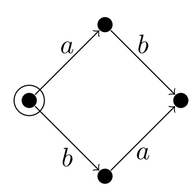
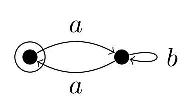
Topics in Concurrency: Problem sheet 1

1. Draw transition systems for the following CCS processes:
   - $(\tau + a!1) \parallel a?X$
   - $(P \parallel a!1 \parallel a!2) \setminus \{a\}$ where $P \stackrel{\text{def}}{=} a?x \to ((x = 1 \to b!X) + P)$
2. Give two distinct *pure CCS* processes that correspond to the initial state of the following transition system:



3. Give a pure CCS term using the alternative syntax for recursion $\mathsf{rec}_i(P_1 = p_1, \dots, P_n = p_n)$ that corresponds to the initial state of the following transition system:



4. Give the least relation on CCS terms that is a bisimulation and contains the two terms that you have given for Question 2.
5. Show that $a.(b + c) \not\sim a.b + a.c + a.(b + c)$ by giving a formula of Hennessy-Milner logic satisfied by one process but not by the other.
6. Give a finite assertion A in Hennessy-Milner logic with the property

   $$p \vDash A \quad \text{iff} \quad p \text{ is strongly bisimilar to the process } a.\mathbf{nil}$$

   for any CCS process p with actions being restricted to the set $\{a, b\}$.
7. Show, by exhibiting bisimulation relations, that

   $$\begin{array}{rcl} (p + q) \setminus L & \sim & p \setminus L + q \setminus L \\ (p + q)[f] & \sim & p[f] + q[f]. \end{array}$$

   Show that if $p \sim p'$ then $p \parallel q \sim p' \parallel q$.

8. Give pure CCS processes p and q such that $(p \parallel q)[f] \not\sim p[f] \parallel q[f]$. Demonstrate that there is no bisimulation relating p and q by giving a formula of Hennessy-Milner logic satisfied by one process and not by the other.

9. A *simulation* between pure CCS terms is defined to be a binary relation S between pure CCS terms such that whenever $(p, q) \in S$ for all actions a and processes p'

$$p \xrightarrow{a} p' \implies \exists q'.\ q \xrightarrow{a}\ \&\ (p', q') \in S.$$

Write $p \leq q$ iff there is a simulation S such that $(p, q) \in S$.

- Give pure CCS terms to show that $p \leq q$ and $q \leq p$ together do not necessarily imply that p and q are strongly bisimilar.
- Consider the fragment of Hennessy-Milner logic

$$A ::= \langle a \rangle A \quad | \quad \bigwedge_{i \in I} A_i$$

where I is a set and a is an action of pure CCS.
Show that

$$p \leq q \quad \text{iff} \quad \forall A.\ p \vDash A \implies q \vDash A.$$