

Introduction to MATLAB

– exercises and solution notes

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Michaelmas 2015 – Part II

Exercise 1: Find a *short* MATLAB expression to build the matrix

$$B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 9 & 7 & 5 & 3 & 1 & -1 & -3 \\ 4 & 8 & 16 & 32 & 64 & 128 & 256 \end{pmatrix}$$

Answer: `b = [1:7; 9:-2:-3; 2.^(2:8)]`

Exercise 2: Give a MATLAB expression that uses only a single matrix multiplication with B to obtain

(a) the sum of columns 5 and 7 of B

Answer: `b * [0 0 0 0 1 0 1]'`

(b) the last row of B

Answer: `[0 0 1] * b`

(c) a version of B with rows 2 and 3 swapped

Answer: `[1 0 0; 0 0 1; 0 1 0] * b`

Exercise 3: Give a MATLAB expression that multiplies two vectors to obtain

(a) the matrix $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}$

Answer: `[1 1 1]' * (1:5)`

(b) the matrix $\begin{pmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \\ 4 & 4 & 4 \end{pmatrix}$

Answer: `(0:4)' * [1 1 1]`

Exercise 4: Modify slide 30 to produce tones of falling frequency instead.

Answer: Replace

```
f = fmin * (fmax/fmin) .^ 1;
```

with

```
f = fmax * (fmin/fmax) .^ 1;
```

Exercise 5:

- (a) Write down the function $g(t)$ that has the shape of a sine wave that increases linearly in frequency from 0 Hz at $t = 0$ s to 5 Hz at $t = 10$ s.

Answer: The instantaneous frequency of function $g(t)$ at time t is

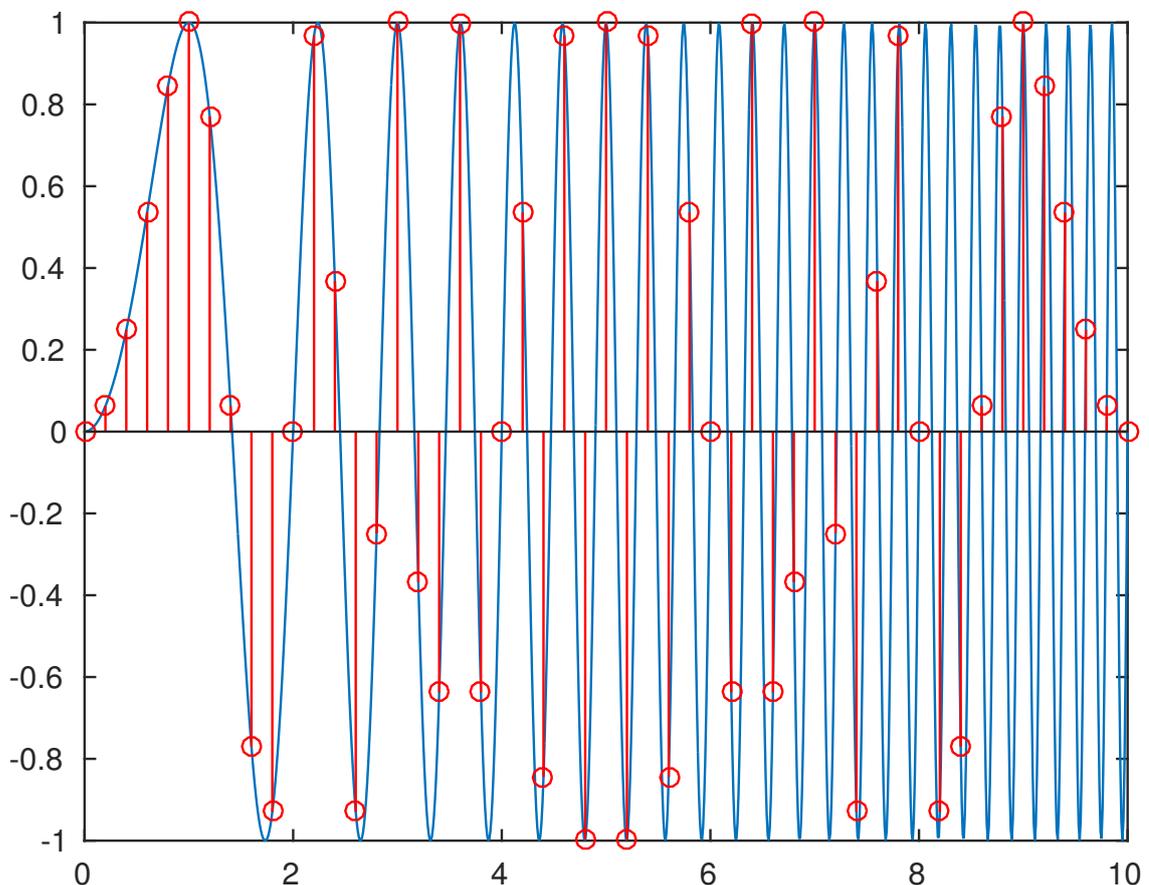
$$f(t) = t \cdot \frac{5 \text{ Hz}}{10 \text{ s}} = \frac{t}{2 \text{ s}^2}$$

and since the phase of a sine wave is 2π times the integrated frequency so far, we get

$$g(t) = \sin\left(2\pi \int_0^t f(t') dt'\right) = \sin\left(2\pi \frac{t^2}{4 \text{ s}^2}\right) = \sin\left(\frac{\pi t^2}{2 \text{ s}^2}\right)$$

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- (b) Plot the graph of this function using MATLAB's `plot` command.
(c) Add to the same figure (this can be achieved using the `hold` command) in a different colour a graph of the same function sampled at 5 Hz, using the `stem` command.

Answer: for (b) and (c)



```

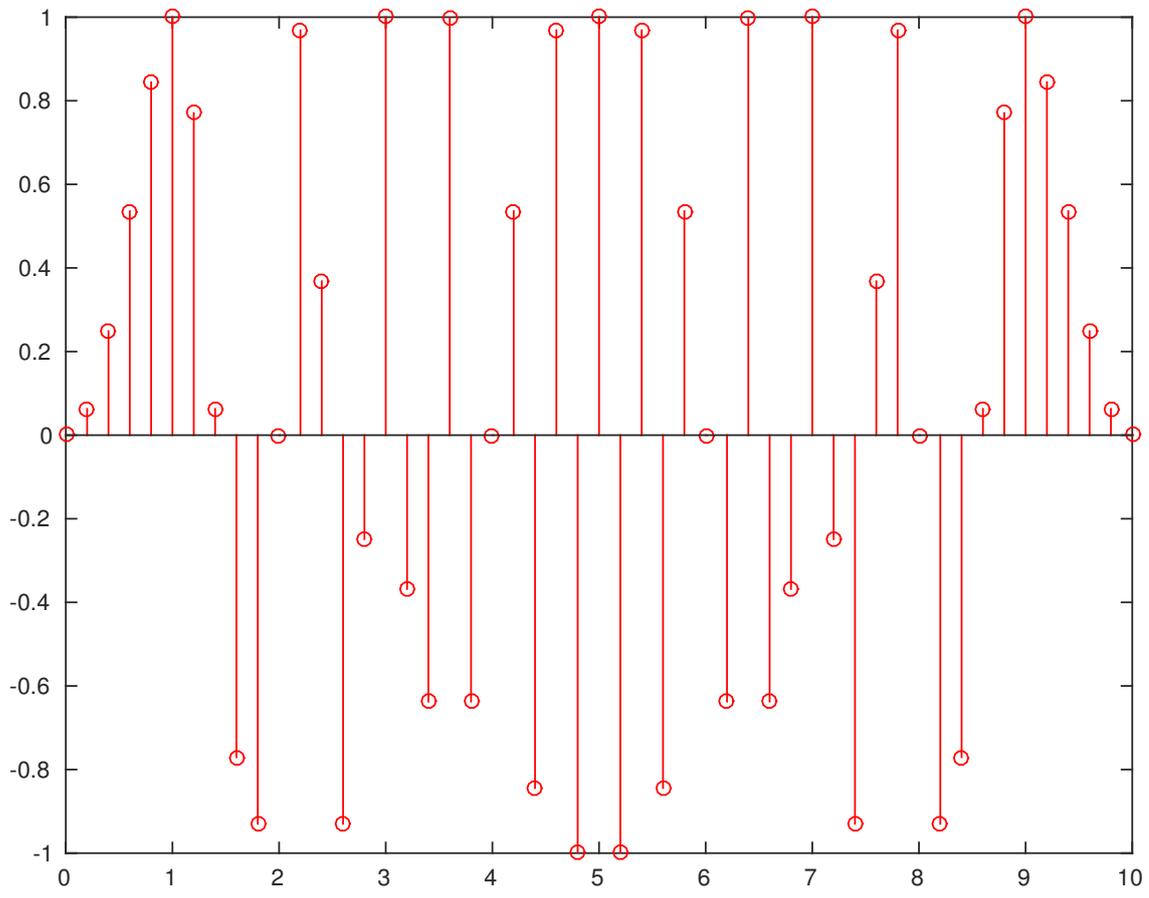
t = 0:0.01:10;
f = sin(pi*t.^2/2);
plot(t,f);
hold;
t2 = 0:1/5:10;
stem(t2, sin(pi*t2.^2/2), 'r');

```

- (d) [Extra credit] Plot the graph from (c) separately. Can you explain its symmetry? [Hints: sampling theorem, aliasing].

Answer: A sine wave with a frequency f larger than half the sampling frequency f_s cannot be distinguished based on the sample values from a sine wave of frequency $f_s - f$. In other words, the sample values would have looked the same had we replaced the instantaneous frequency $f(t)$ with $f_s/2 - |f_s/2 - f(t)|$, and the latter is symmetric around $f_s/2$, which is in this graph 2.5 Hz and occurs at $t = 5$ s.

[The above is of course just a hand-waving argument, but shall be sufficient for this exercise. There are actually a few more conditions fulfilled here that lead to the exact symmetry of the plot. Firstly, since we started sampling at $t = 0$ s with $f_s = 5$ Hz, the positions of the sample values end up being symmetric around $t = 5$ s. Secondly, at the symmetry point $t = 5$ s, the sine wave was at a symmetric peak from where increasing or decreasing the phase has the same result.]



Exercise 6: Use MATLAB to write an audio waveform (8 kHz sampling frequency) that contains a sequence of nine tones with frequencies 659, 622, 659, 622, 659, 494, 587, 523, and 440 Hz. Then add to this waveform a copy of itself in which every other sample has been multiplied by -1 . Play the waveform, write it to a WAV file, and use the `specgram` command to plot its spectrogram with correctly labelled time and frequency axis.

Answer:

```
f = [659 622 659 622 659 494 587 523 440];
fs = 8000; % sampling frequency
d = 0.5; % duration per tone
t = 0:1/fs:d-1/fs;
w = sin(2 * pi * f' * t)/2;
w = w'; w = w(:)';
w = [w, w .* (mod((1:length(w)), 2) * 2 - 1)];
audiowrite('matlab_answer-2.wav', w, fs);
specgram(w, [], fs);
```

