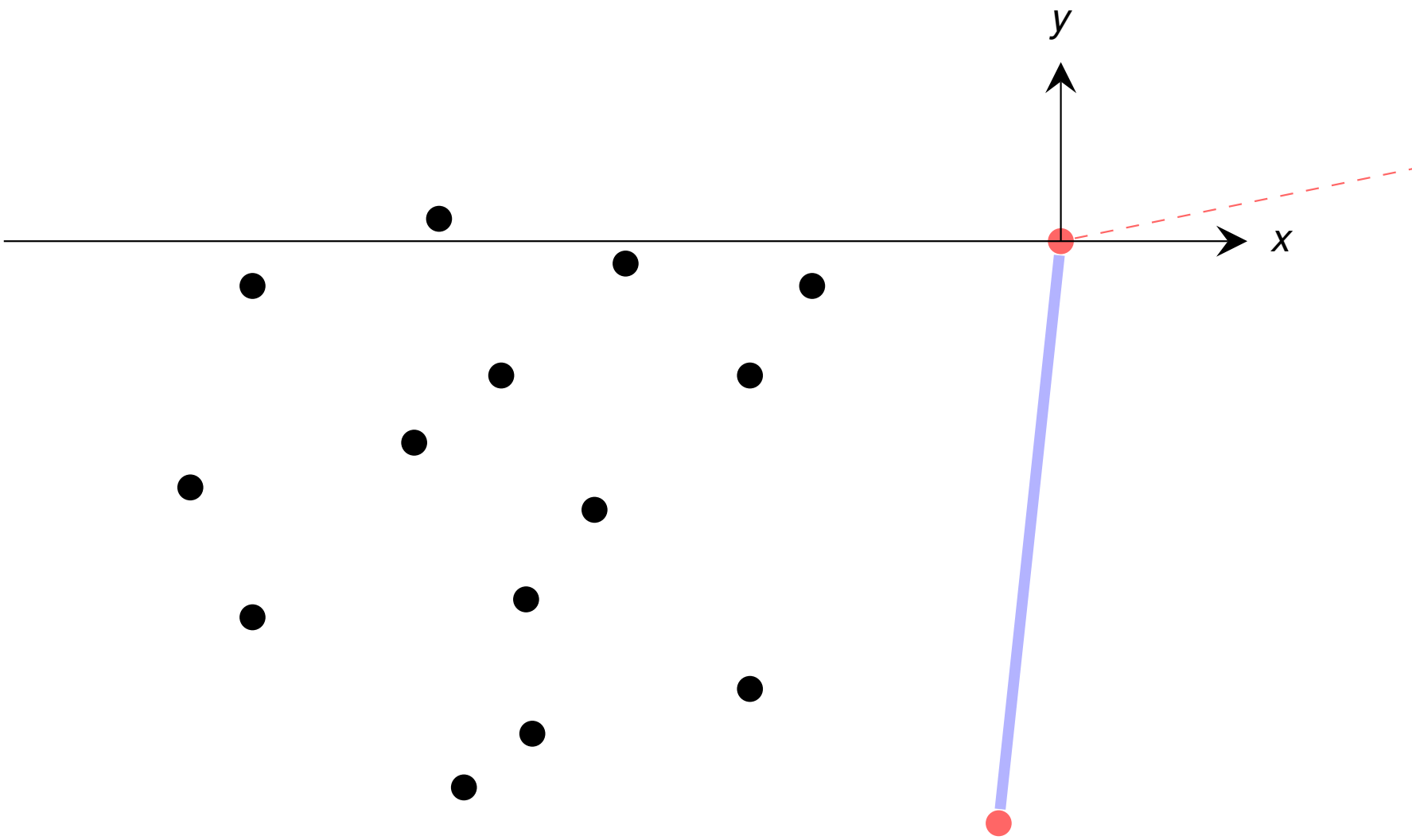

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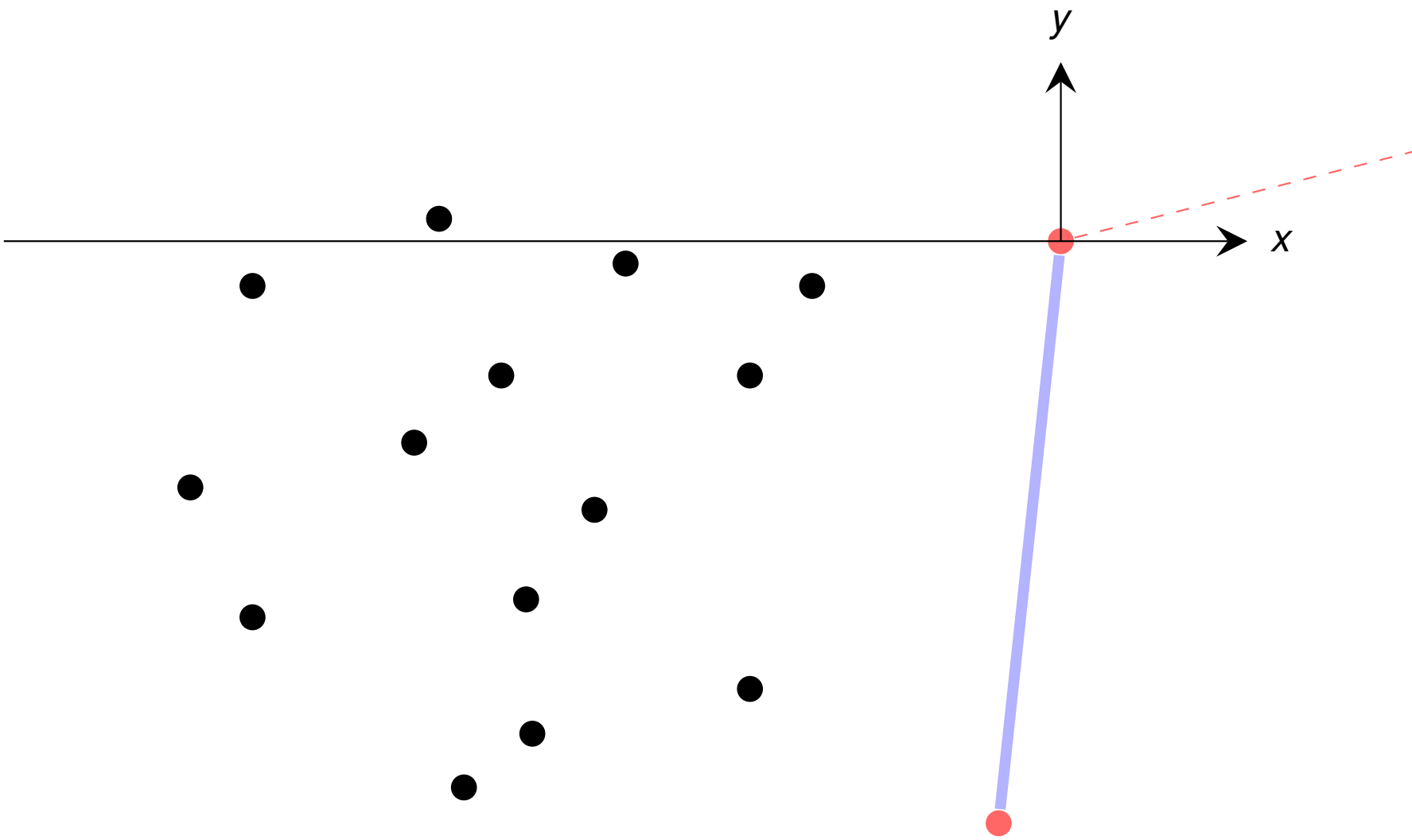

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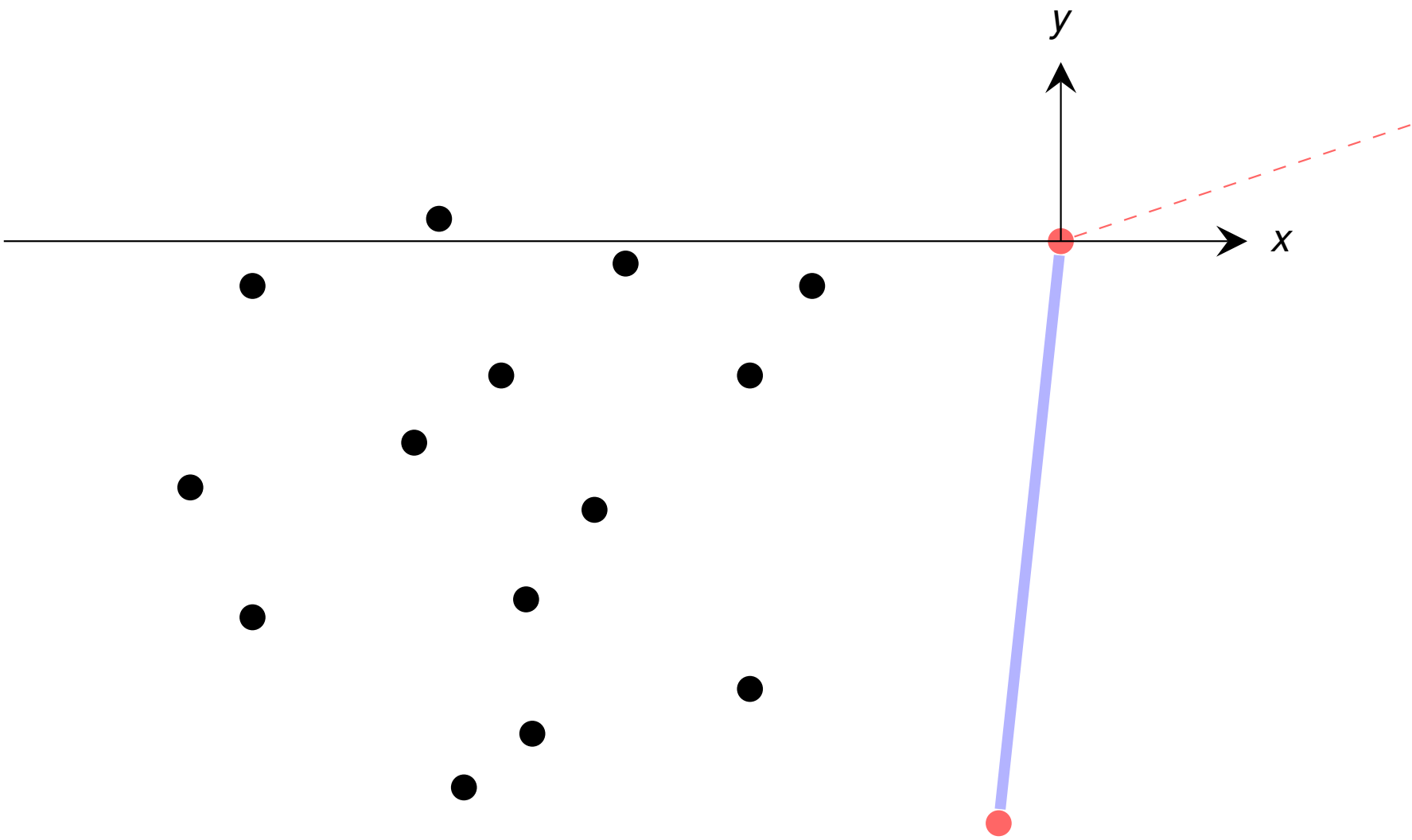

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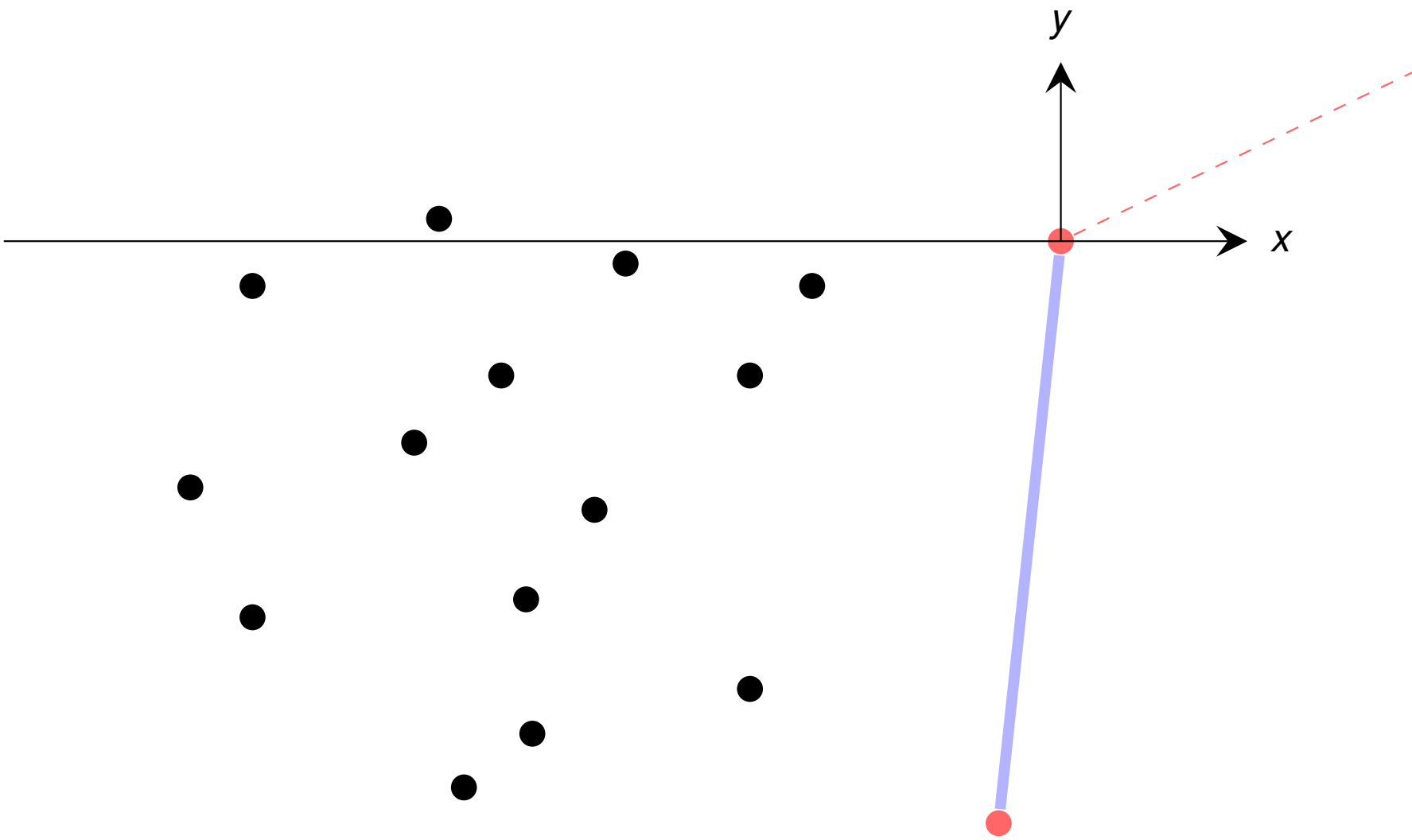

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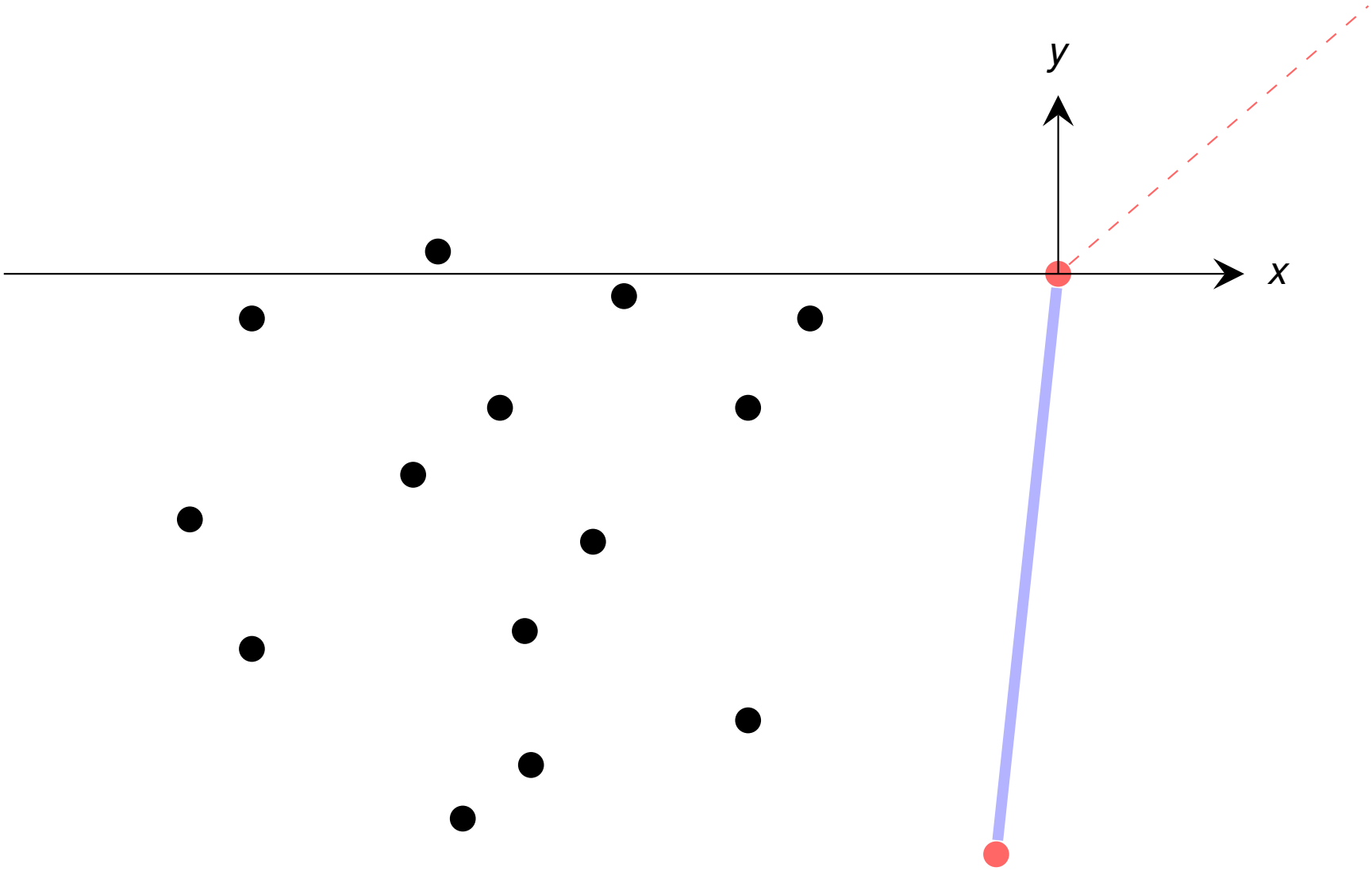

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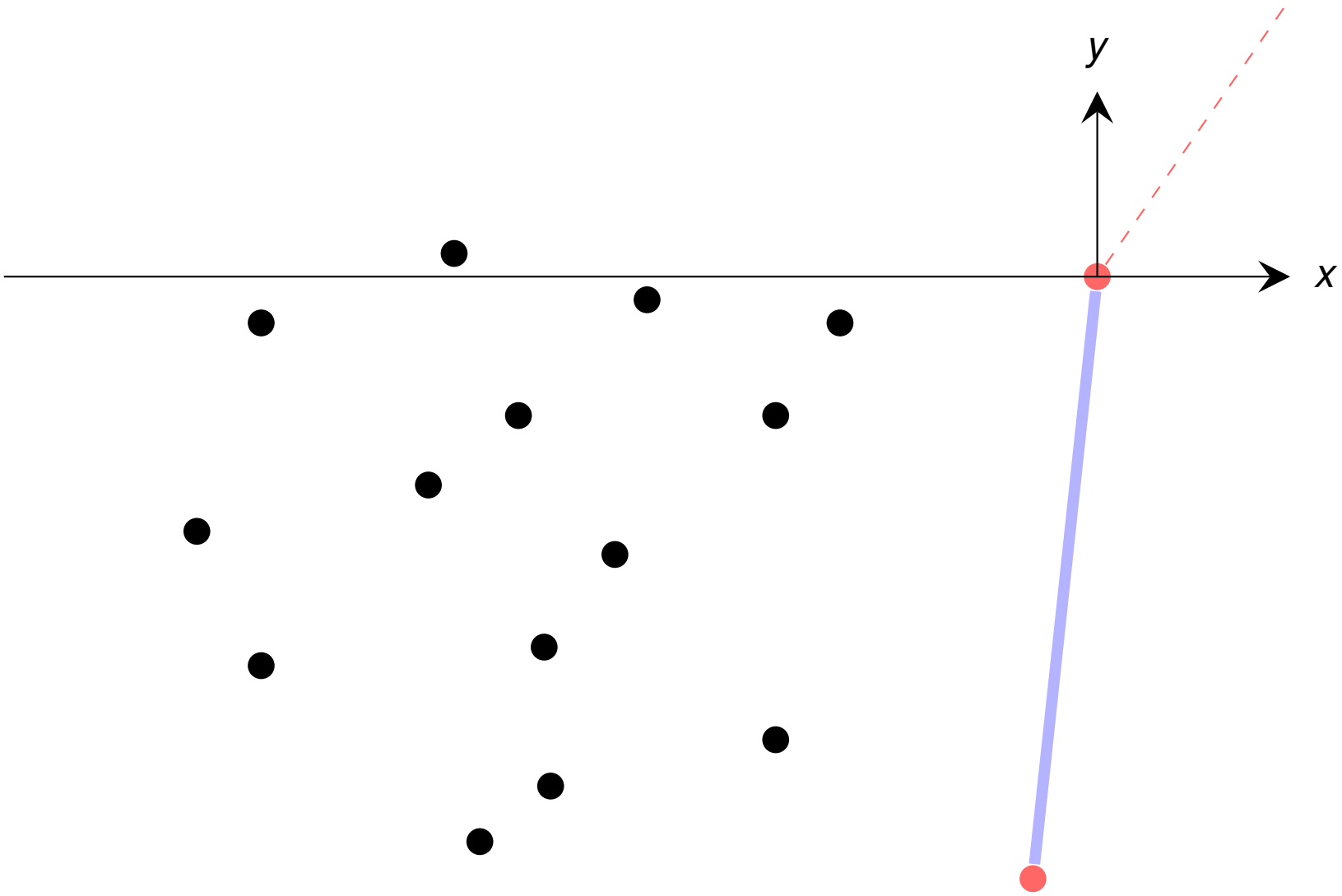

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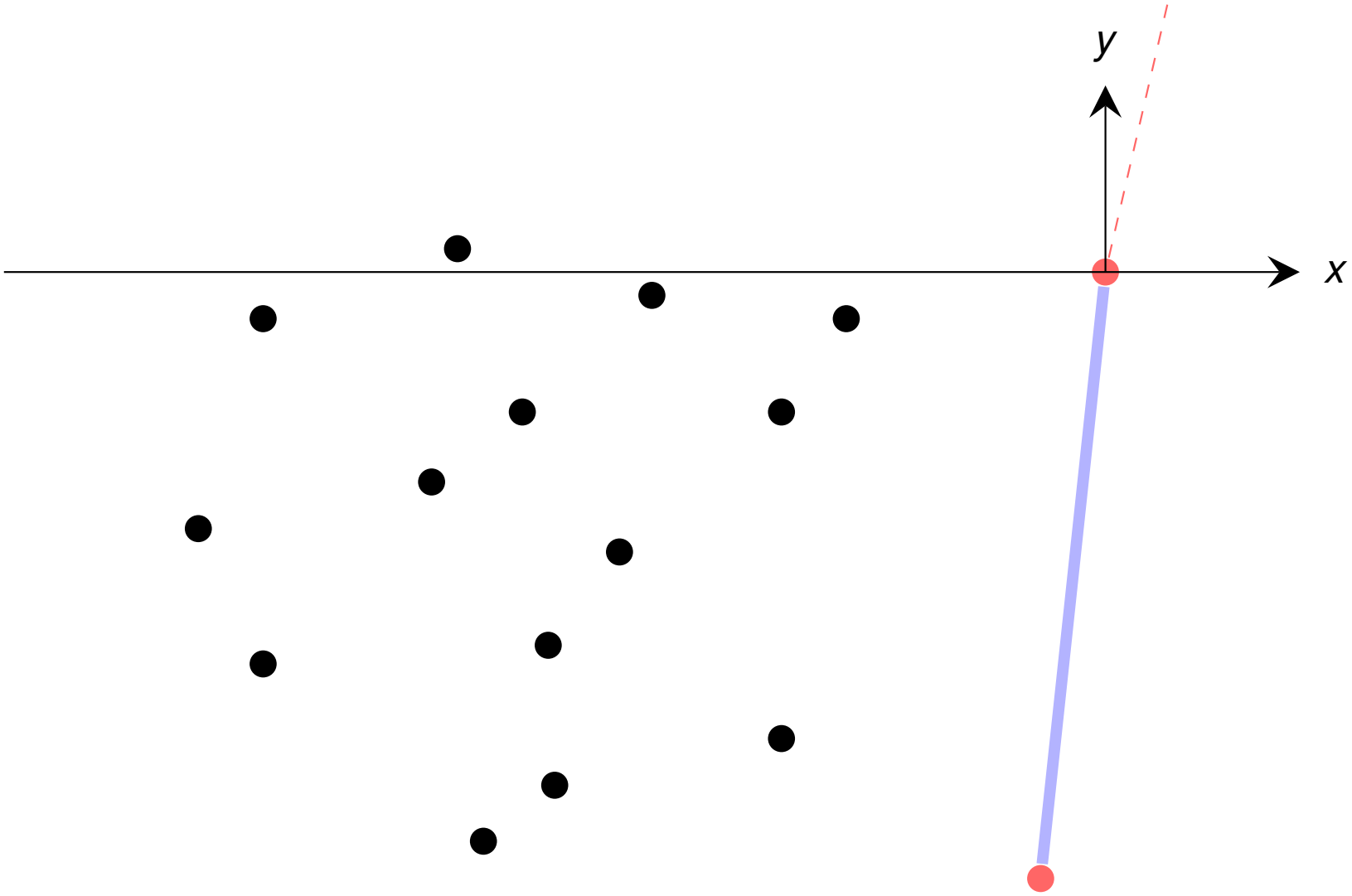

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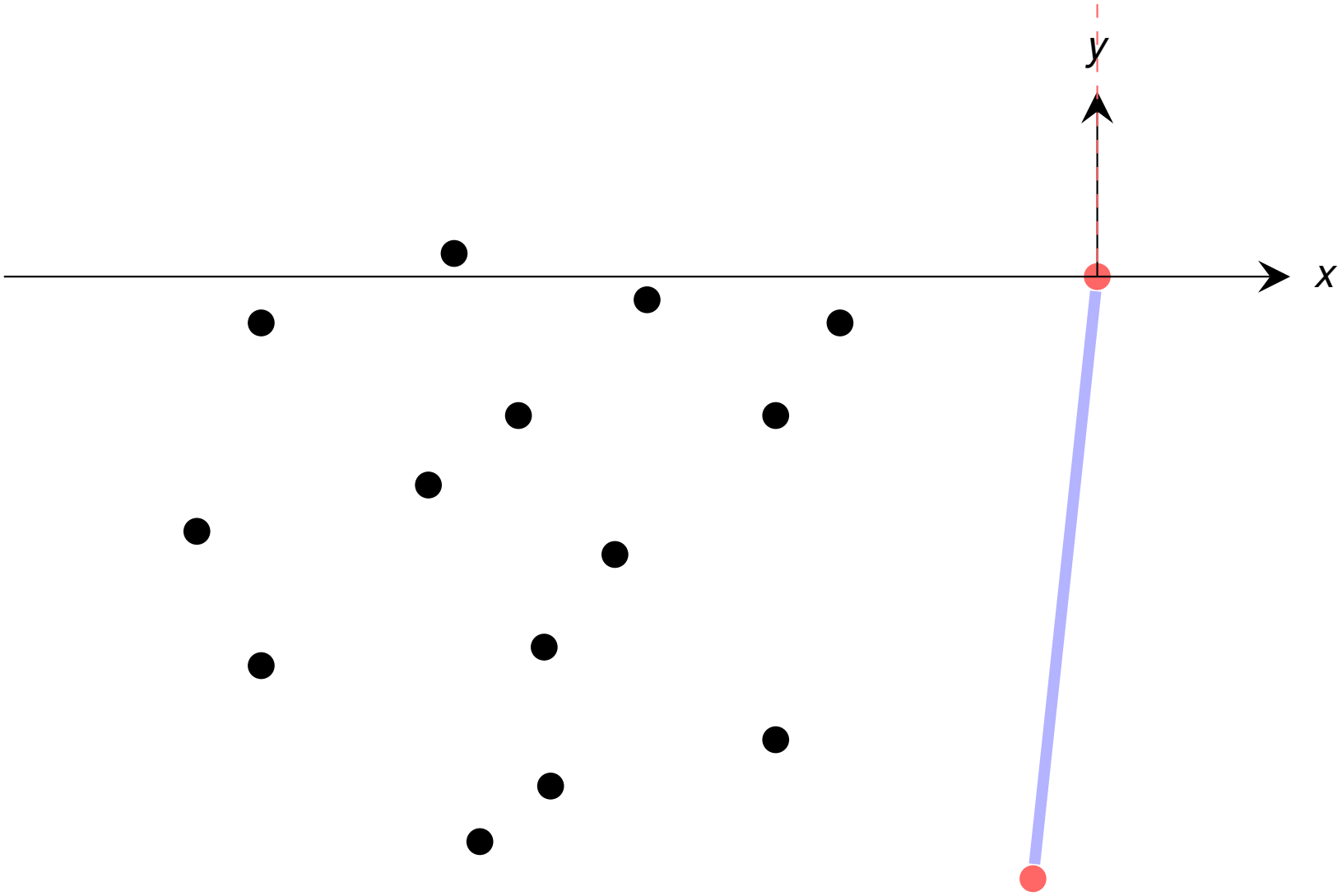

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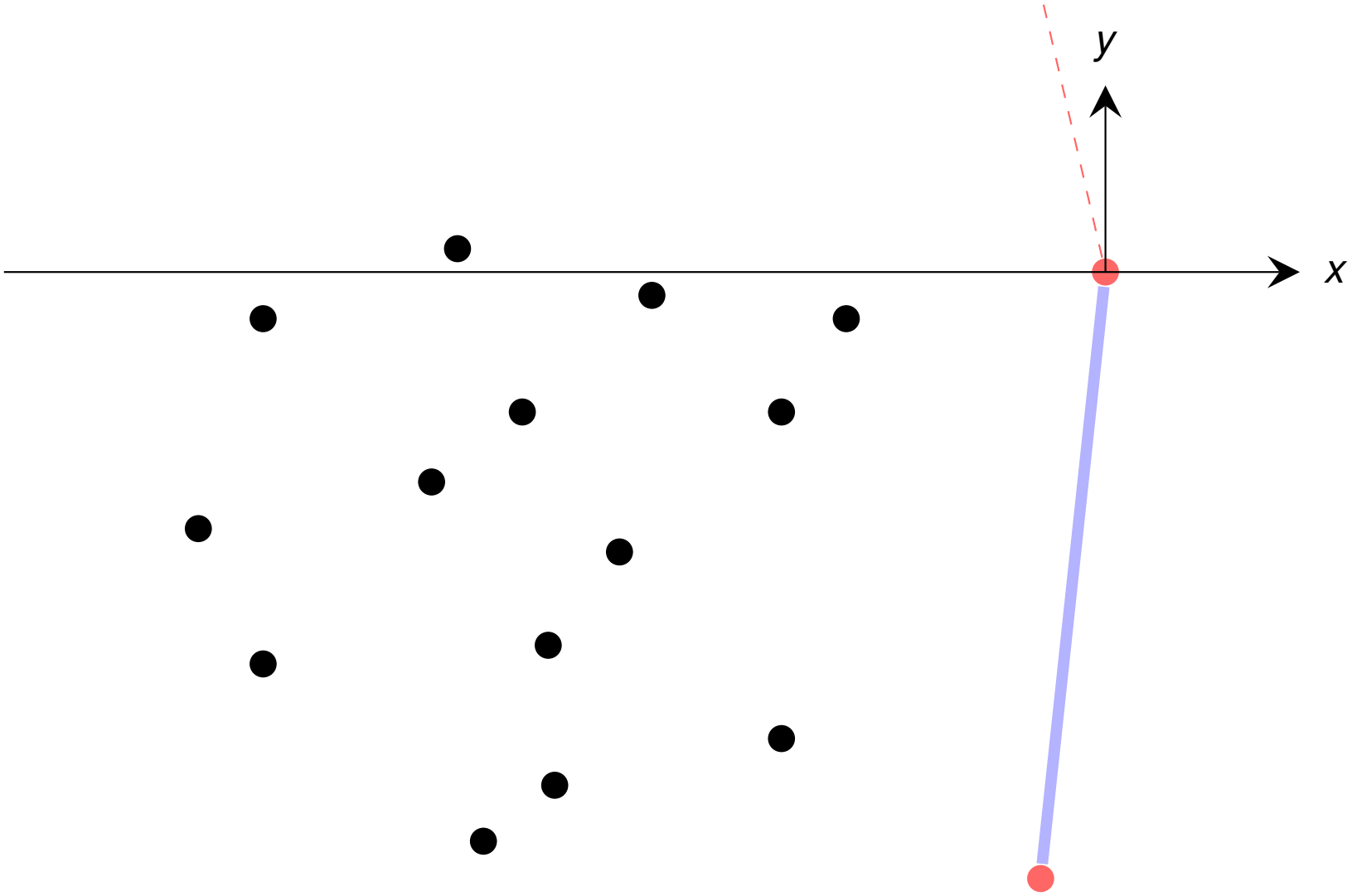

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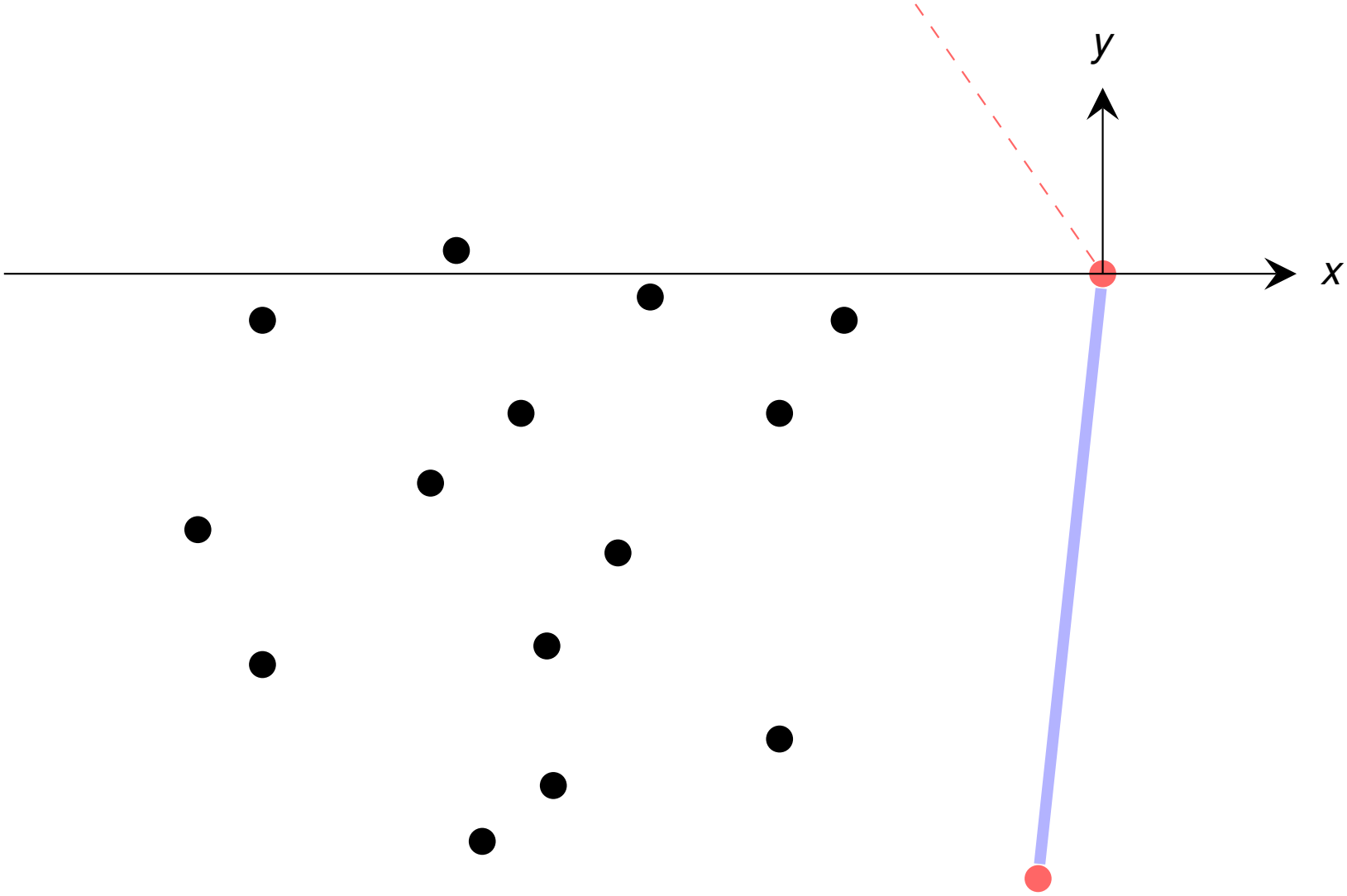

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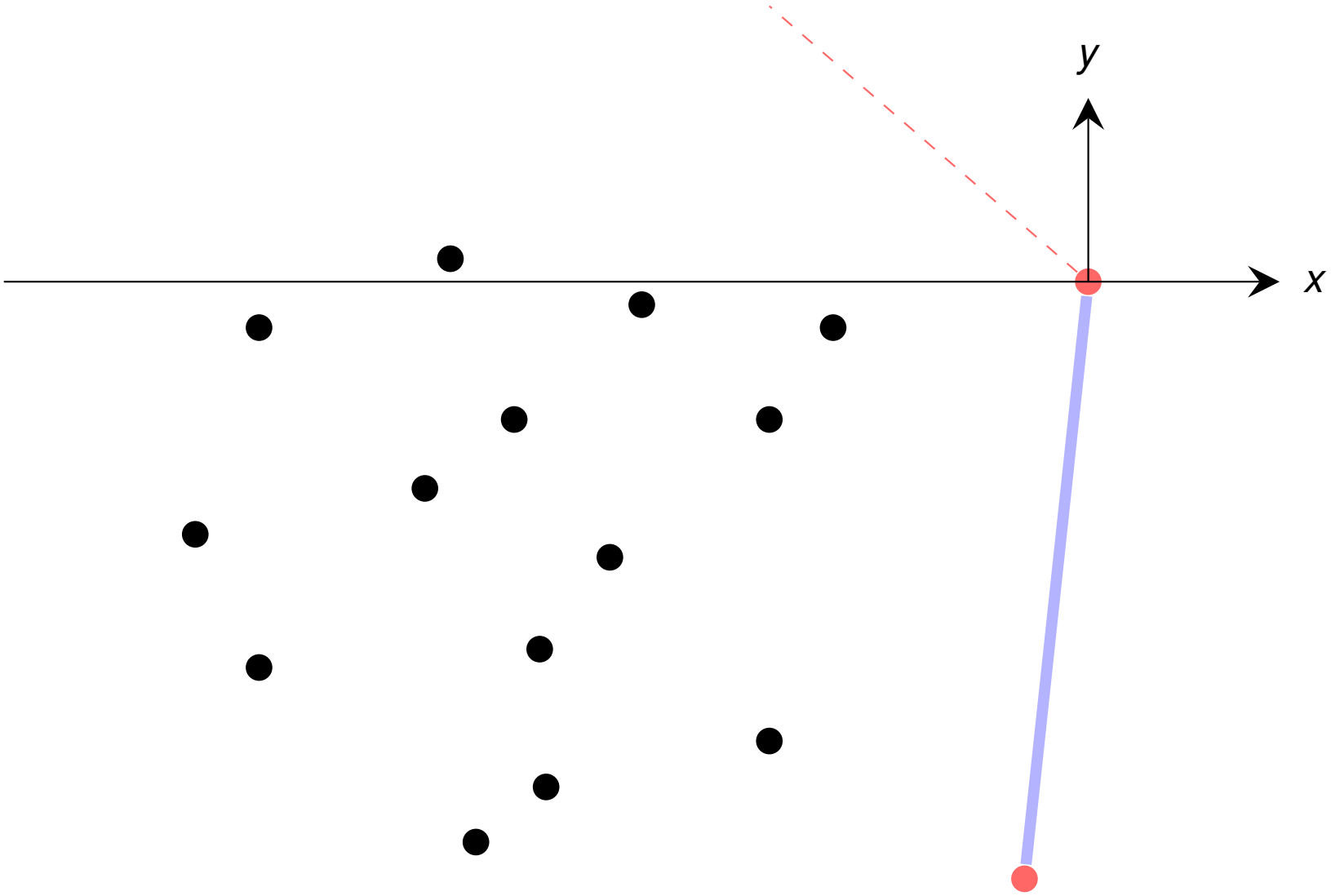

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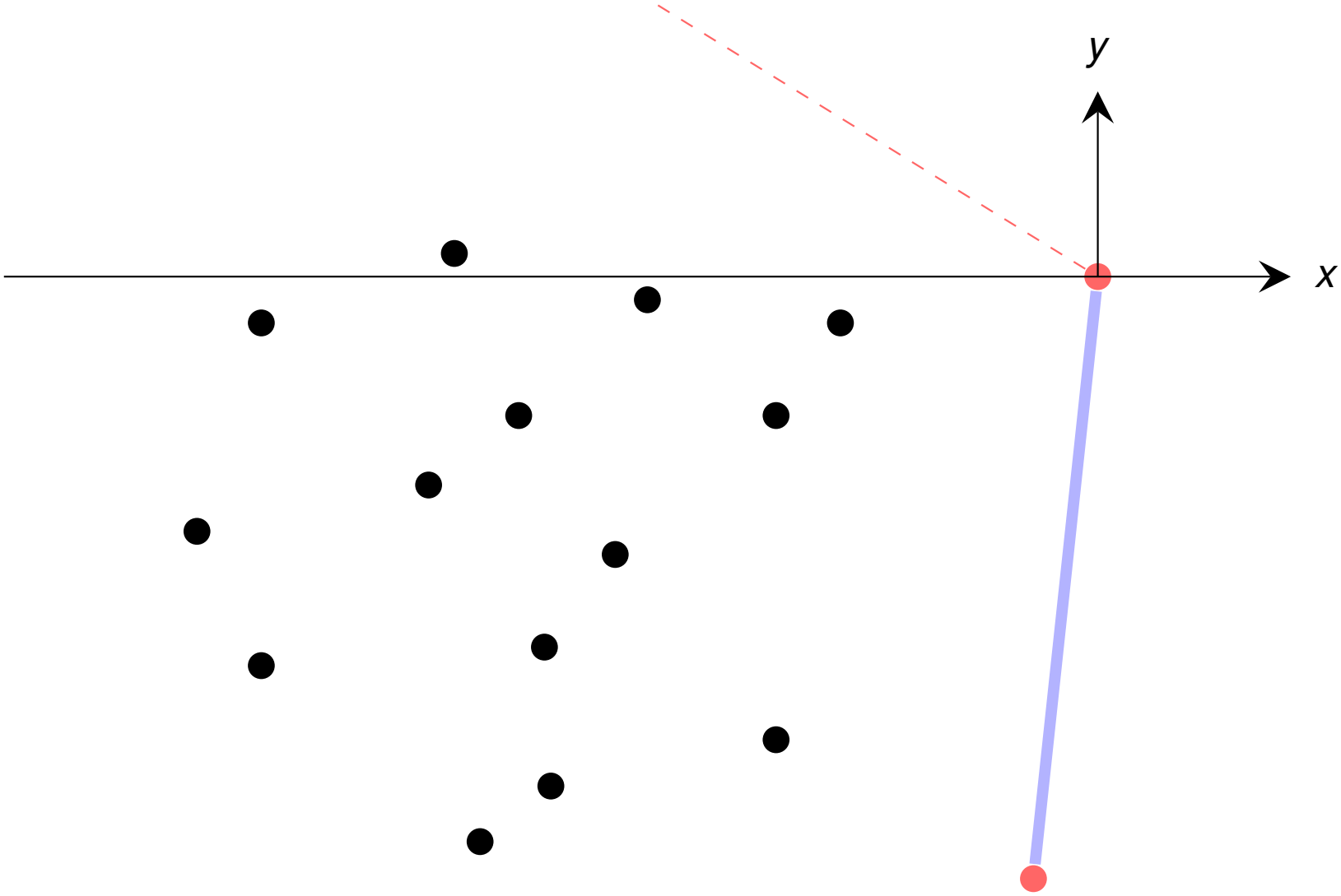

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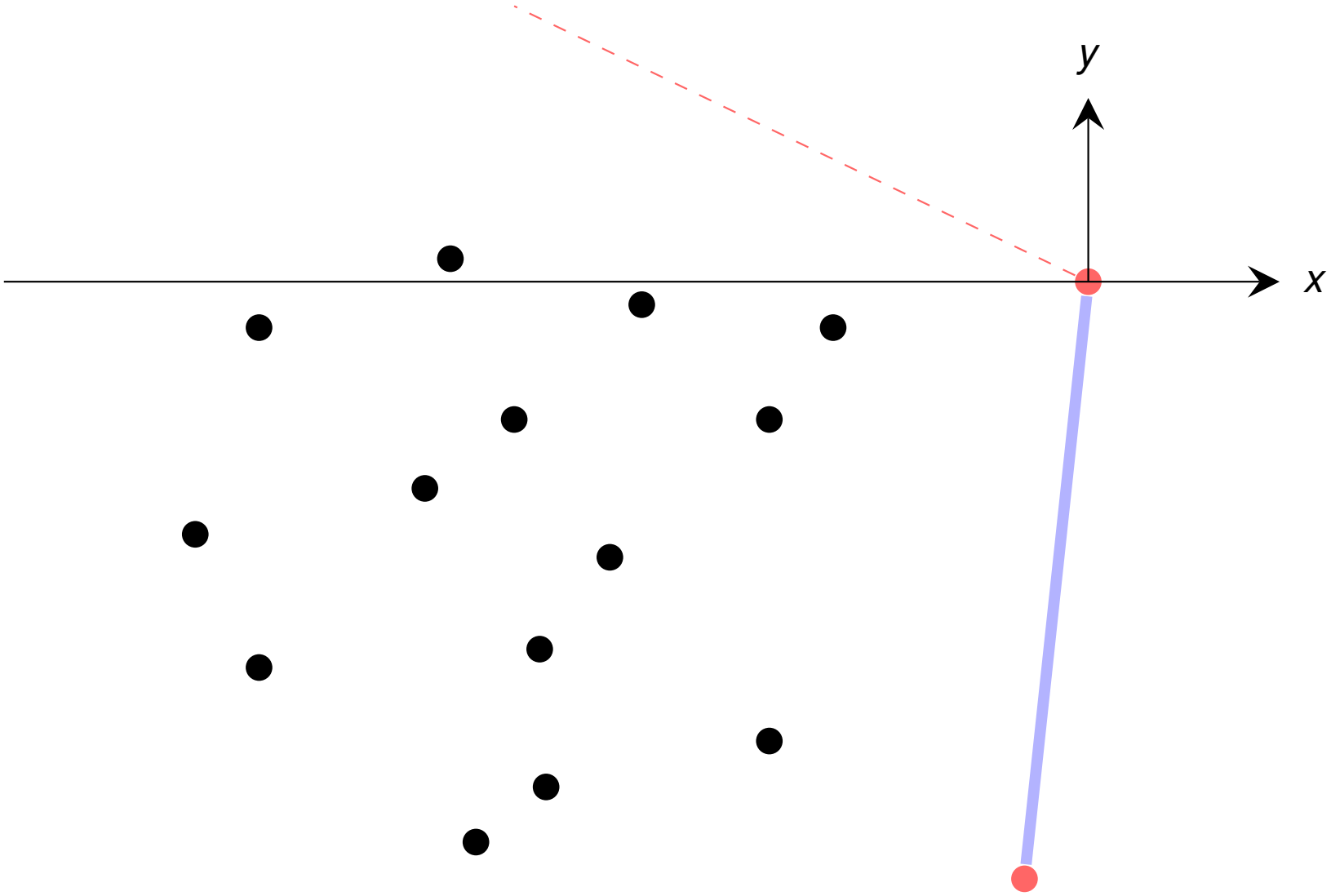

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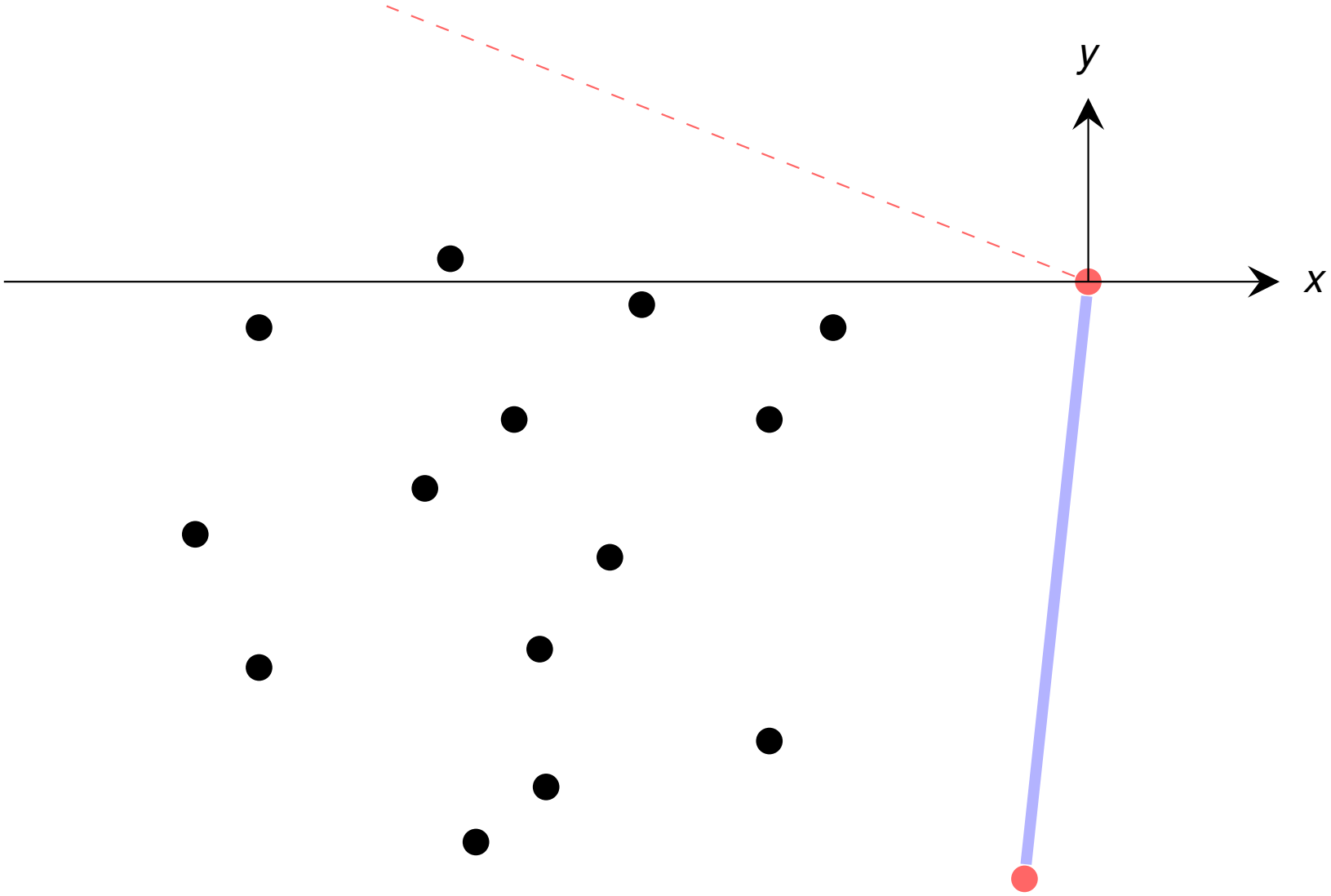

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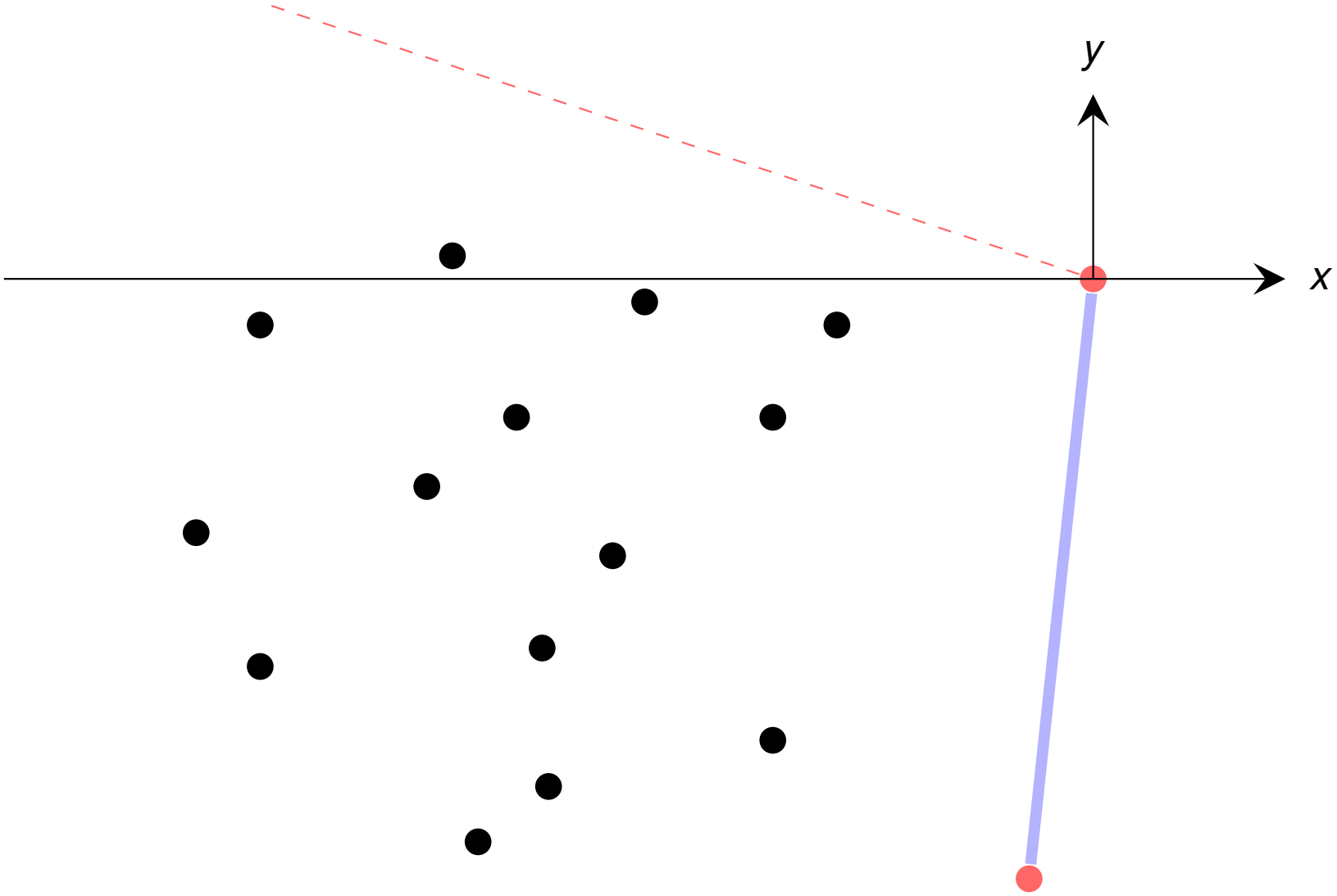

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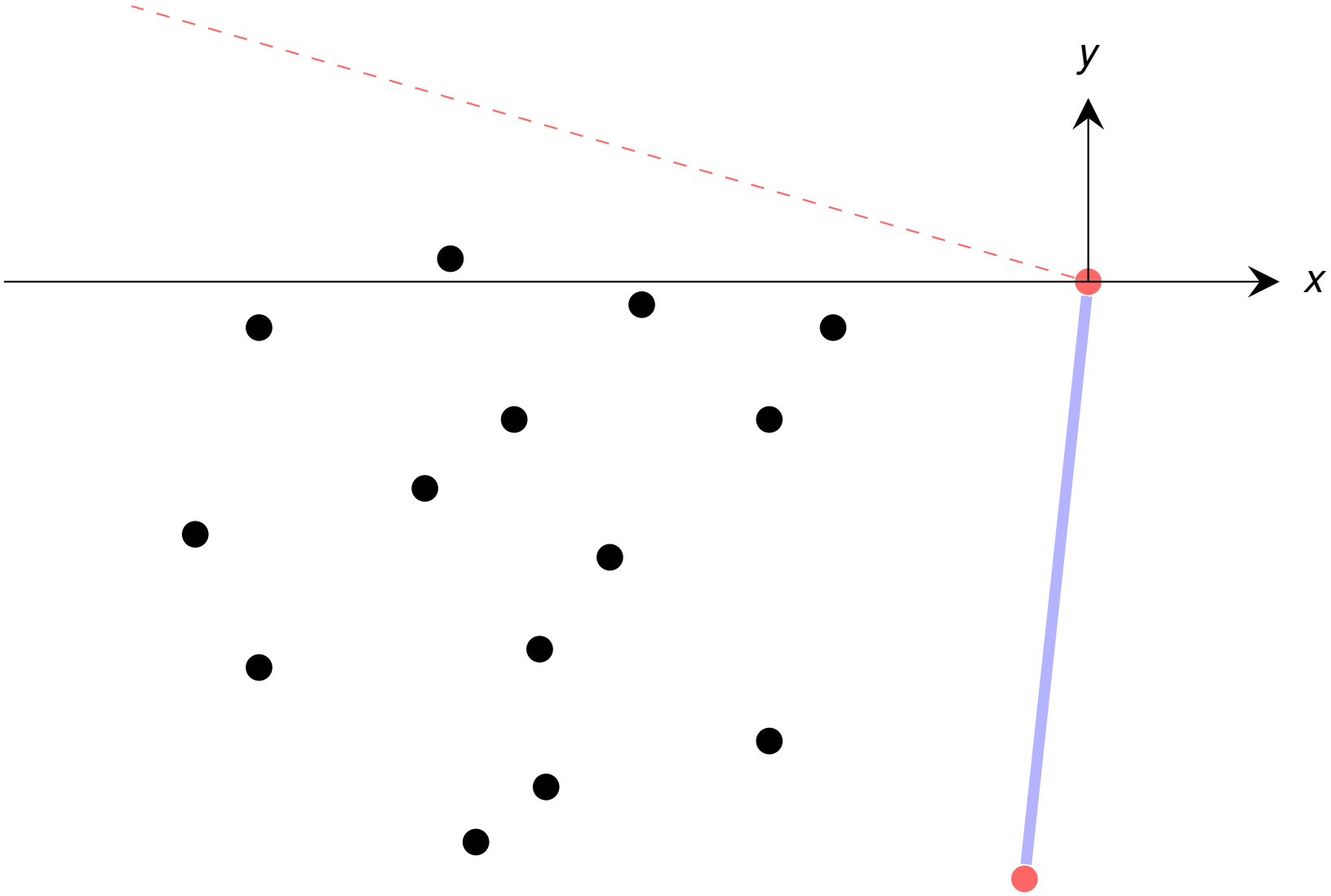

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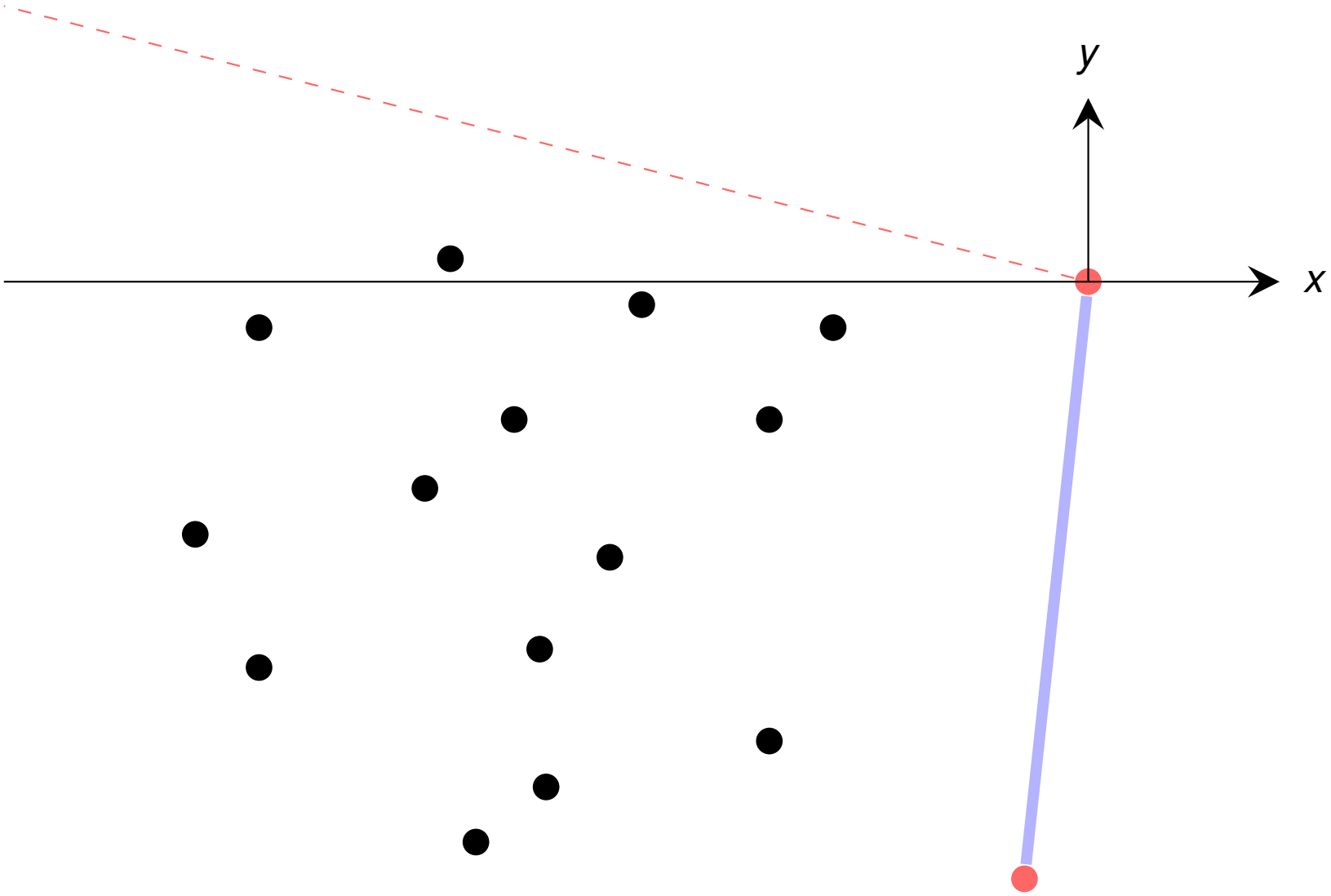

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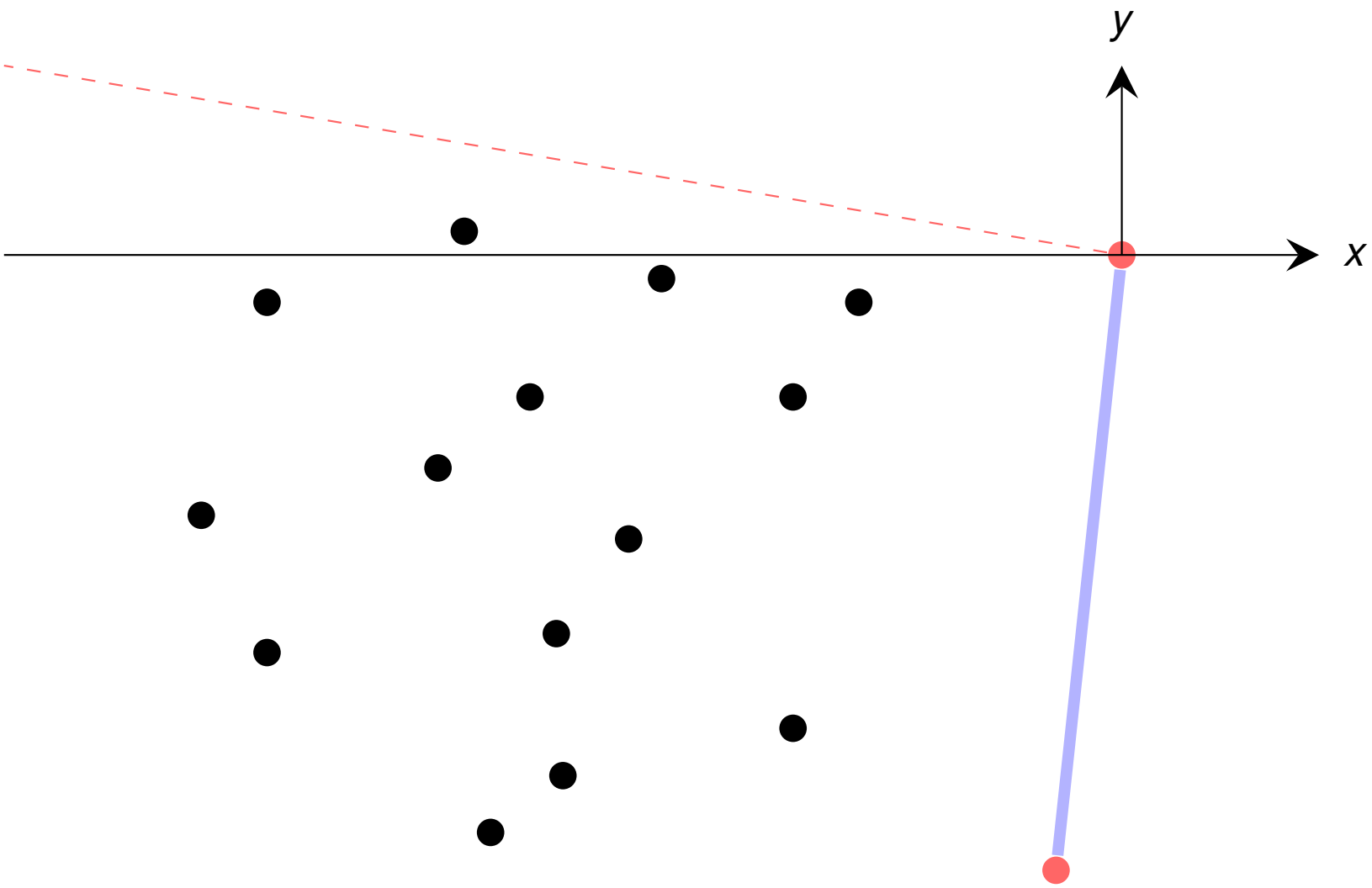

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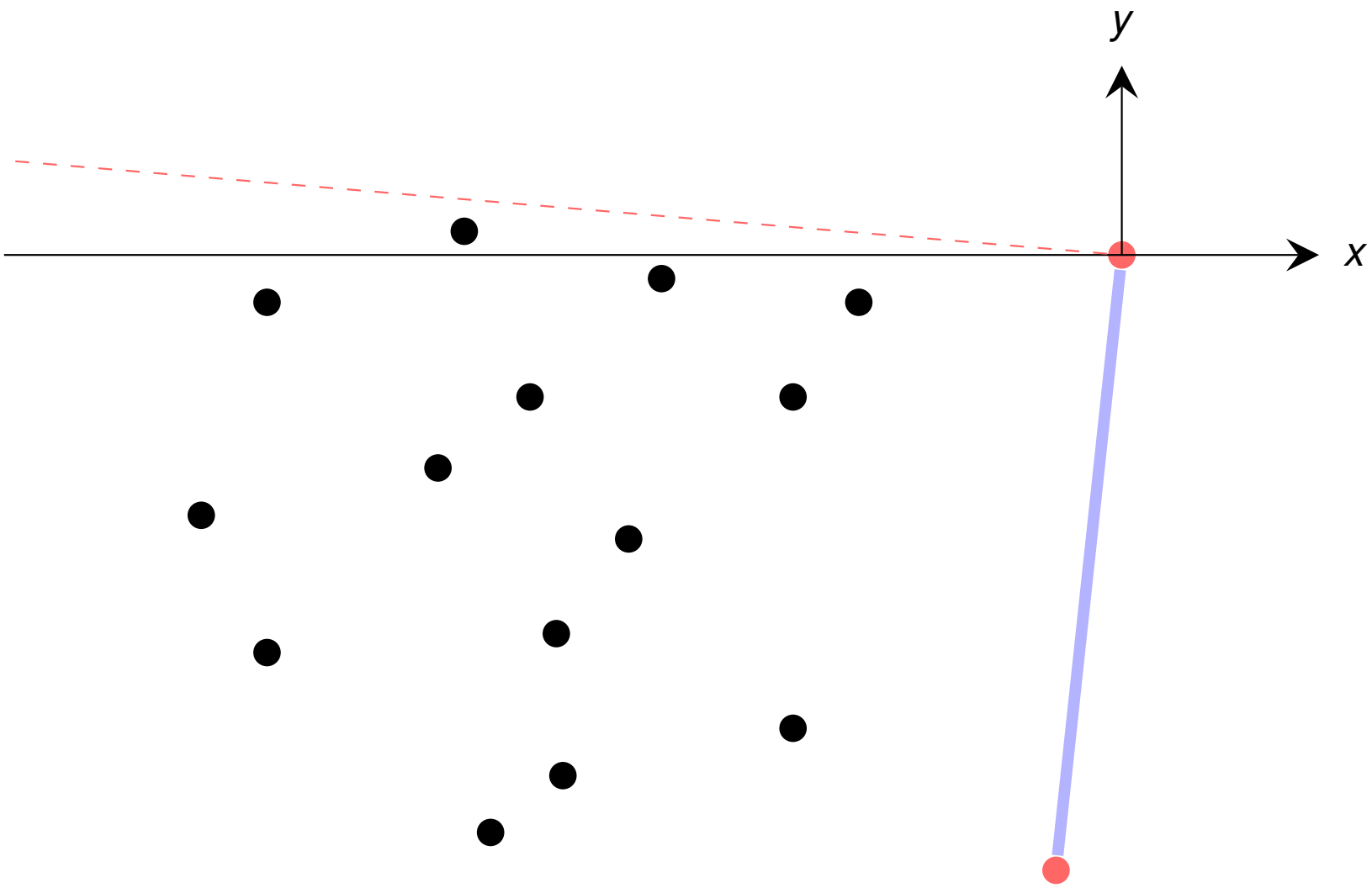

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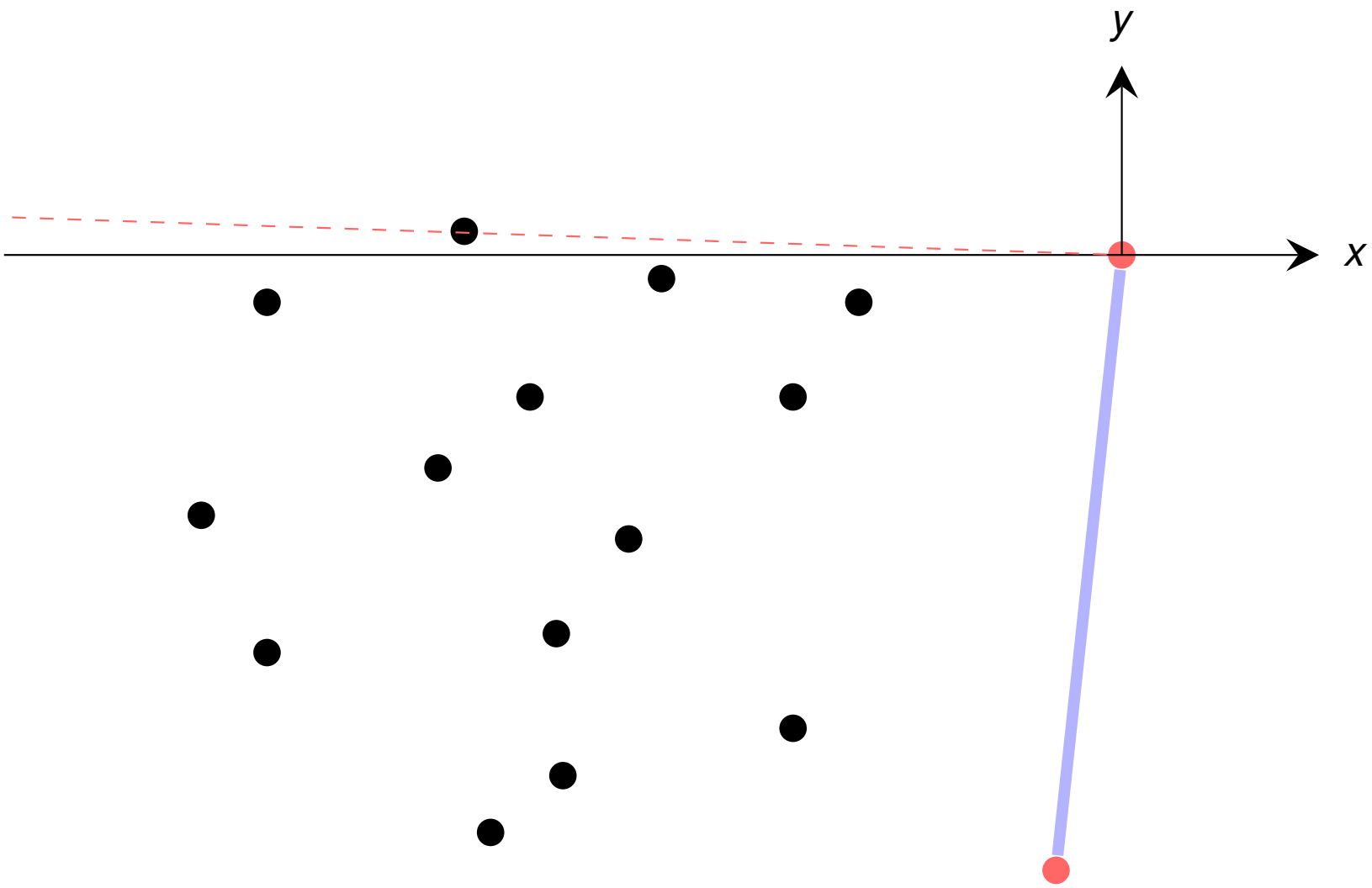

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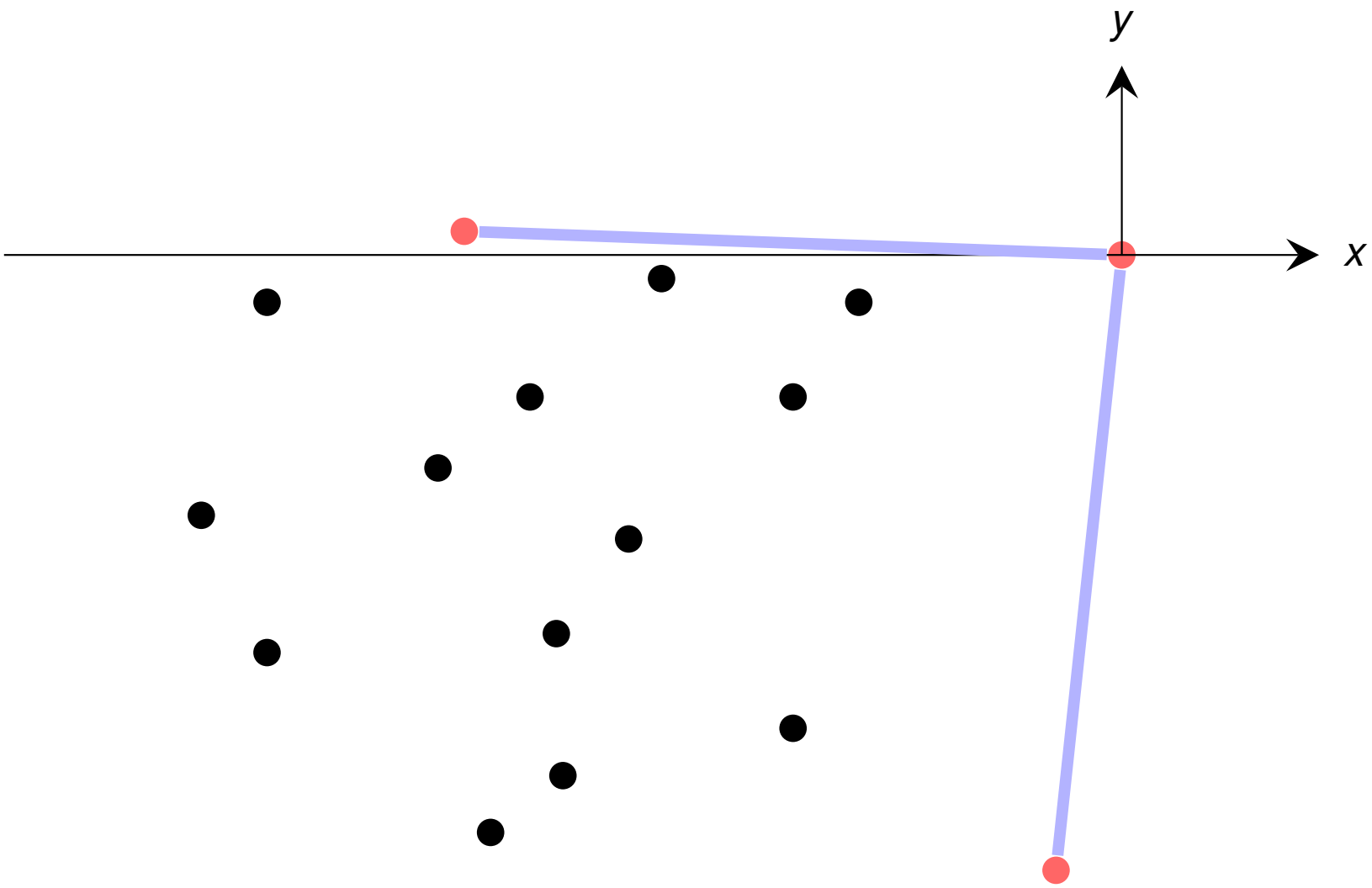

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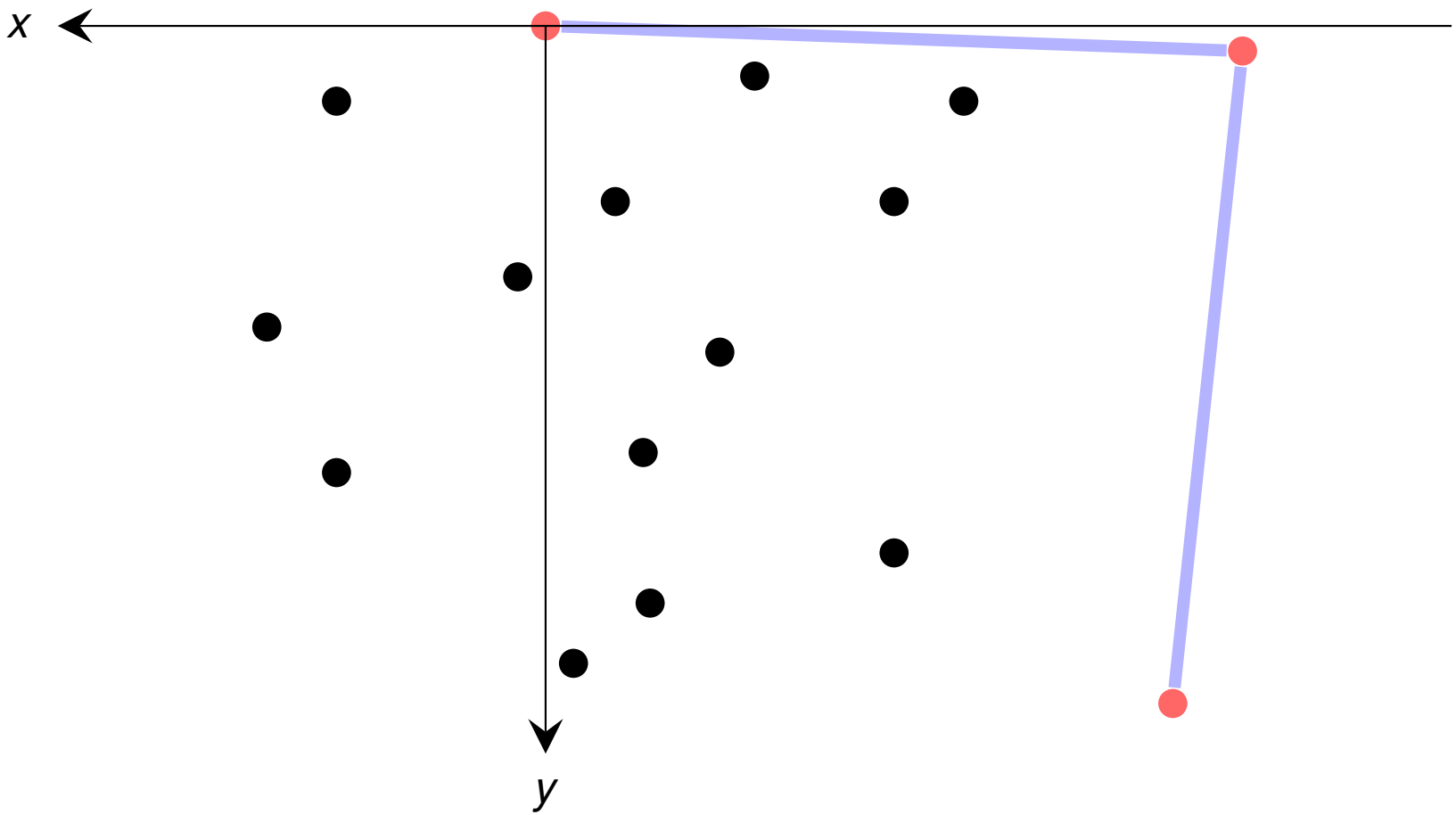

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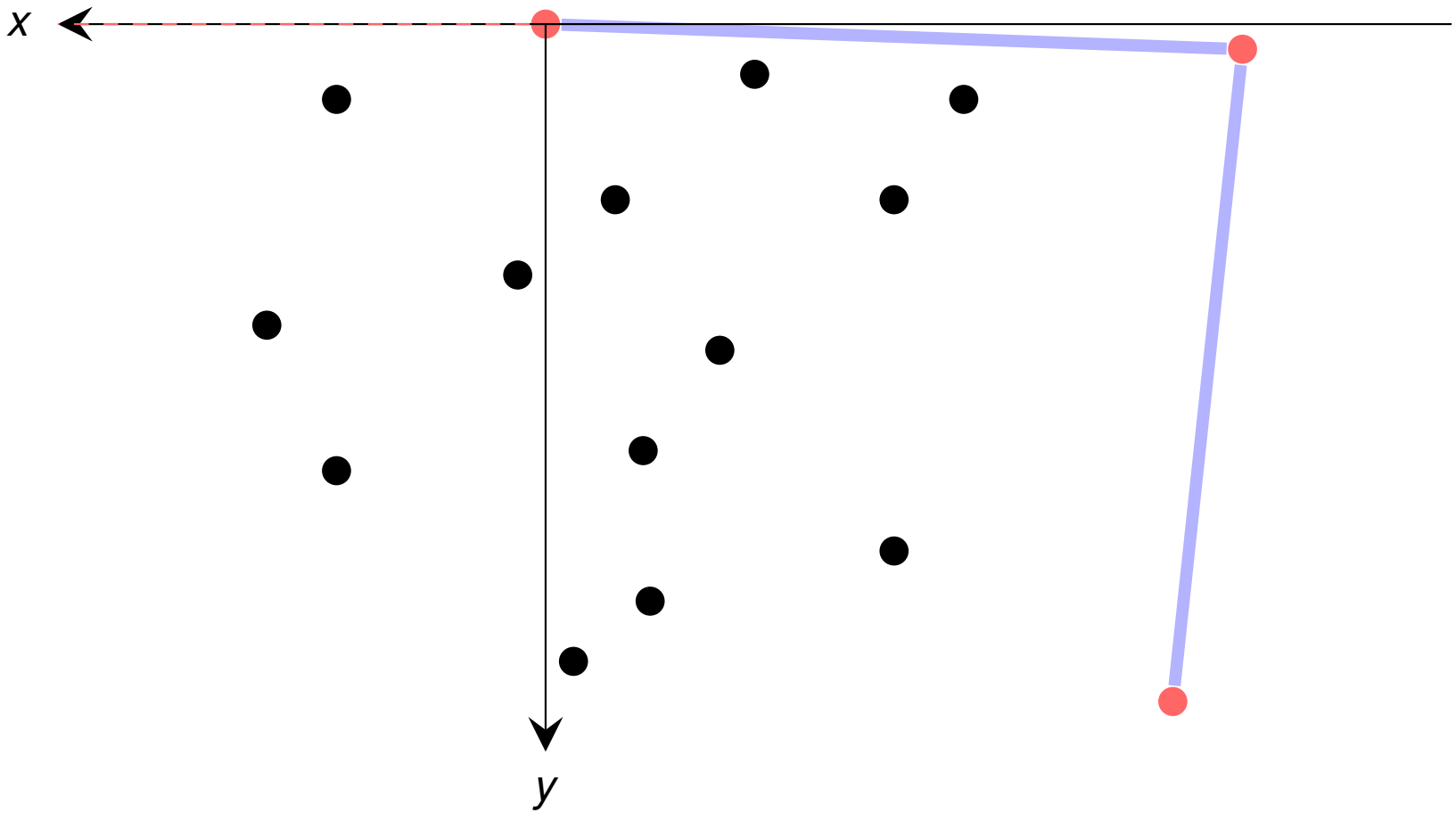

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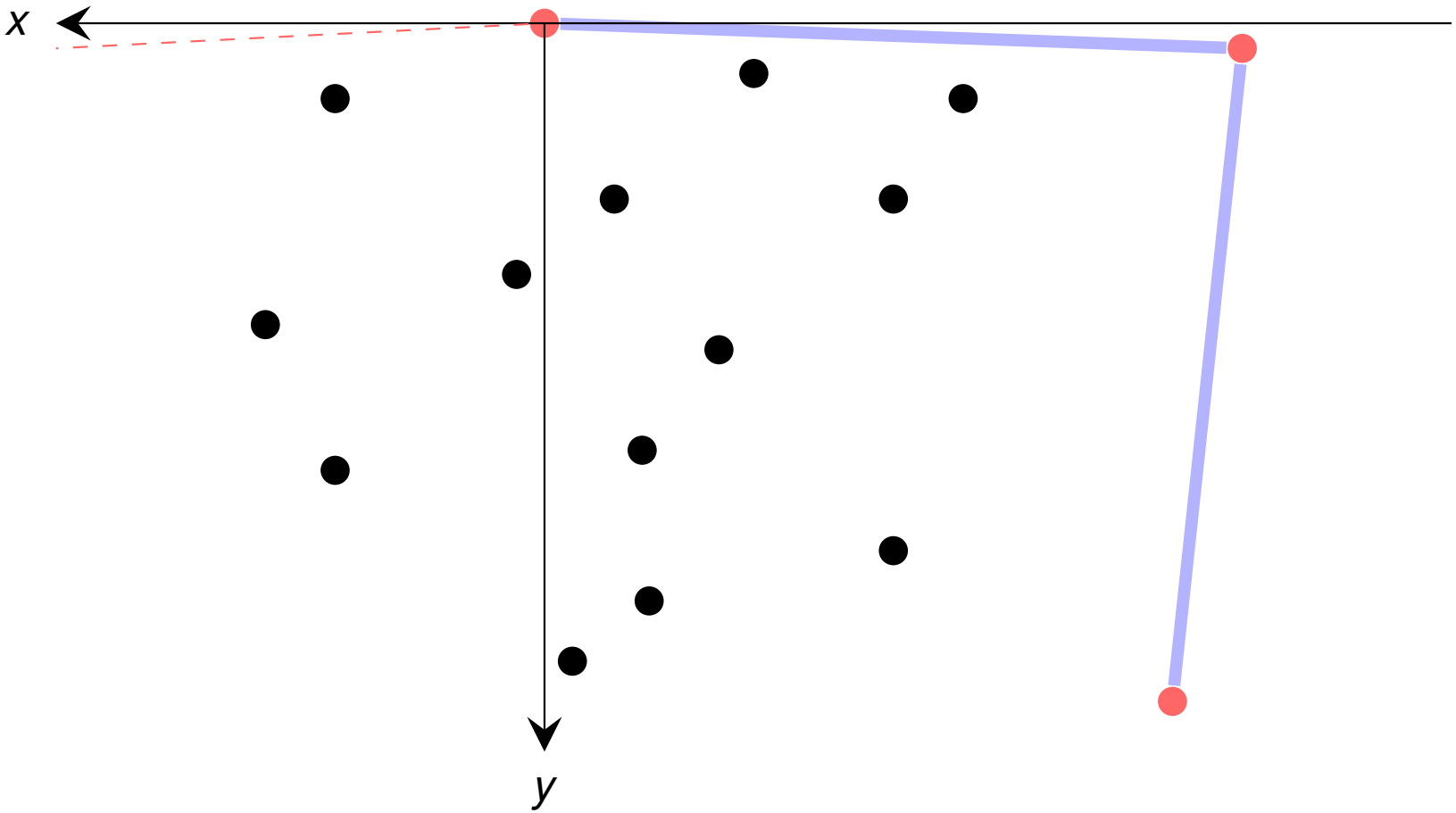

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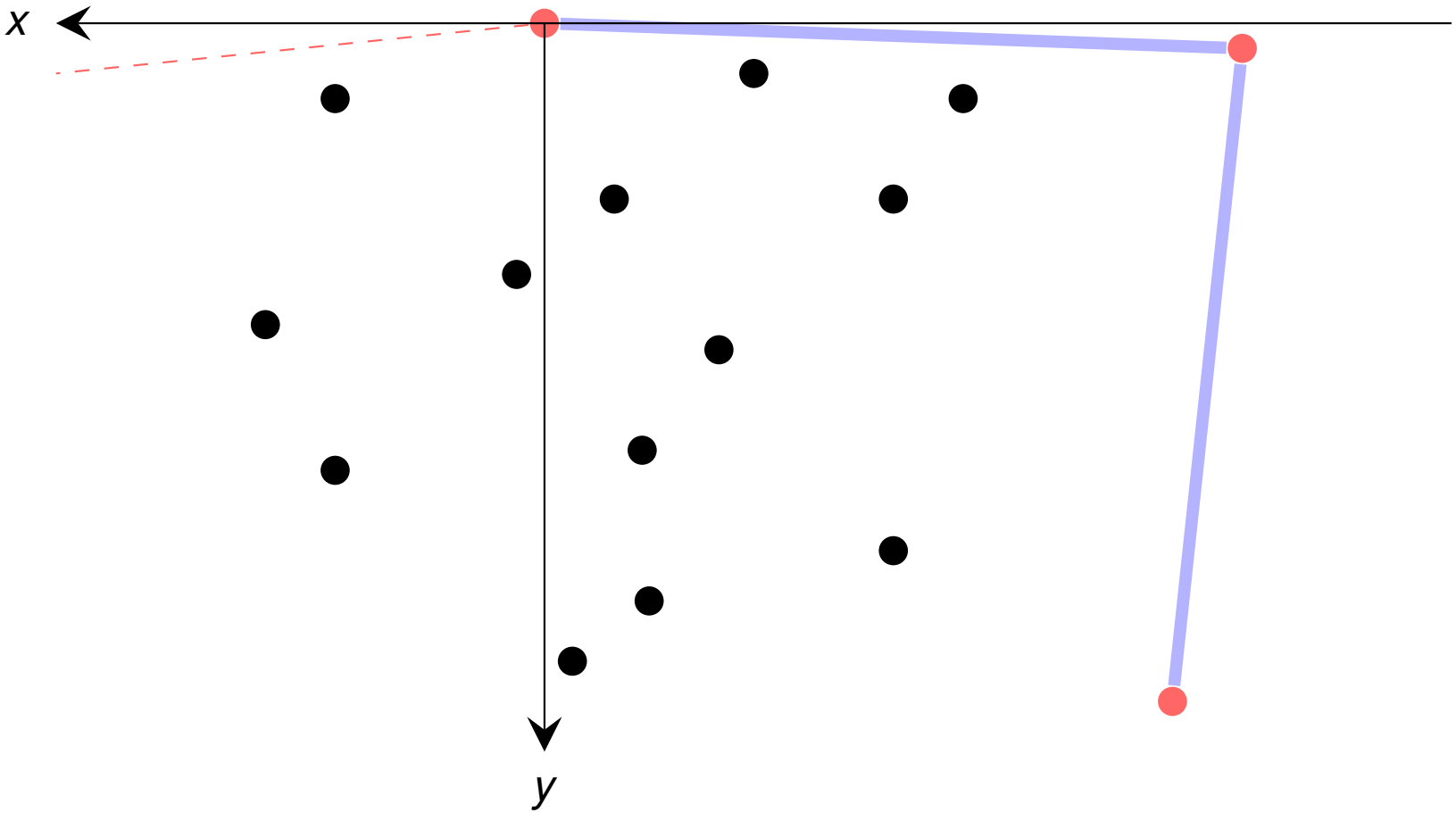

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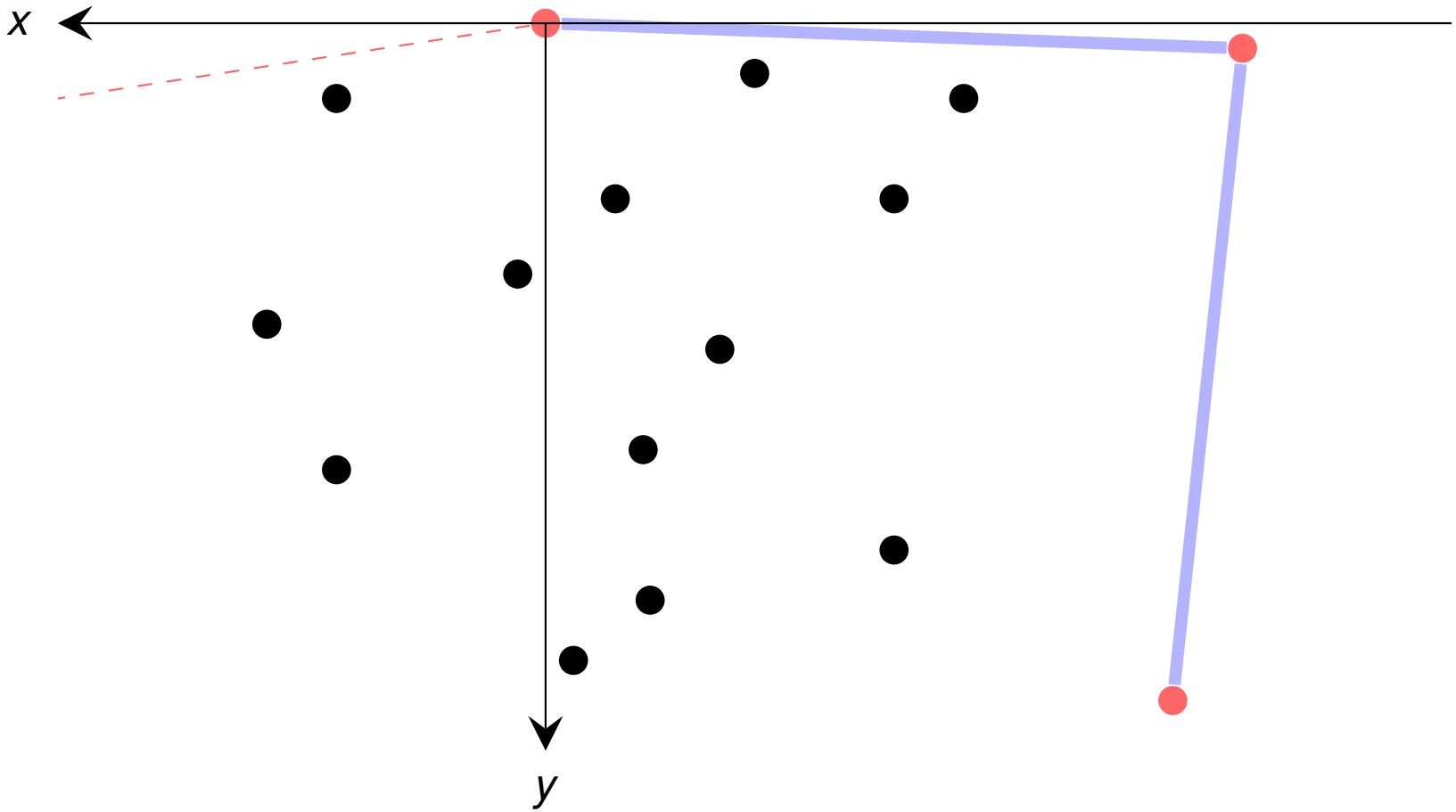

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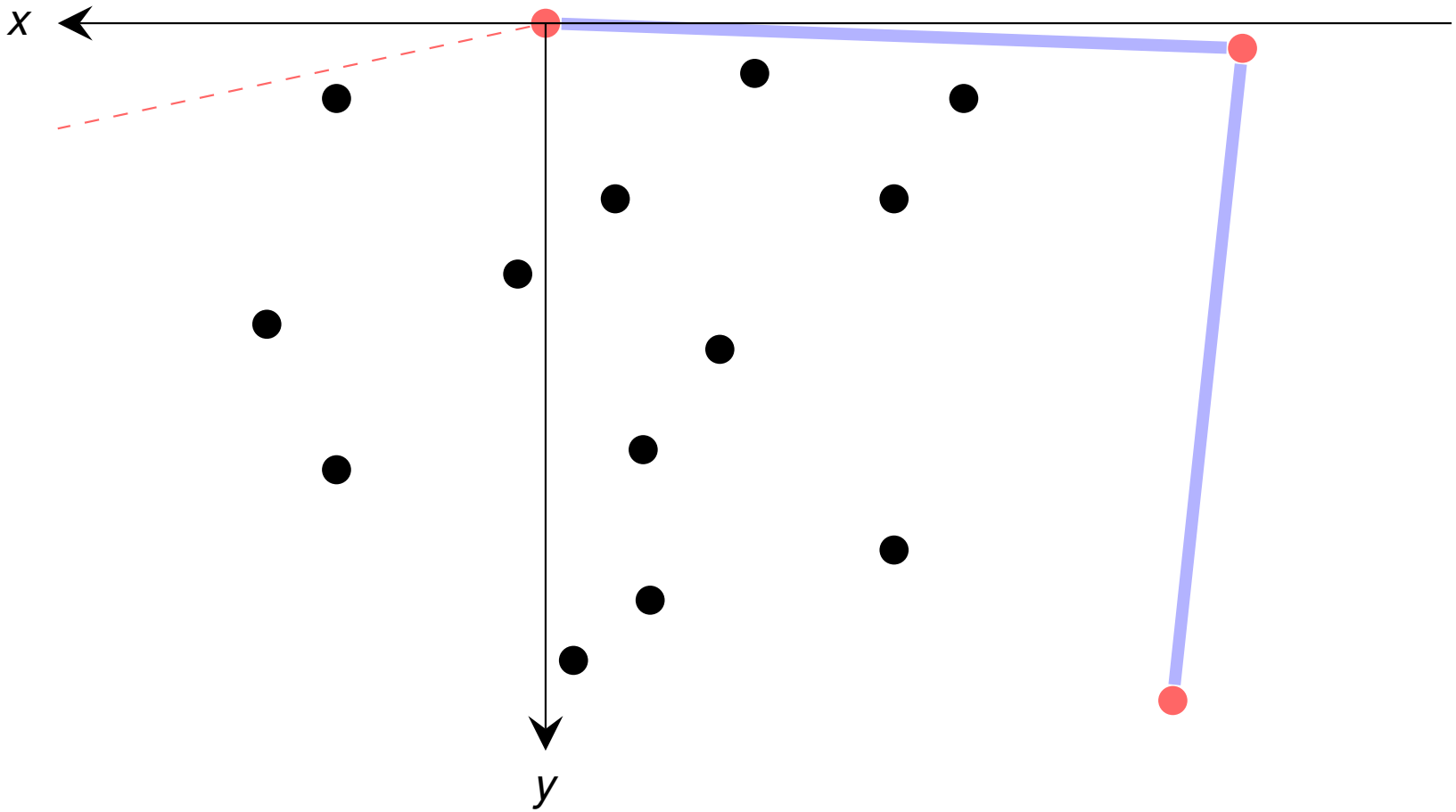

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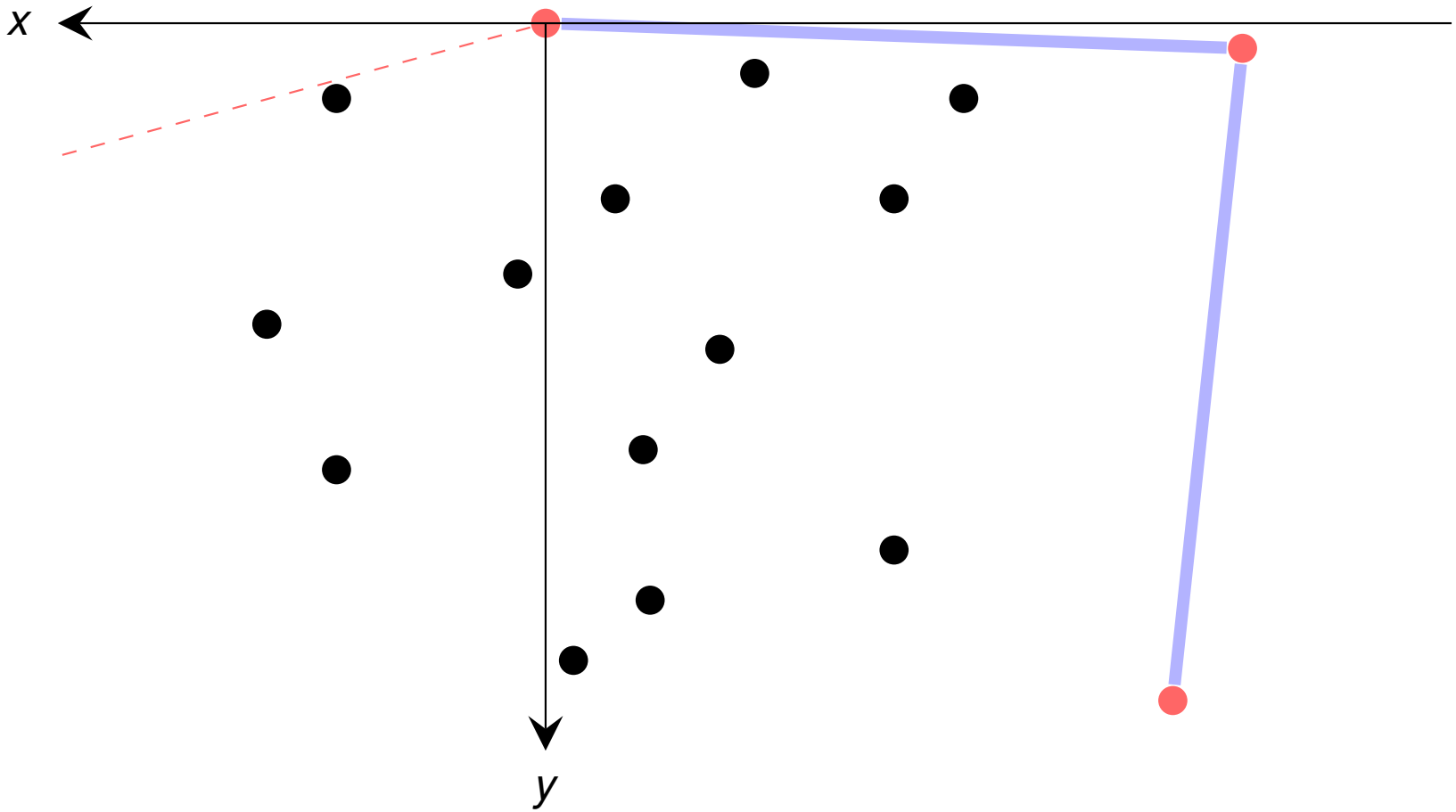

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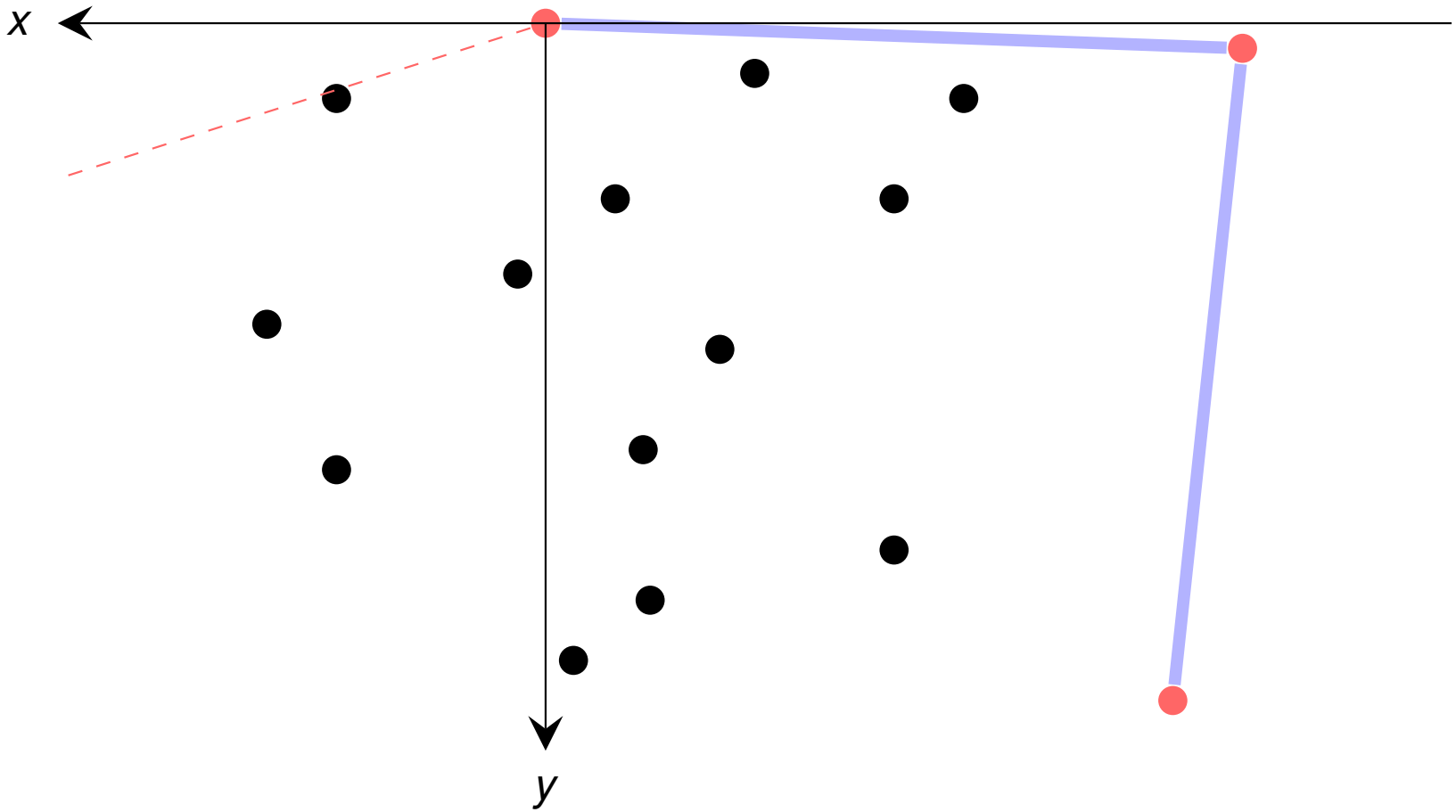

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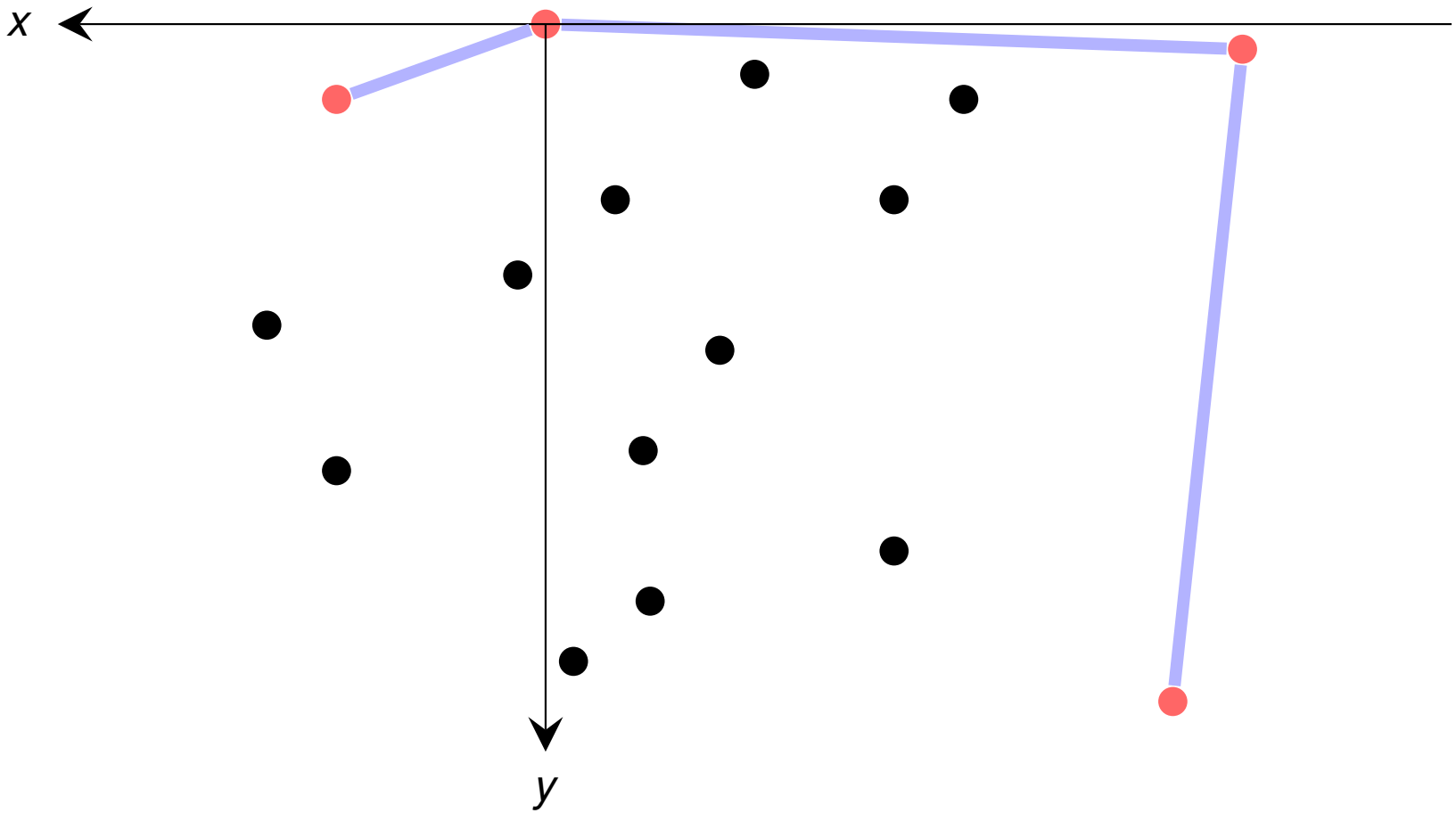

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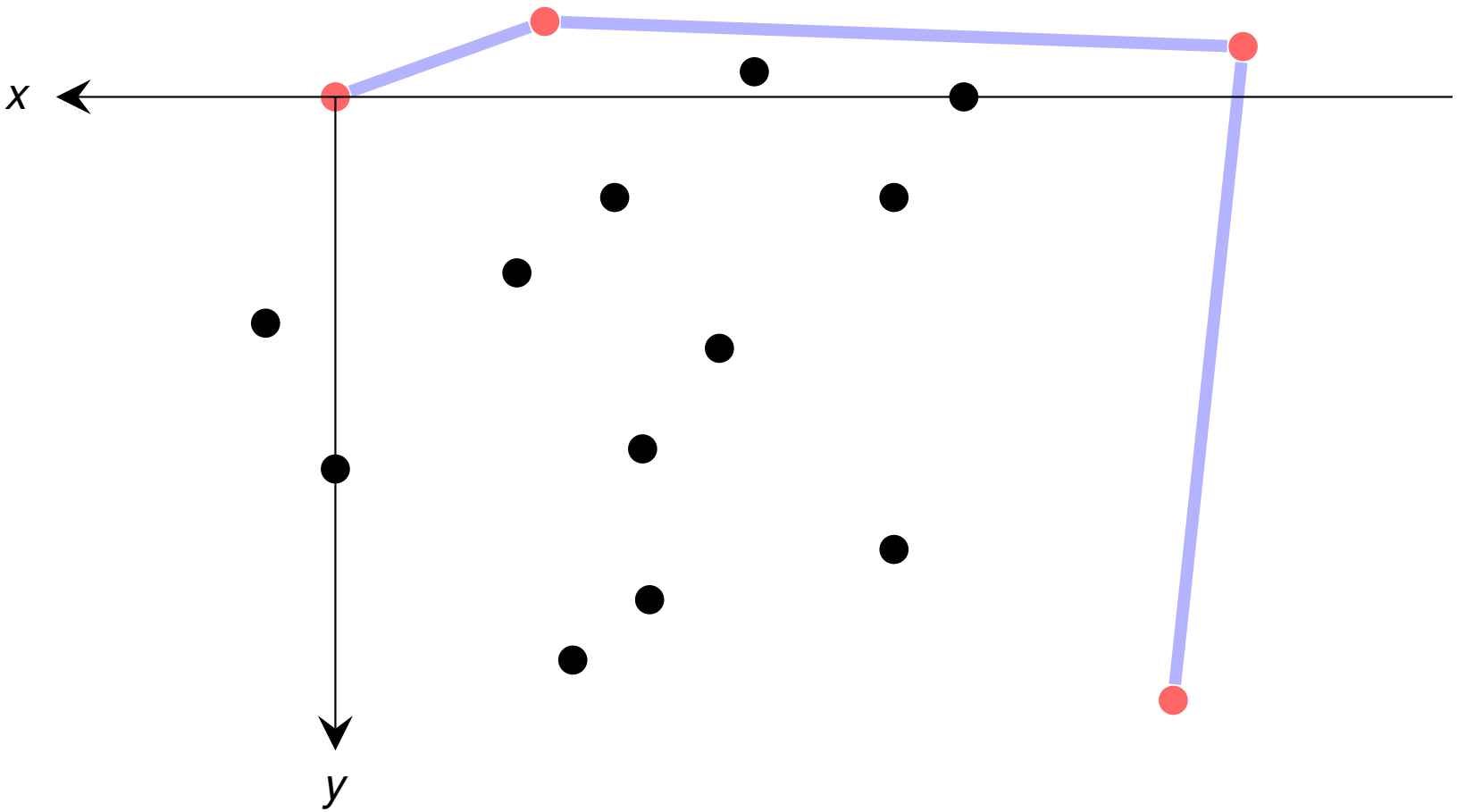

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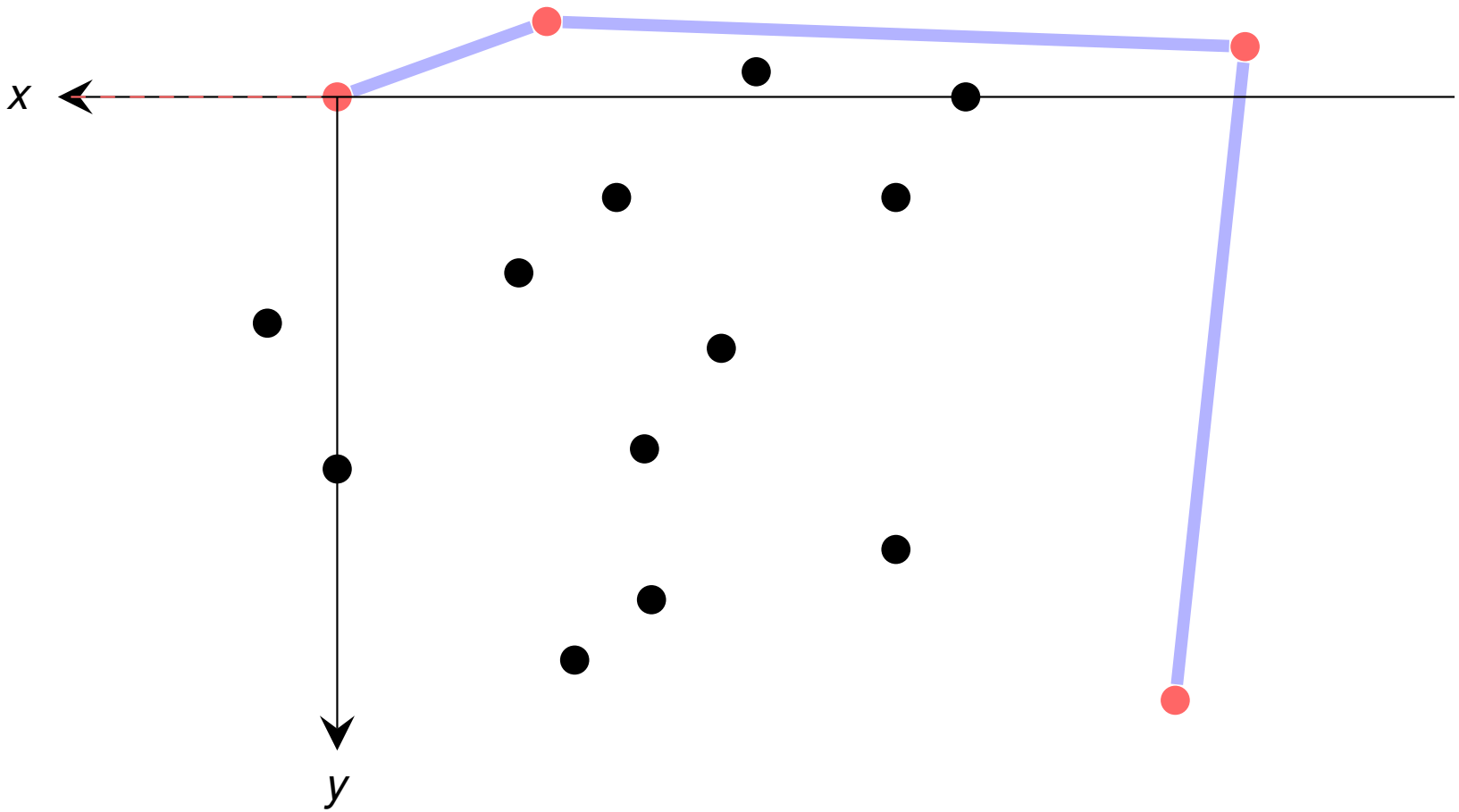

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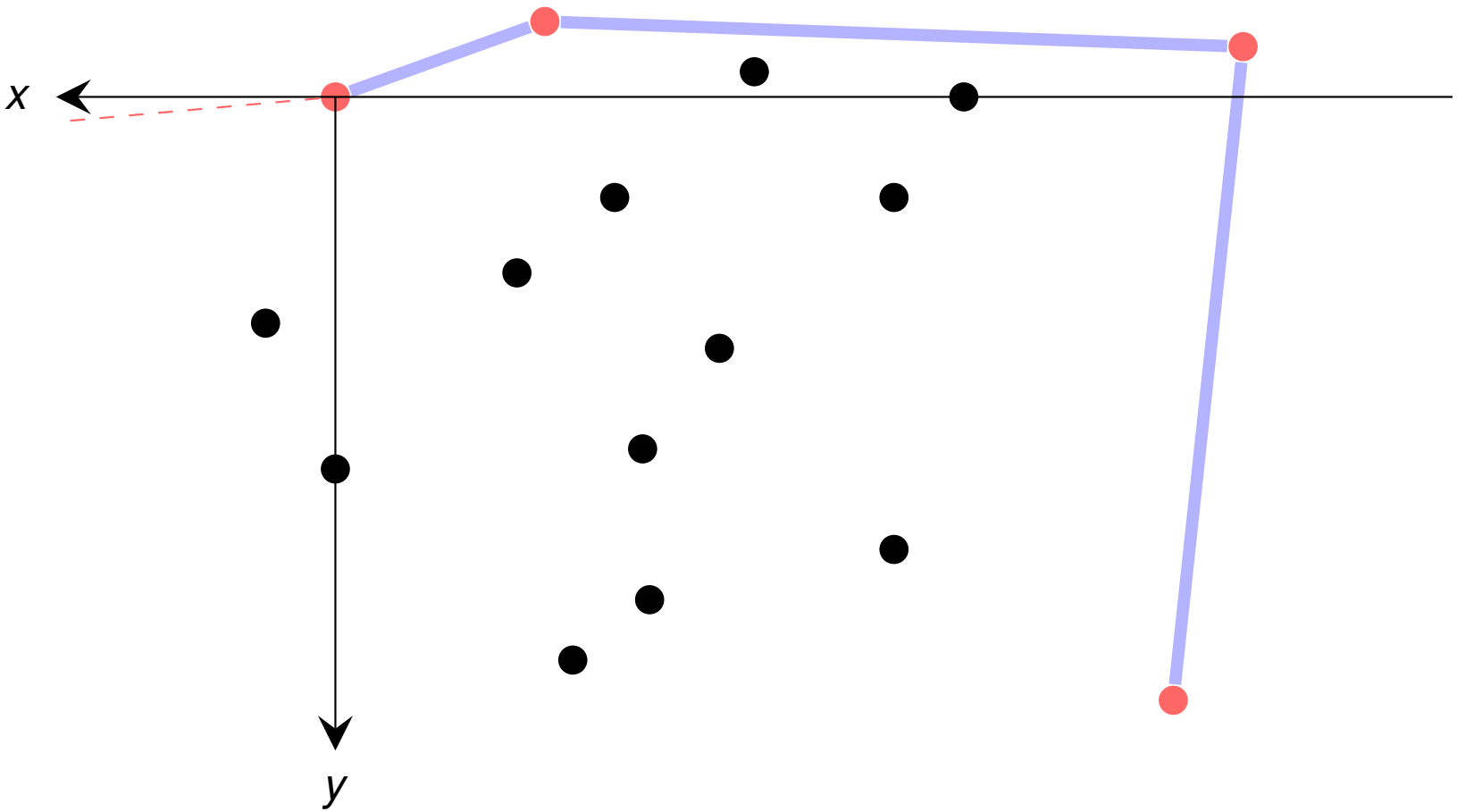

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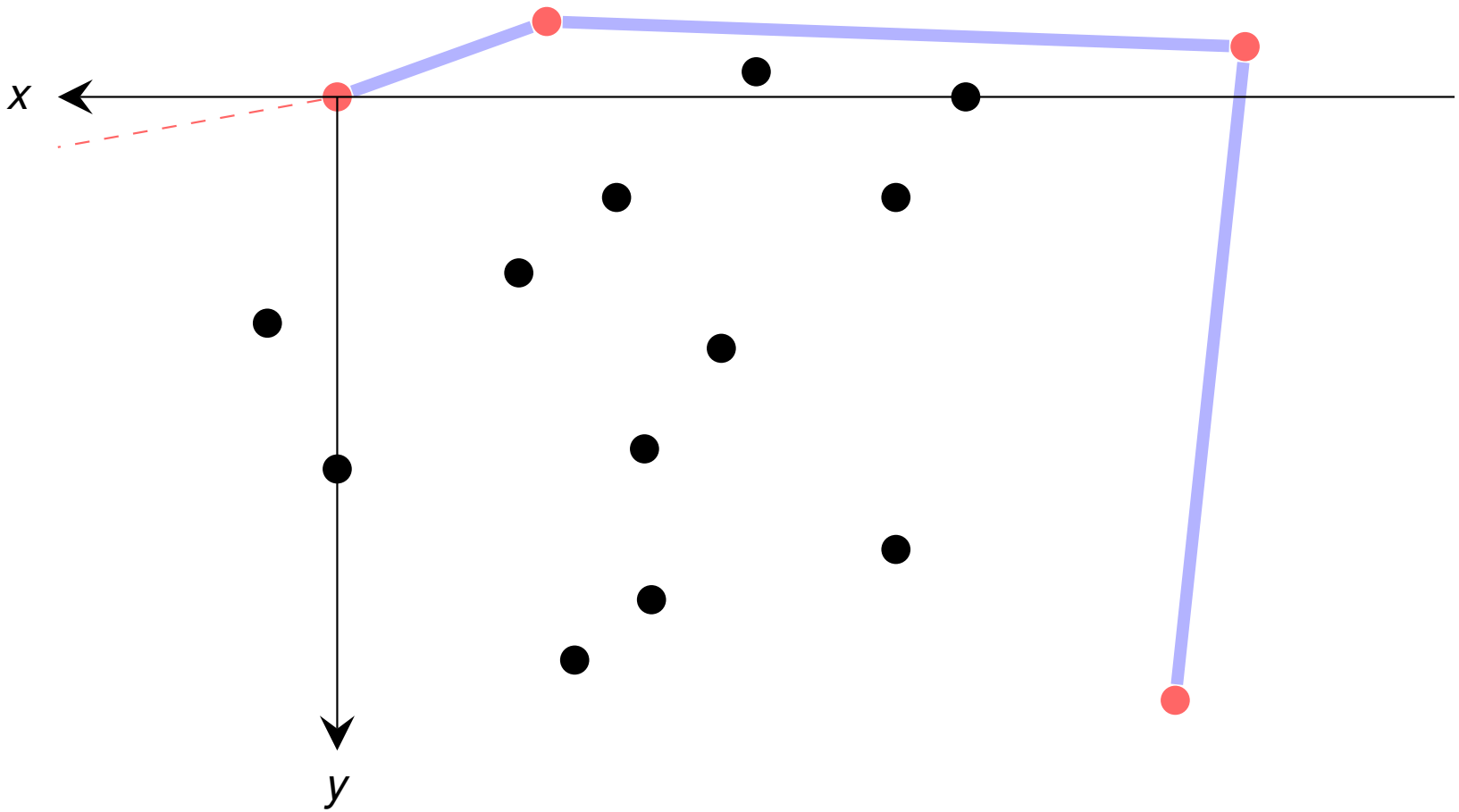

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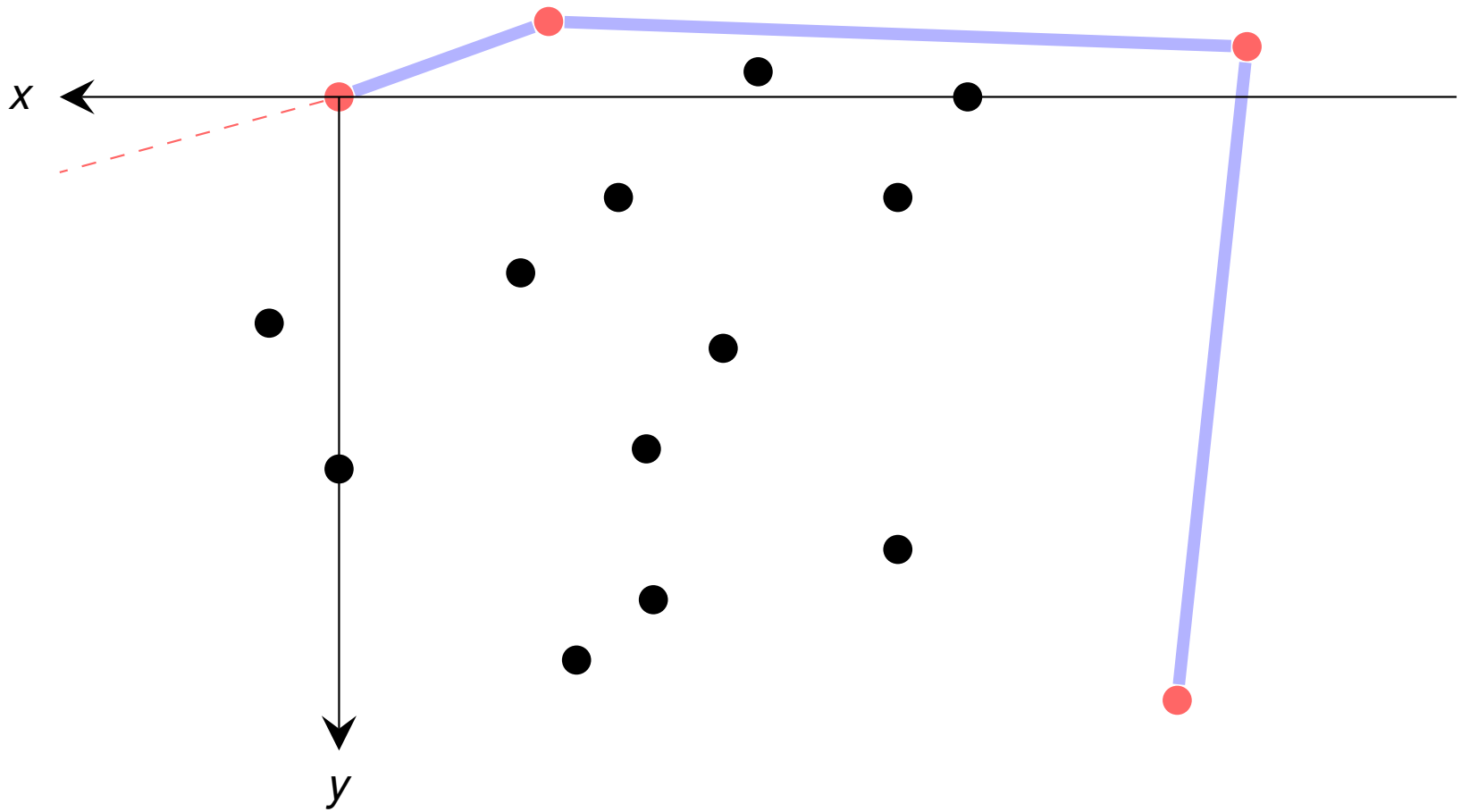

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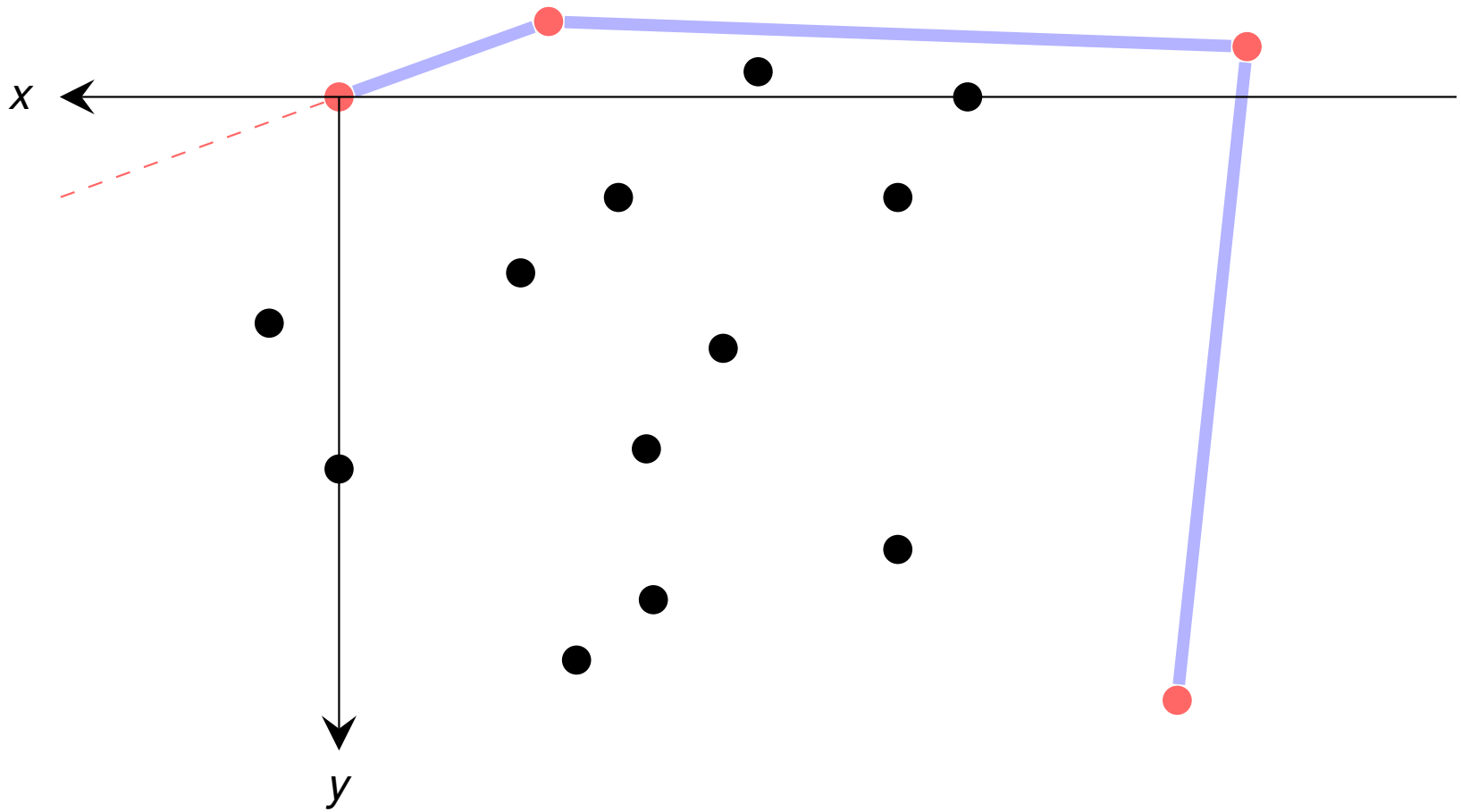

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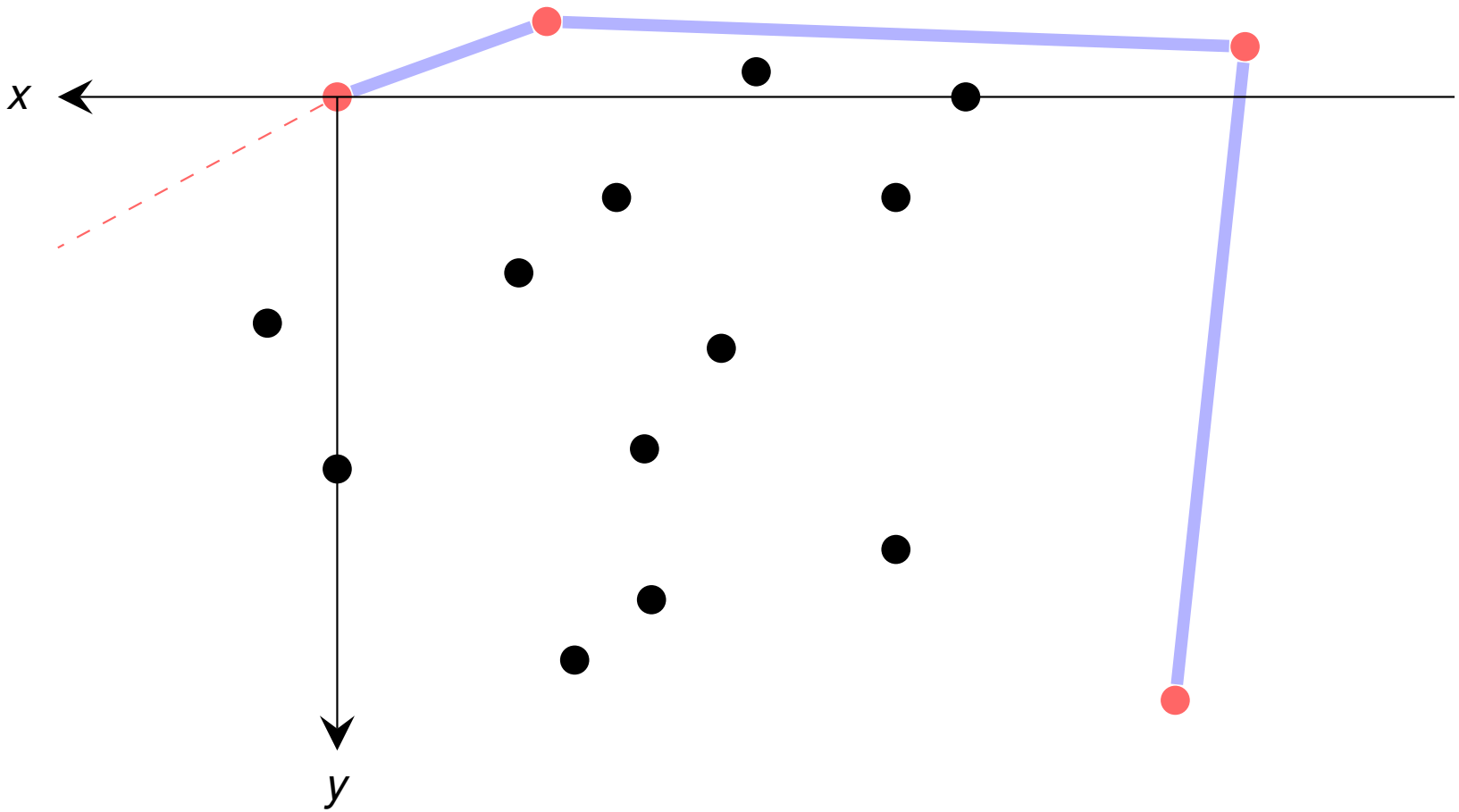

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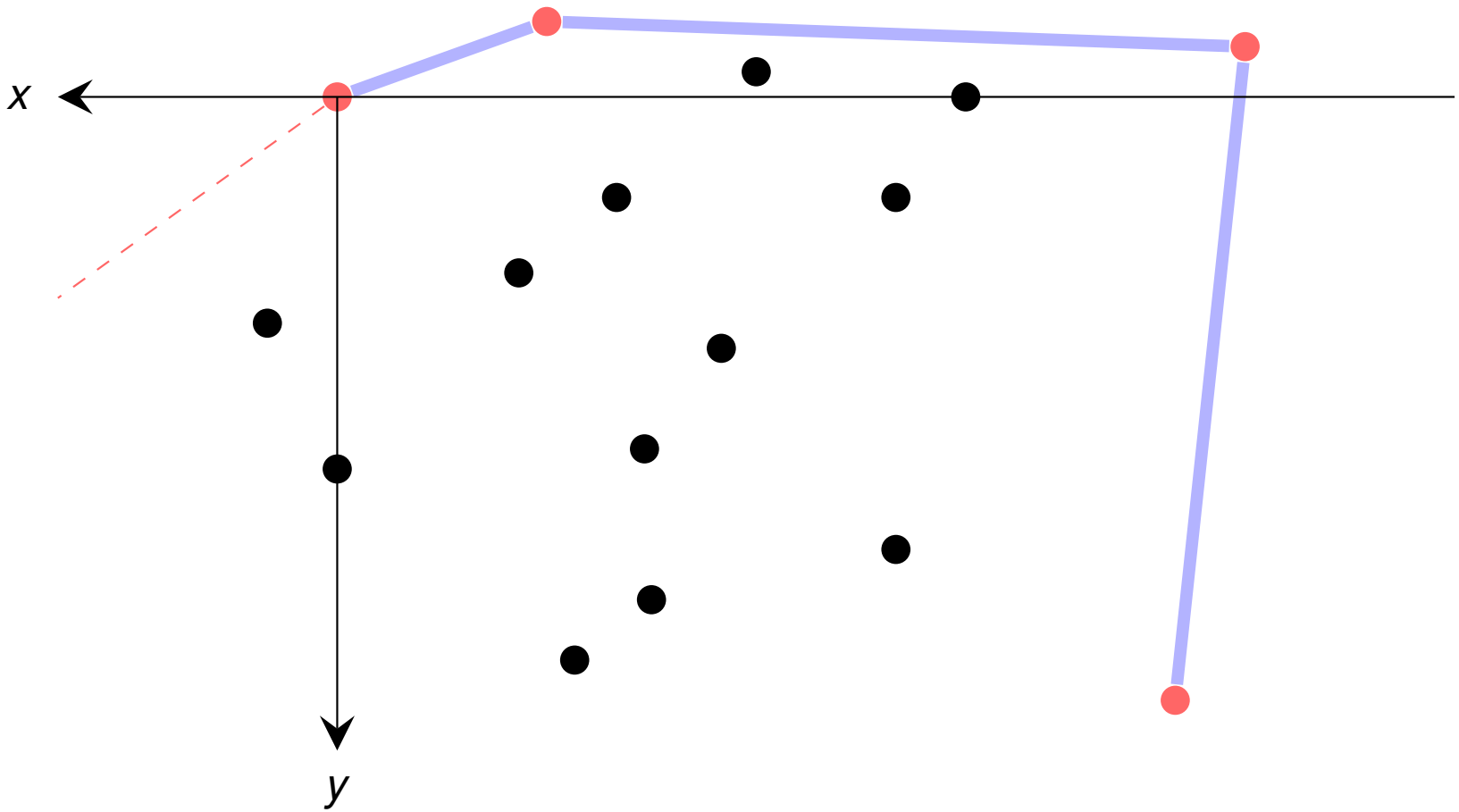

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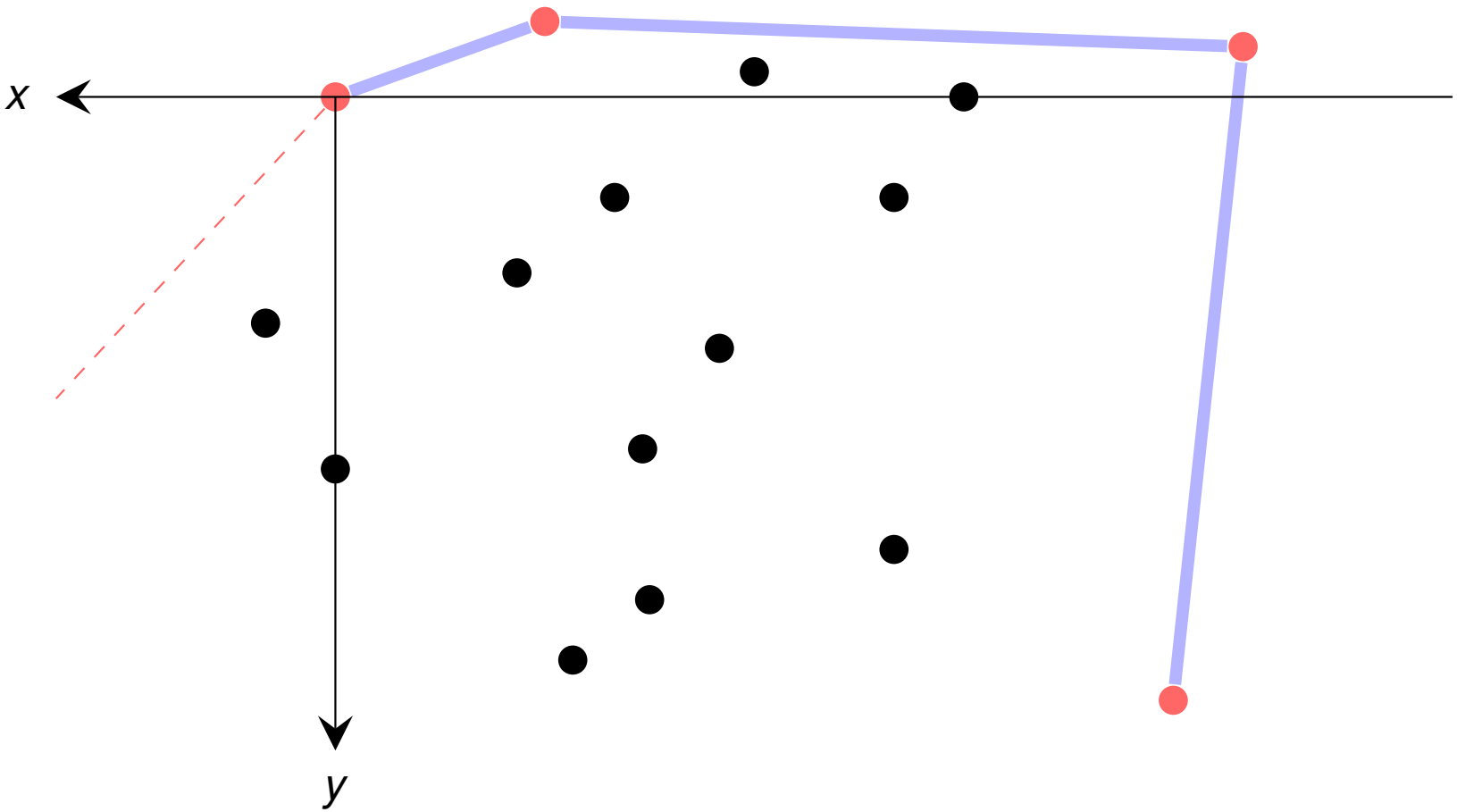

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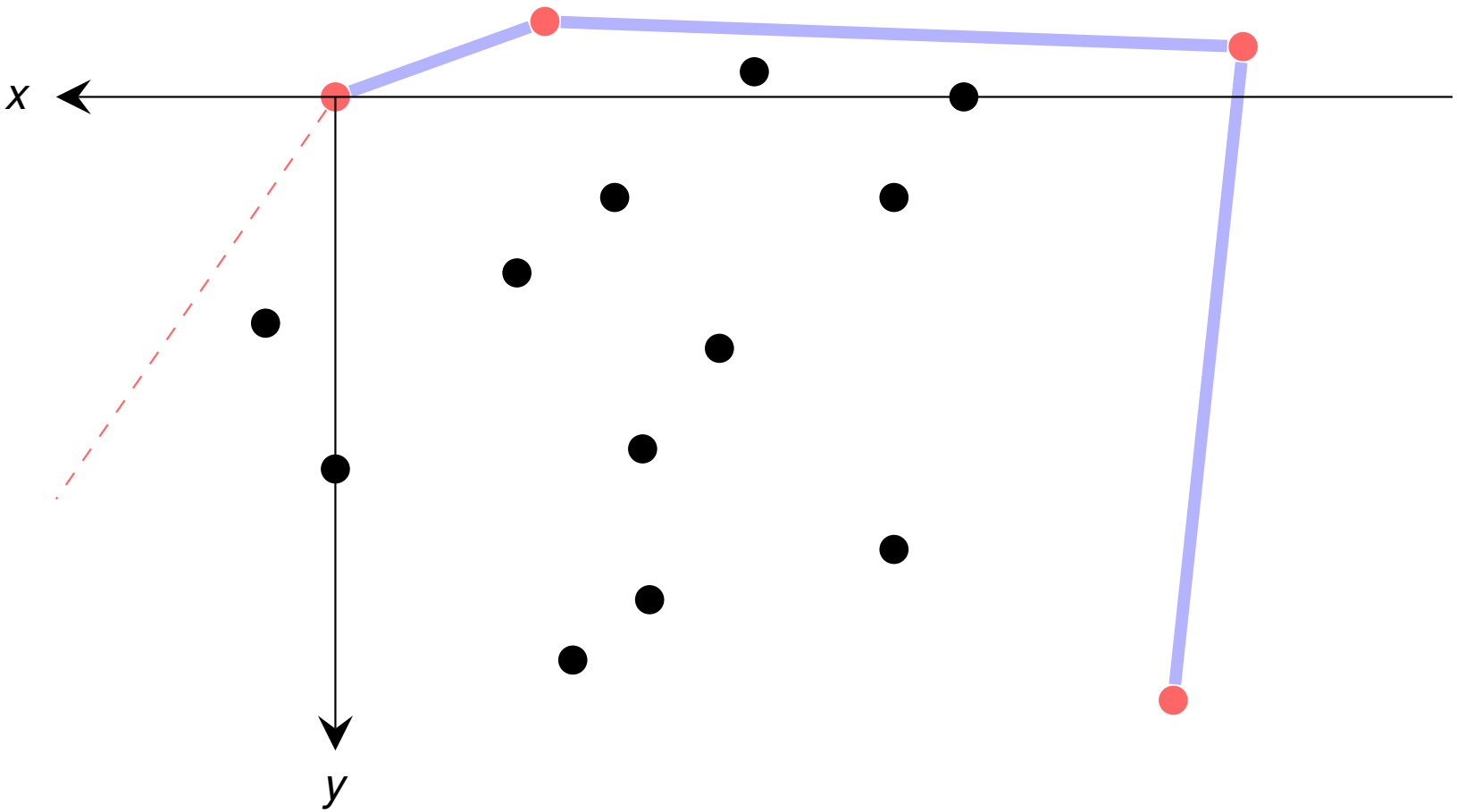

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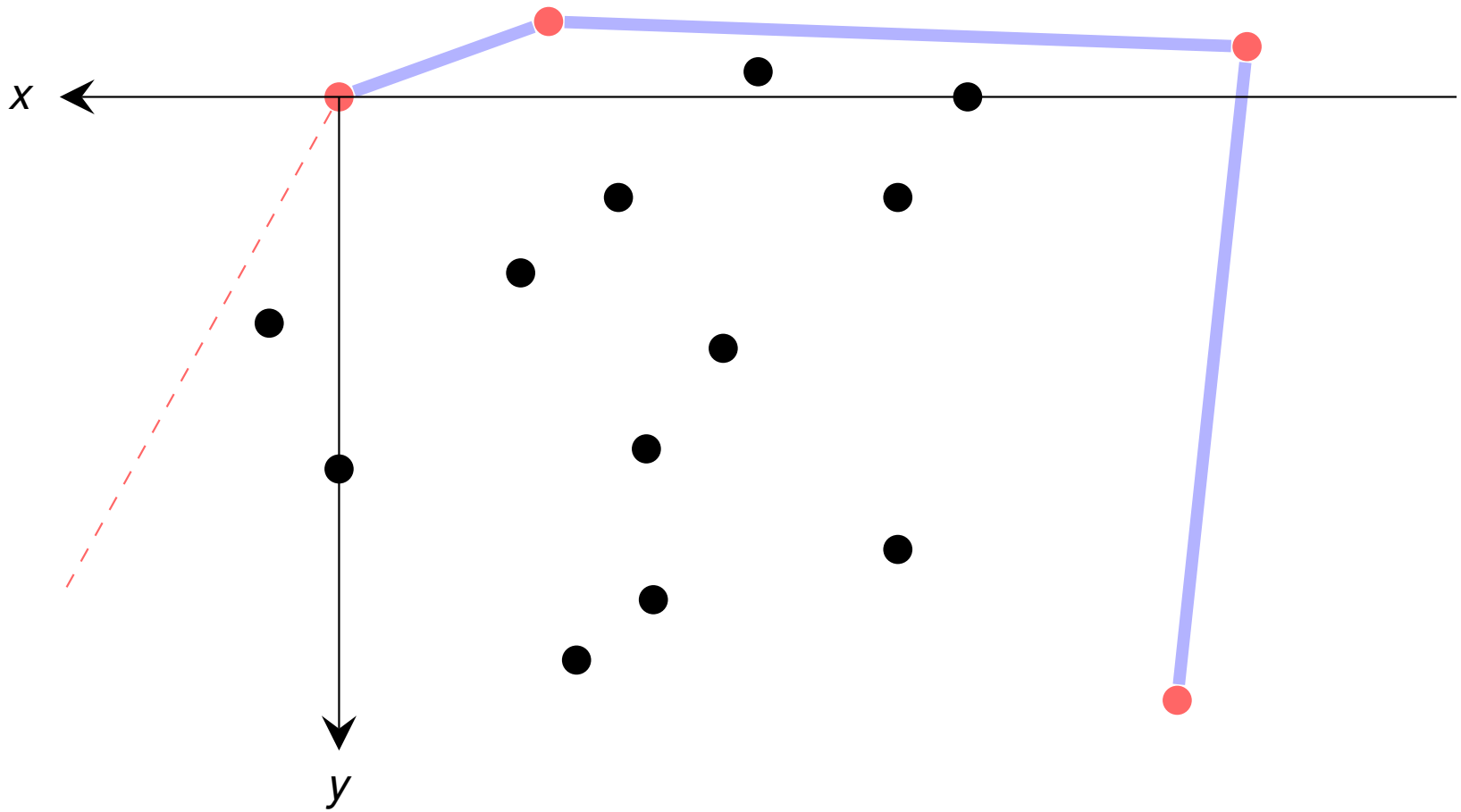

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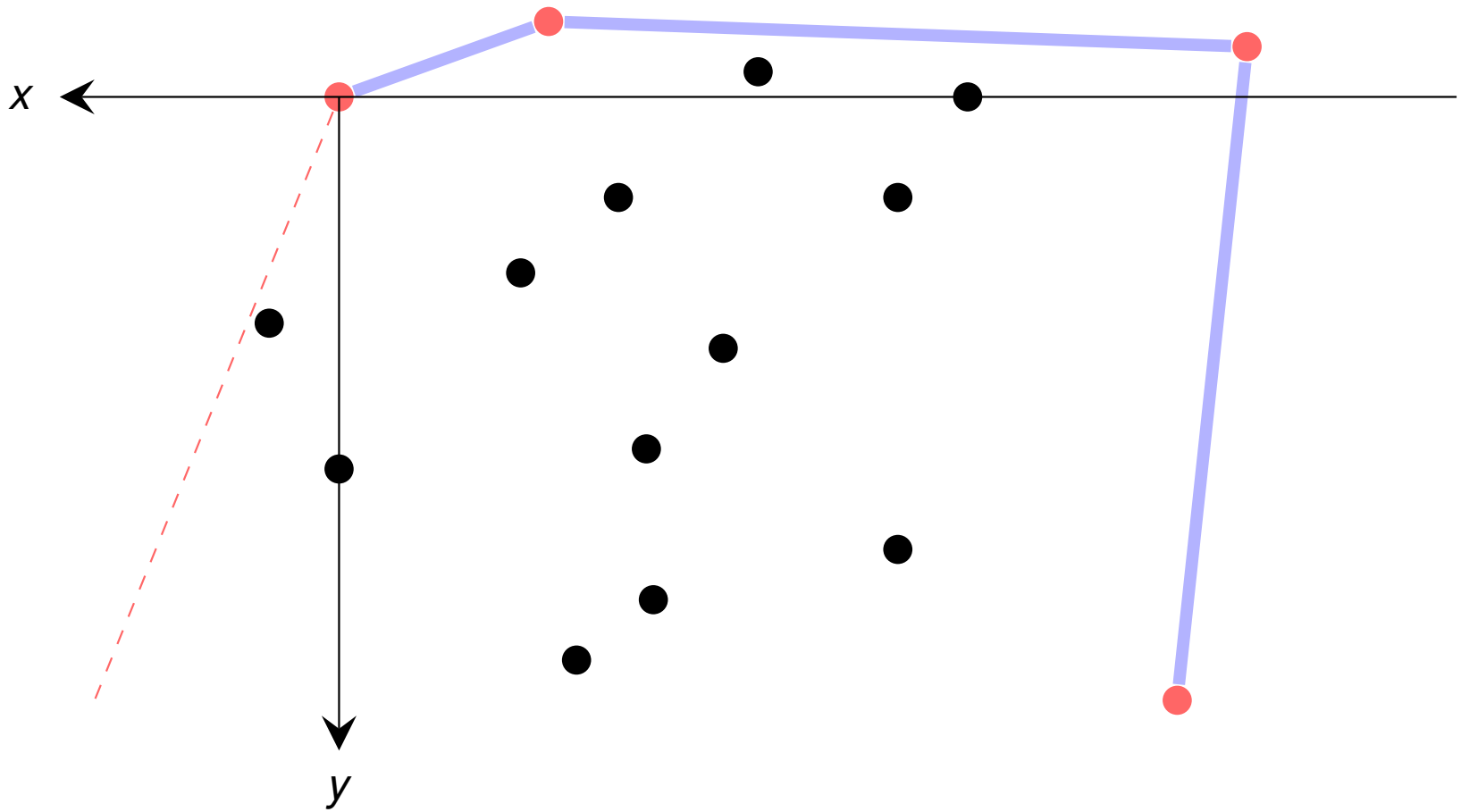

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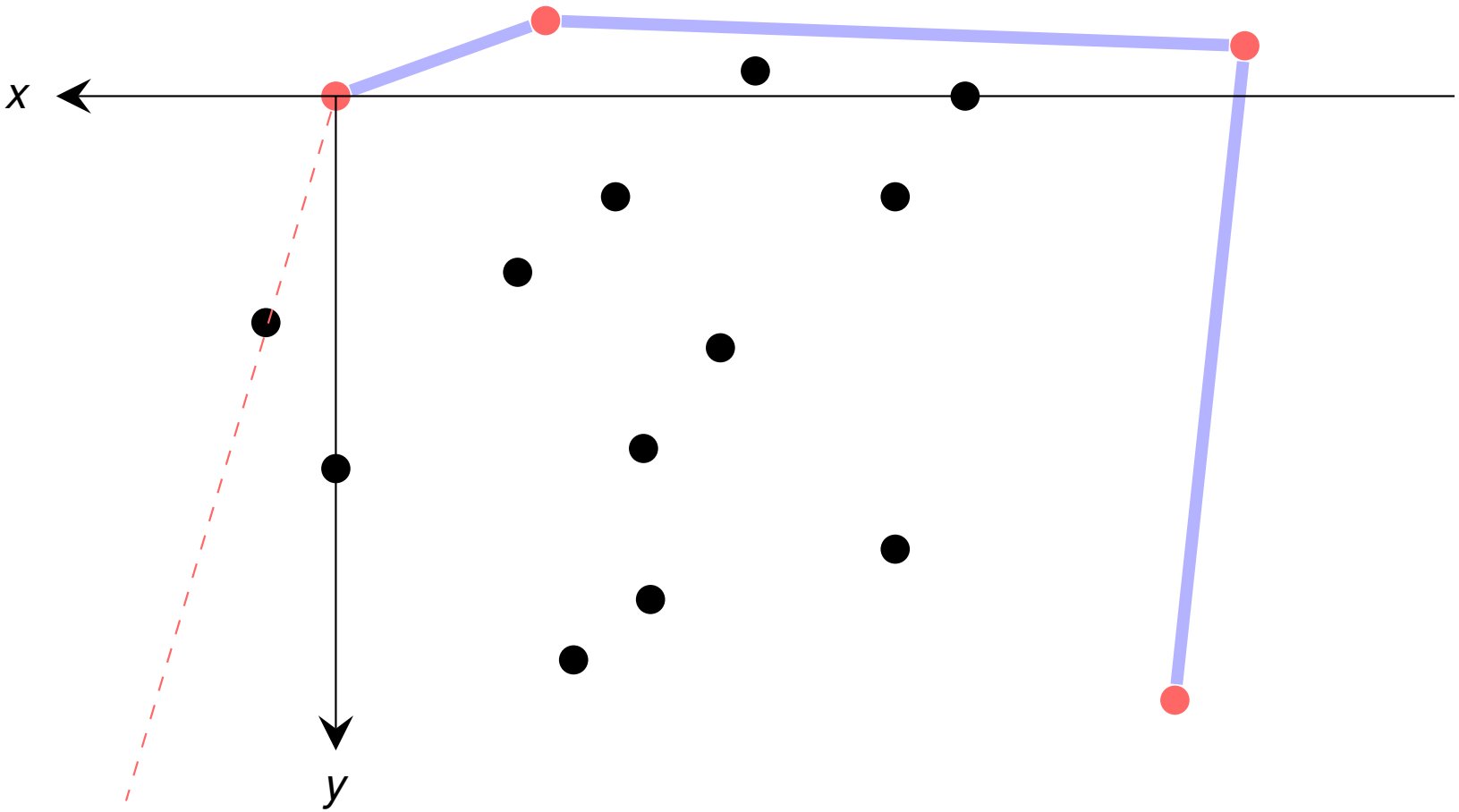

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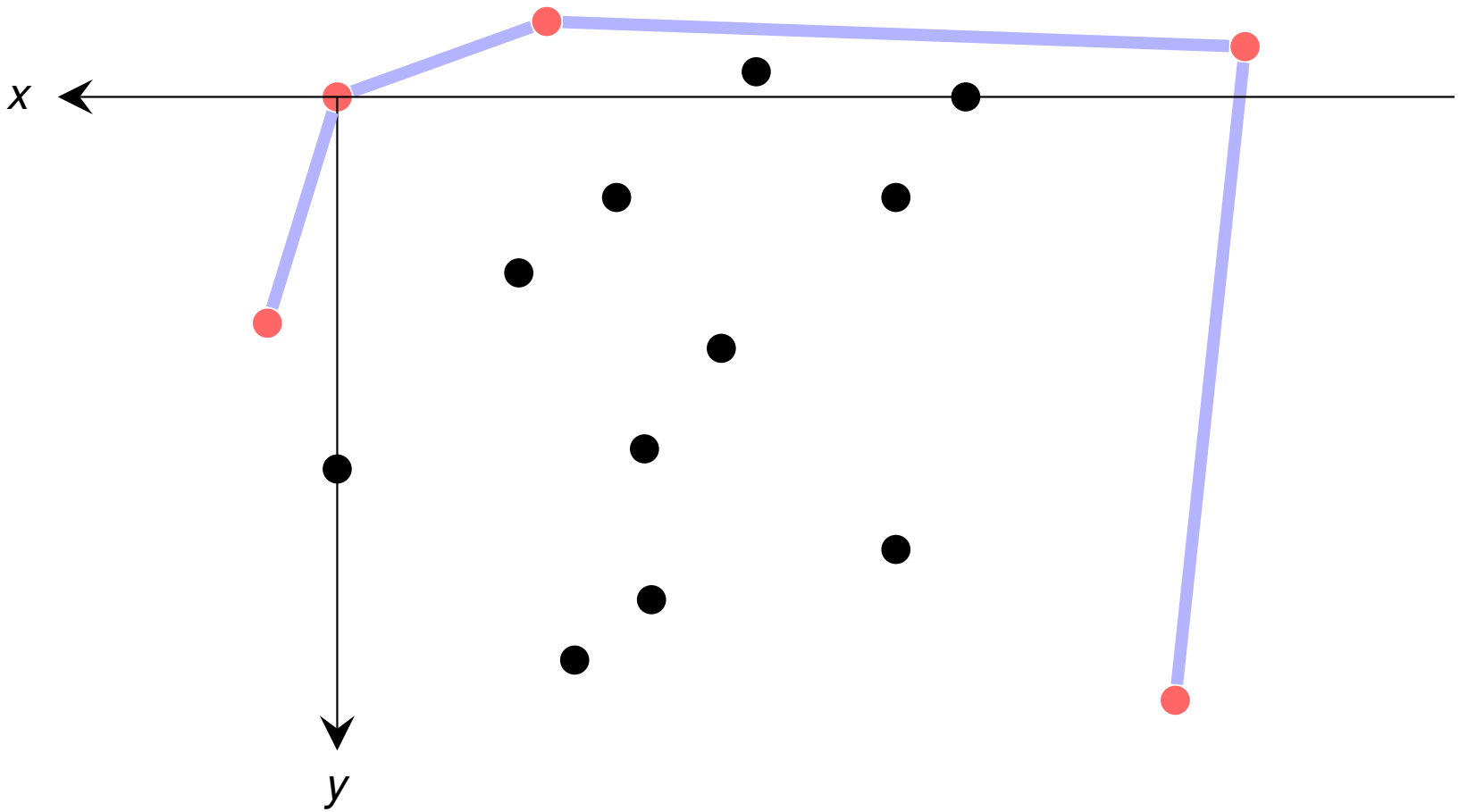

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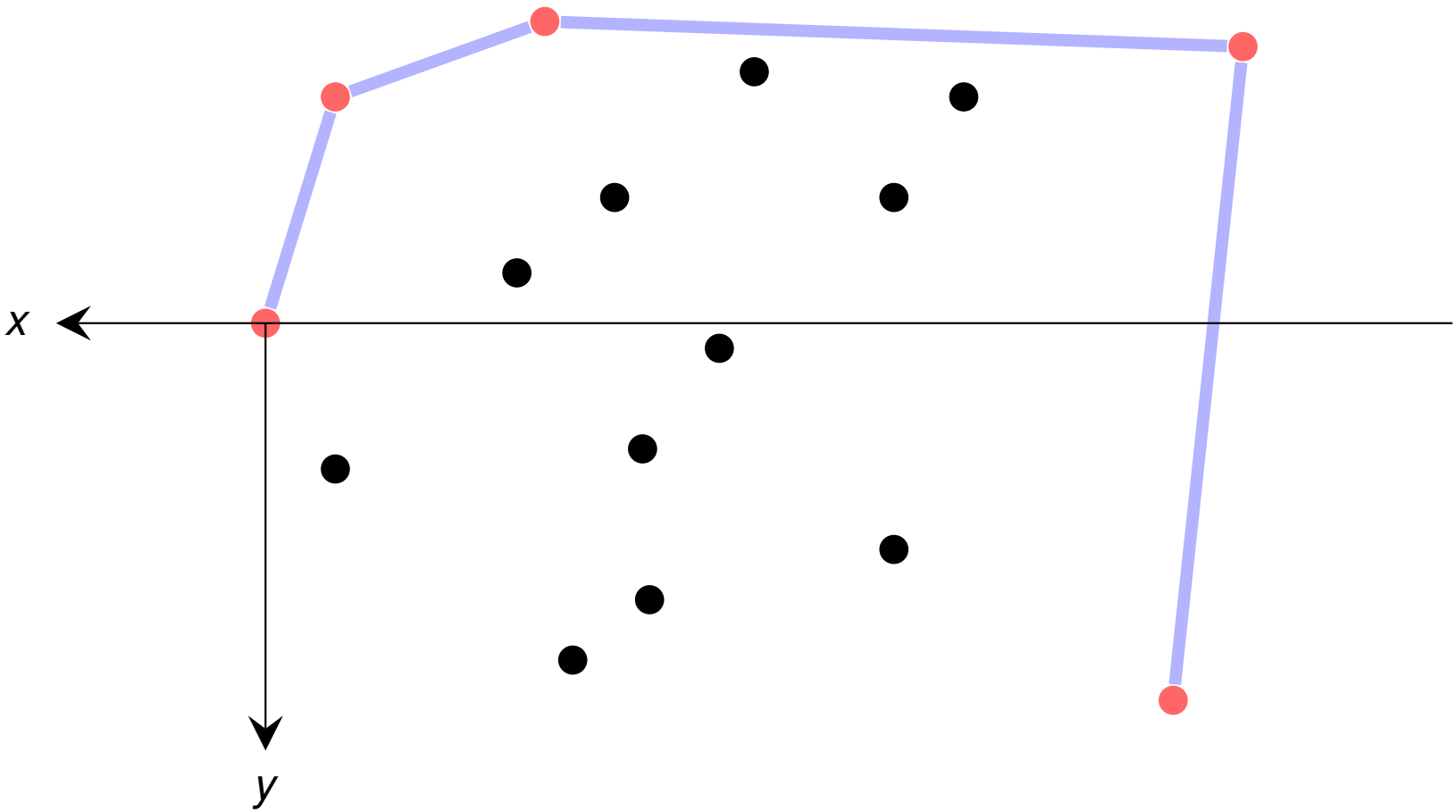

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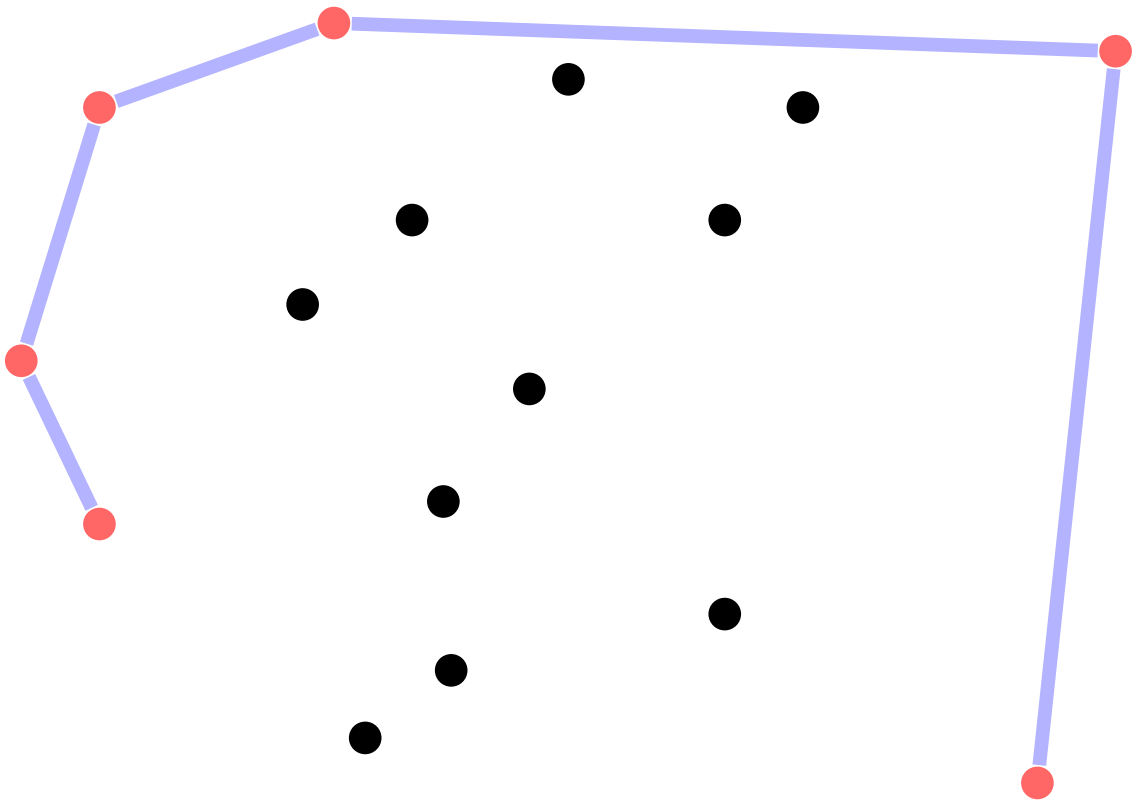

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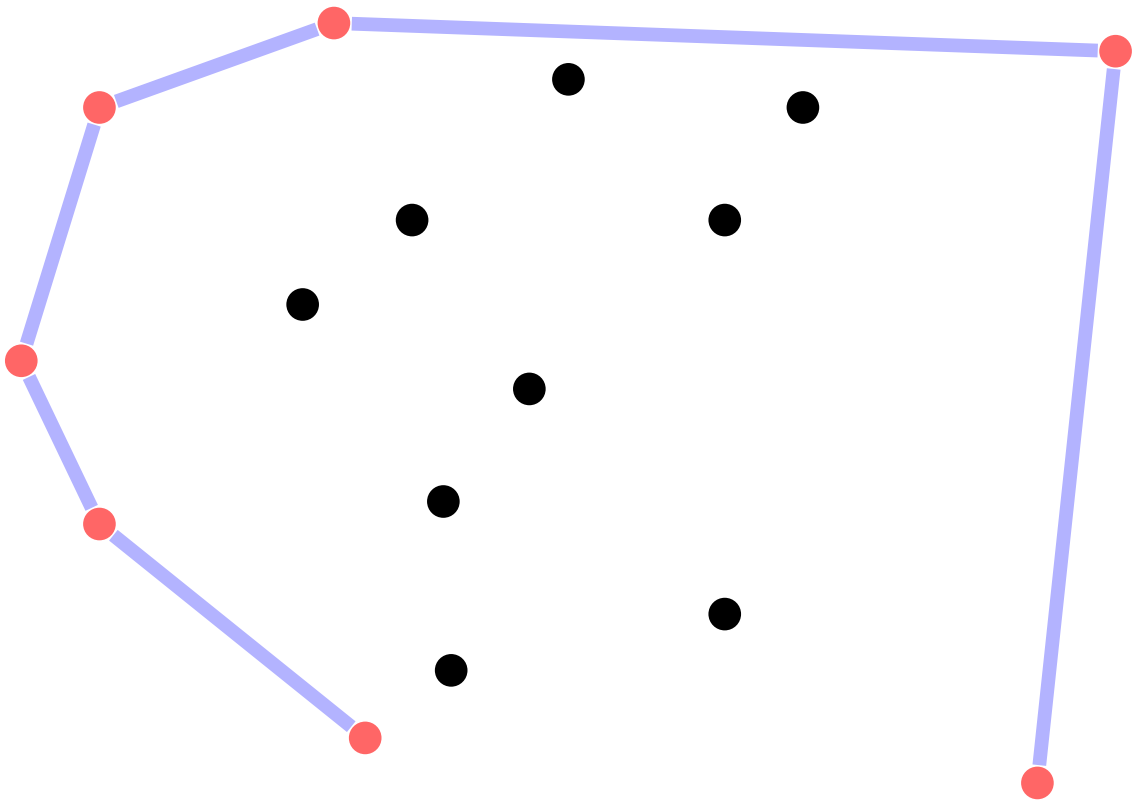
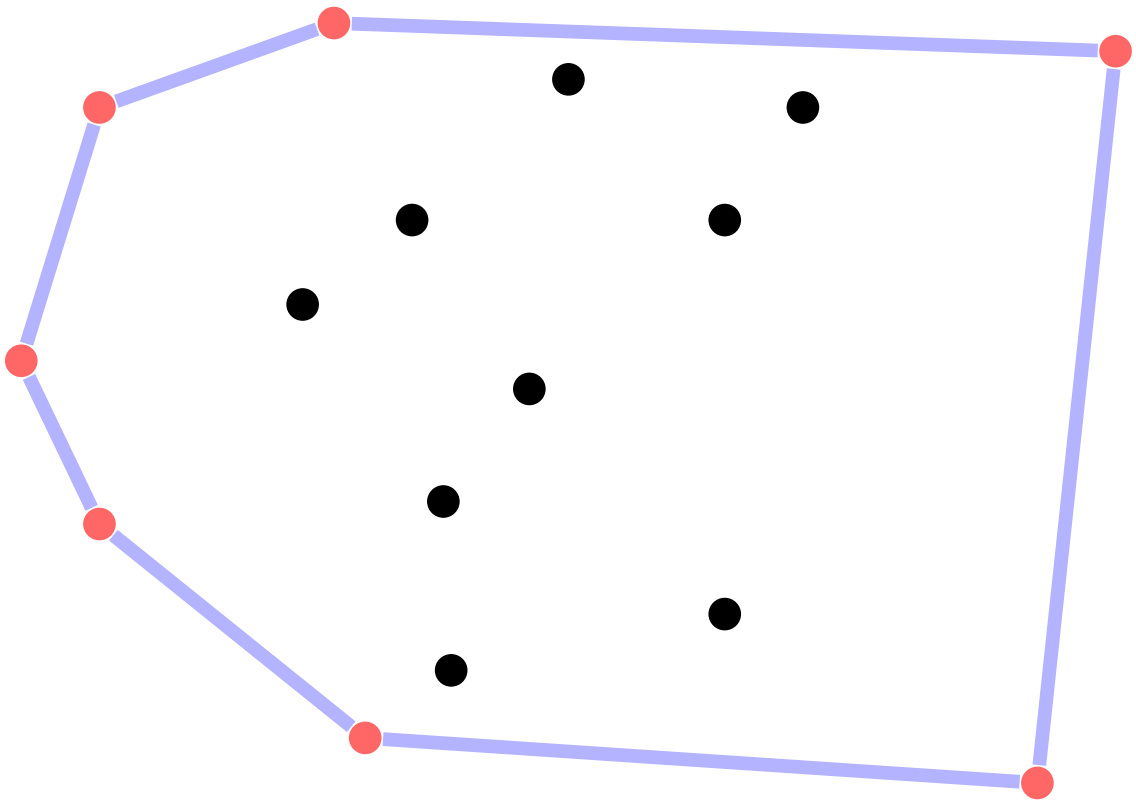


Execution of Jarvis' March



Computing Convex Hull: Summary

Graham's Scan

- natural backtracking algorithm
- cross-product avoids computing polar angles
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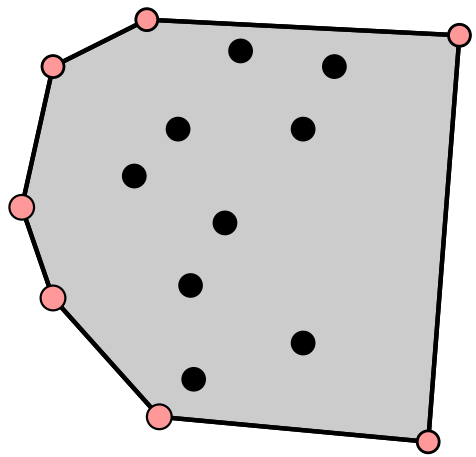

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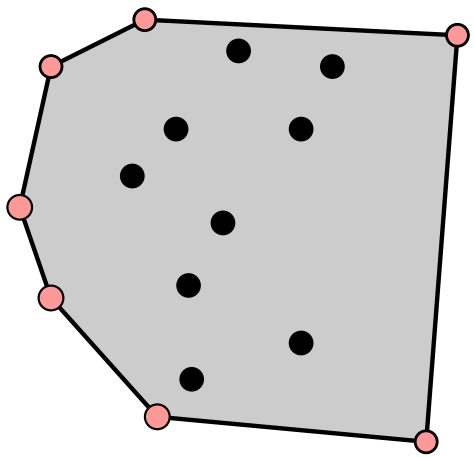
Improves Graham's scan only if $h = O(\log n)$

There exists an algorithm with $O(n \log h)$ runtime!



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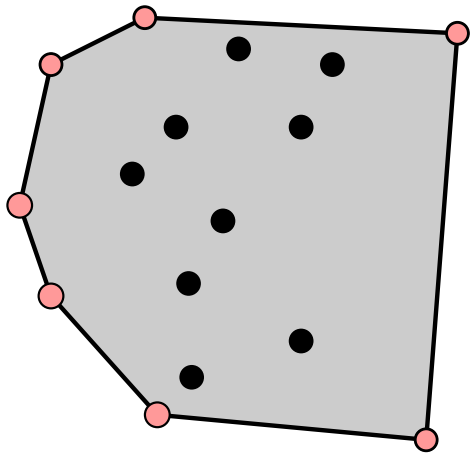

Lessons Learned

- cross product very powerful tool
- need to take care of degenerate cases, numerical precision

The End

Thank you for attending this course &
Best wishes for the rest of your Tripos!

- Don't forget to visit the online feedback page!
- Please send comments on the slides (typos, criticsm, praise etc.) to: **`tms41@cam.ac.uk`**