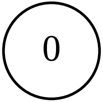
Triggering a cascading cut of length $k = 6$ in a Fibonacci heap

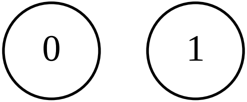
Step 1 - Add nodes

First, we will add enough items to allow for a path from root to leaf with $k = 6$ intermediate nodes to exist. This means $2^{k+1} = 2^7 = 128$ nodes. We will add one more than that in order to have only a single (large) tree after Step 2.

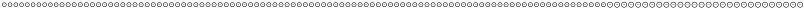
Step 1 - Add nodes - Insert(0)



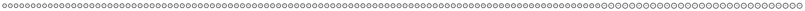
Step 1 - Add nodes - INSERT(1)



Step 1 - Add nodes - 126 INSERTs later



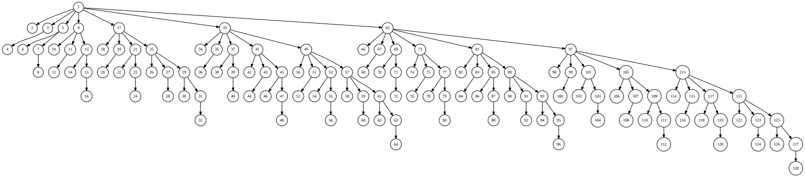
Step 1 - Add nodes - INSERT(128)



Step 2 - Consolidation

We now perform an EXTRACT-MIN operation to force a consolidation step which will merge our singleton trees into a single tree with depth $k + 1 = 7$.

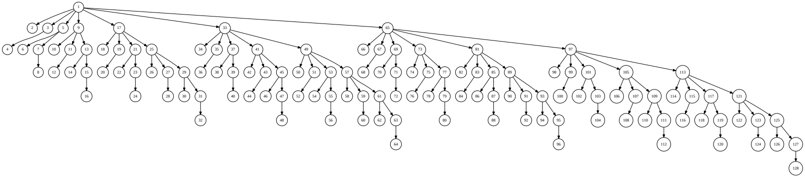
Step 2 - Consolidation - Result



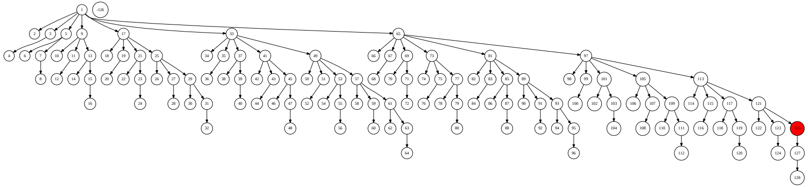
Step 3 - Strategic cuts

We will perform $k - 1 = 5$ Decrease-Key operations on $k - 1 = 5$ carefully chosen nodes. These nodes will be cut from the start tree and appended to the root list. As a result of this, certain nodes in the start tree will be marked. This prepares the cascading cut which we will trigger in the next step.

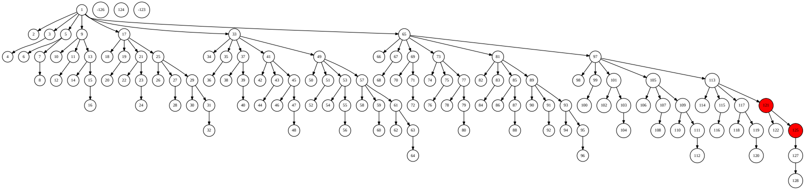
Step 3 - Strategic cuts - Before first cut



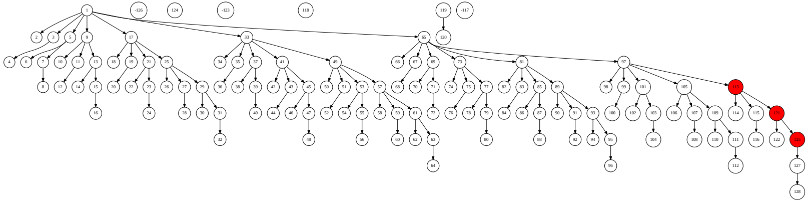
Step 3 - Strategic cuts - After DECREASE-KEY(126,-126)



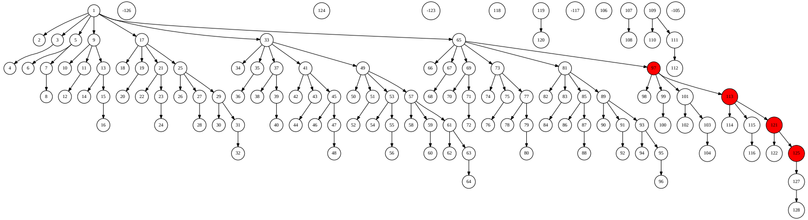
Step 3 - Strategic cuts - After Decrease-Key(123,-123)



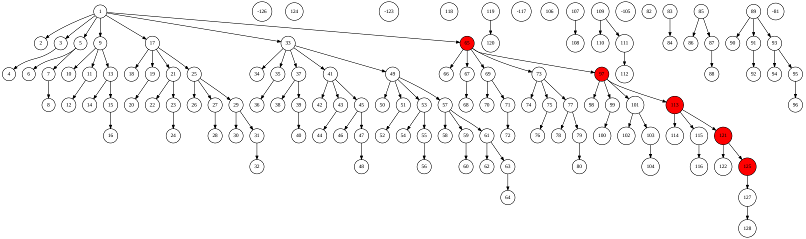
Step 3 - Strategic cuts - After Decrease-Key(117,-117)



Step 3 - Strategic cuts - After DECREASE-KEY(105,-105)



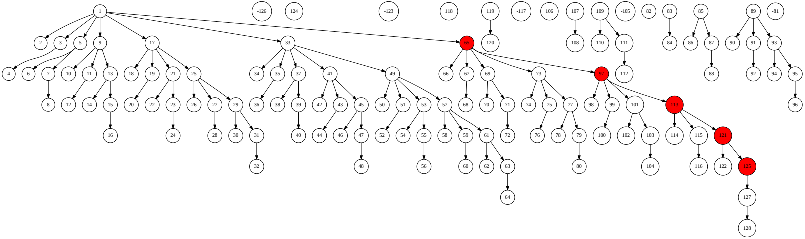
Step 3 - Strategic cuts - After DECREASE-KEY(81,-81)



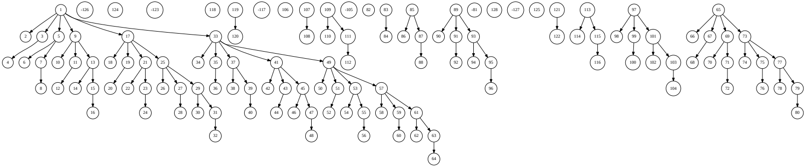
Step 4 - Triggering the cascading cut

Lastly we need to trigger our cascading cut. This is done by removing (ie. Decrease-Key so that it violates the heap property) a child (in this case item 127) of the bottommost node in the chain of marked nodes we have set up in Step 3.

Step 4 - Triggering the cascading cut - Before



Step 4 - Triggering the cascading cut - After



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