Lecture 8: Linkage algorithms and web search

Information Retrieval
Computer Science Tripos Part II

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[1]Based on slides from Simone Teufel and Ronan Cummins

Upcoming today

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- How to compute PageRank

Overview

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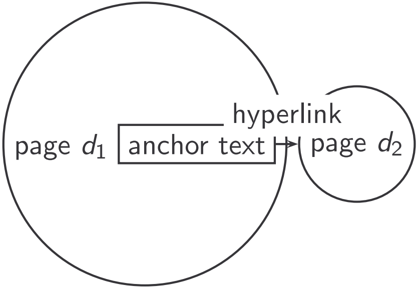

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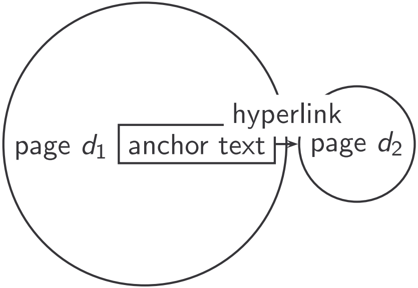

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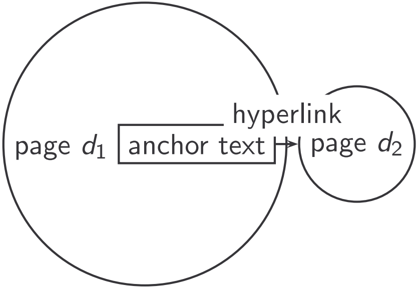

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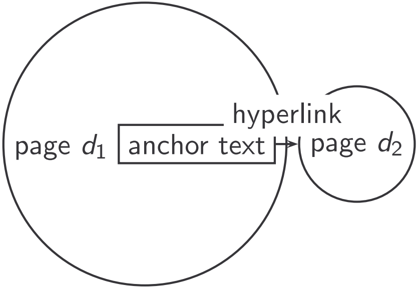

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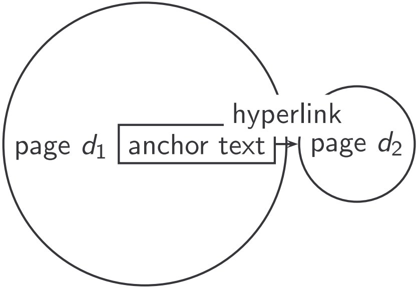

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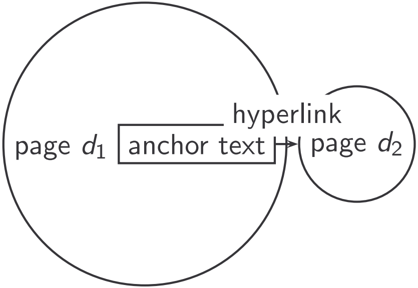

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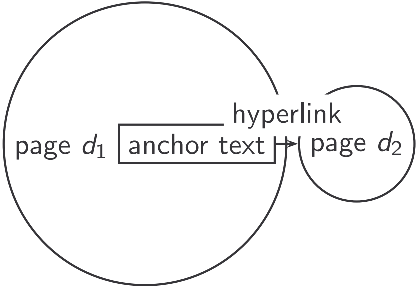

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- Searching on [anchor text $\rightarrow d_2$] is better for the query *IBM*.
  - In this representation, the page with the most occurrences of *IBM* is www.ibm.com.

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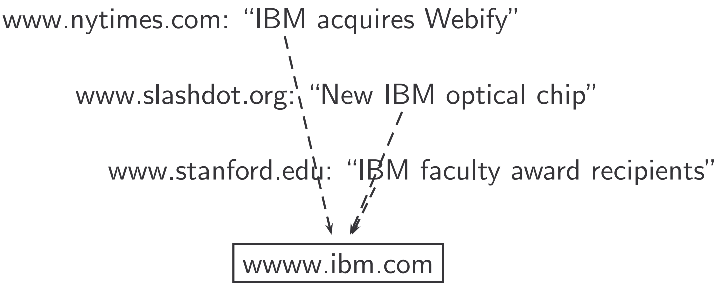

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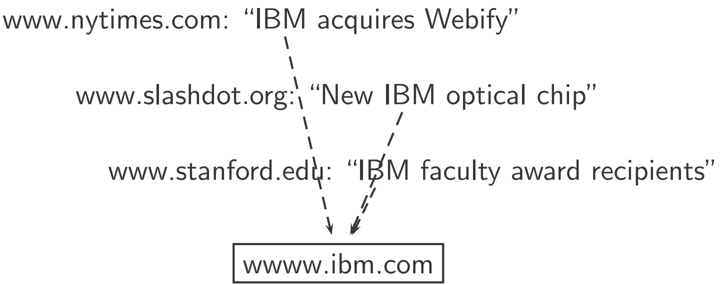

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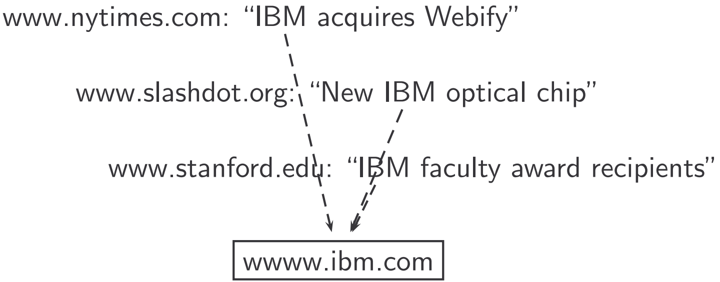

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- Anchor text can be weighted more highly than document text. (based on Assumptions 1&2)

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- Defused Google bombs: [who is a failure?], [evil empire]

A historic google bomb

Web Images Groups News Froogle Local **more »**

miserable failure | Search | Advanced Search | Preferences

Web Results **1** - **10** of about **969,000** for **miserable failure**. (**0.06** seconds)

Biography of President George W. Bush
Biography of the president from the official White House web site.
www.whitehouse.gov/president/gwbbio.html - 29k - Cached - Similar pages
Past Presidents - Kids Only - Current News - President
More results from www.whitehouse.gov »

Welcome to MichaelMoore.com!
Official site of the gadfly of corporations, creator of the film Roger and Me and the television show The Awful Truth. Includes mailing list, message board, ...
www.michaelmoore.com/ - 35k - Sep 1, 2005 - Cached - Similar pages

BBC NEWS | Americas | '**Miserable failure**' links to Bush
Web users manipulate a popular search engine so an unflattering description leads to the president's page.
news.bbc.co.uk/2/hi/americas/3298443.stm - 31k - Cached - Similar pages

Google's (and Inktomi's) **Miserable Failure**
A search for **miserable failure** on Google brings up the official George W. Bush biography from the US White House web site. Dismissed by Google as not a ...
searchenginewatch.com/sereport/article.php/3296101 - 45k - Sep 1, 2005 - Cached - Similar pages

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- Next: PageRank algorithm for computing weighted citation frequency on the web

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- PageRank = long-term visit rate = steady state probability

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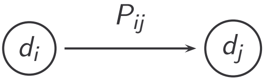
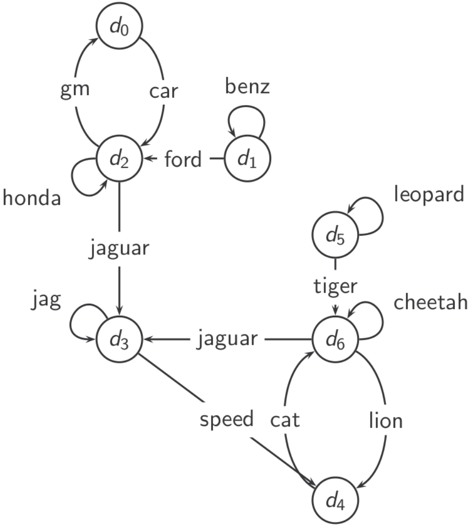
- Clearly, for all i, $\sum_{j=1}^{N} P_{ij} = 1$



Example web graph



Link matrix for example

	d_0	d_1	d_2	d_3	d_4	d_5	d_6
d_0	0	0	1	0	0	0	0
d_1	0	1	1	0	0	0	0
d_2	1	0	1	1	0	0	0
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d_6	0	0	0	1	1	0	1

Transition probability matrix P for example

	d_0	d_1	d_2	d_3	d_4	d_5	d_6
d_0	0.00	0.00	1.00	0.00	0.00	0.00	0.00
d_1	0.00	0.50	0.50	0.00	0.00	0.00	0.00
d_2	0.33	0.00	0.33	0.33	0.00	0.00	0.00
d_3	0.00	0.00	0.00	0.50	0.50	0.00	0.00
d_4	0.00	0.00	0.00	0.00	0.00	0.00	1.00
d_5	0.00	0.00	0.00	0.00	0.00	0.50	0.50
d_6	0.00	0.00	0.00	0.33	0.33	0.00	0.33

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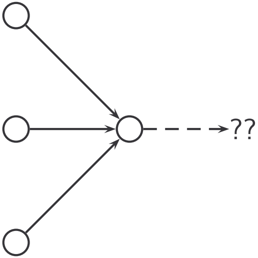
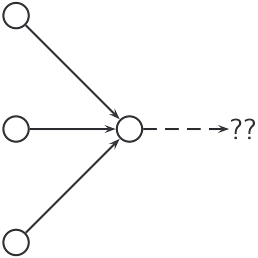
- The web graph must correspond to an ergodic Markov chain.
- First a special case: The web graph must not contain dead ends.

Dead ends





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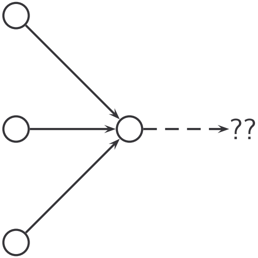

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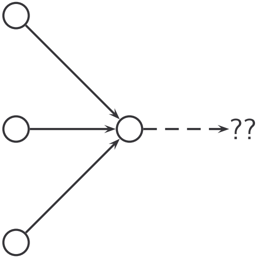

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- Random walk can get stuck in dead ends.
- If there are dead ends, long-term visit rates are not well-defined (or non-sensical).

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- Note: "jumping" from dead end is independent of teleportation rate.

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- But even without dead ends, a graph may not have well-defined long-term visit rates.
- More generally, we require that the Markov chain be ergodic.

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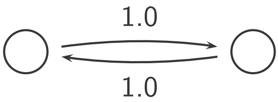
- A non-ergodic Markov chain:



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- $\Rightarrow$ Web-graph+teleporting has a steady-state probability distribution.

Ergodic Markov chains

- Theorem: For any ergodic Markov chain, there is a unique long-term visit rate for each state.
- This is the steady-state probability distribution.
- Over a long time period, we visit each state in proportion to this rate.
- It doesn't matter where we start.
- Teleporting makes the web graph ergodic.
- $\Rightarrow$ Web-graph+teleporting has a steady-state probability distribution.
- $\Rightarrow$ Each page in the web-graph+teleporting has a PageRank.

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- Next: how to compute PageRank

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- $\sum x_i = 1$

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- So from $\vec{x}$, our next state is distributed as $\vec{x}P$.

Steady state in vector notation

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- (We use $\vec{\pi}$ to distinguish it from the notation for the probability vector $\vec{x}$.)
- π_i is the long-term visit rate (or PageRank) of page i.
- So we can think of PageRank as a very long vector – one entry per page.

Steady-state distribution: Example

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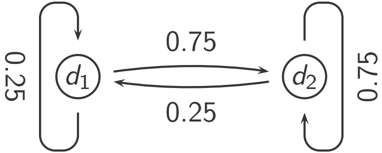
What is the PageRank / steady state in this example?



Steady-state distribution: Example

	x_1 $P_t(d_1)$	x_2 $P_t(d_2)$		
			$P_{11} = 0.25$	$P_{12} = 0.75$
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t_0				
t_1				

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			$P_{11} = 0.25$ $P_{12} = 0.75$ $P_{21} = 0.25$ $P_{22} = 0.75$
t_0	0.25	0.75	
t_1	0.25		

$$
\begin{aligned}
P_t(d_1) &= P_{t-1}(d_1) \cdot P_{11} + P_{t-1}(d_2) \cdot P_{21} \\
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PageRank vector $= \vec{\pi} = (\pi_1, \pi_2) = (0.25, 0.75)$

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- All transition probability matrices have largest eigenvalue 1.

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- Algorithm: multiply $\vec{x}$ by increasing powers of P until convergence.
- This is called the power method.
- Recall: regardless of where we start, we eventually reach the steady state $\vec{\pi}$.
- Thus: we will eventually (in asymptotia) reach the steady state.

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t_0	0	1			$= \vec{x}P$
t_1					$= \vec{x}P^2$
t_2					$= \vec{x}P^3$
t_3					$= \vec{x}P^4$
					$\dots$
t_∞					$= \vec{x}P^\infty$

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t_1	0.3	0.7	0.24	0.76	$= \vec{x}P^2$
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t_2	0.24	0.76	0.252	0.748	$= \vec{x}P^3$
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			...		...
t_∞					$= \vec{x}P^\infty$

$$P_t(d_1) = P_{t-1}(d_1) * P_{11} + P_{t-1}(d_2) * P_{21}$$
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Computing PageRank: Power method

	x_1 $P_t(d_1)$	x_2 $P_t(d_2)$			
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PageRank vector $= \vec{\pi} = (\pi_1, \pi_2) = (0.25, 0.75)$

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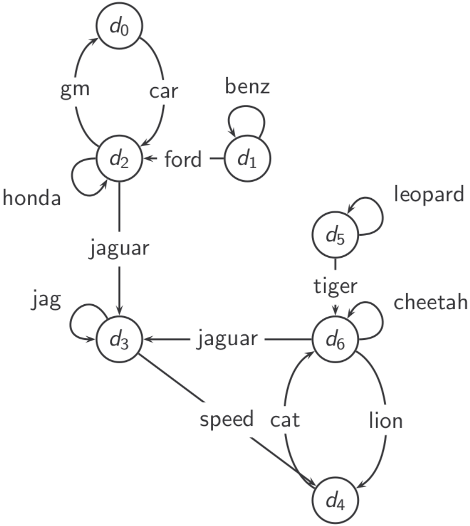
  - Clearly not desirable
- In practice: rank according to weighted combination of raw text match, anchor text match, PageRank & other factors

Example web graph



Transition (probability) matrix

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	d_0	d_1	d_2	d_3	d_4	d_5	d_6
d_0	0.00	0.00	1.00	0.00	0.00	0.00	0.00
d_1	0.00	0.50	0.50	0.00	0.00	0.00	0.00
d_2	0.33	0.00	0.33	0.33	0.00	0.00	0.00
d_3	0.00	0.00	0.00	0.50	0.50	0.00	0.00
d_4	0.00	0.00	0.00	0.00	0.00	0.00	1.00
d_5	0.00	0.00	0.00	0.00	0.00	0.50	0.50
d_6	0.00	0.00	0.00	0.33	0.33	0.00	0.33

Transition matrix with teleporting ($\alpha = 0.14$)

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	d_0	d_1	d_2	d_3	d_4	d_5	d_6
d_0	0.02	0.02	0.88	0.02	0.02	0.02	0.02
d_1	0.02	0.45	0.45	0.02	0.02	0.02	0.02
d_2	0.31	0.02	0.31	0.31	0.02	0.02	0.02
d_3	0.02	0.02	0.02	0.45	0.45	0.02	0.02
d_4	0.02	0.02	0.02	0.02	0.02	0.02	0.88
d_5	0.02	0.02	0.02	0.02	0.02	0.45	0.45
d_6	0.02	0.02	0.02	0.31	0.31	0.02	0.31

Power method vectors $\vec{x}P^k$

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	$\vec{x}$	$\vec{x}P^1$	$\vec{x}P^2$	$\vec{x}P^3$	$\vec{x}P^4$	$\vec{x}P^5$	$\vec{x}P^6$	$\vec{x}P^7$	$\vec{x}P^8$	$\vec{x}P^9$	$\vec{x}P^{10}$	$\vec{x}P^{11}$	$\vec{x}P^{12}$	$\vec{x}P^{13}$
d_0	0.14	0.06	0.09	0.07	0.07	0.06	0.06	0.06	0.06	0.05	0.05	0.05	0.05	0.05
d_1	0.14	0.08	0.06	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
d_2	0.14	0.25	0.18	0.17	0.15	0.14	0.13	0.12	0.12	0.12	0.12	0.11	0.11	0.11
d_3	0.14	0.16	0.23	0.24	0.24	0.24	0.24	0.25	0.25	0.25	0.25	0.25	0.25	0.25
d_4	0.14	0.12	0.16	0.19	0.19	0.20	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.21
d_5	0.14	0.08	0.06	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
d_6	0.14	0.25	0.23	0.25	0.27	0.28	0.29	0.29	0.30	0.30	0.30	0.30	0.31	0.31

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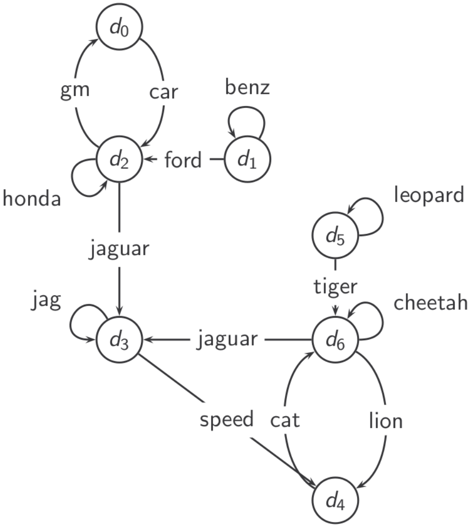

Example web graph

	PageRank
d_0	0.05
d_1	0.04
d_2	0.11
d_3	0.25
d_4	0.21
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PageRank(d_2) $<$ PageRank(d_6): why?

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- Addressing link spam is difficult and crucial.

Overview

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Link Analysis

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- And also Hyperlink-Induced Topic Search (HITS)

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- PageRank can be viewed as the stationary distribution of a Markov chain
- Power iteration is *one simple method* of calculating the stationary distribution
- Topic sensitive variants exist

Reading

- MRS Chapter 21, excluding 21.3.
- MRS 21.3 on HITS algorithm – *optional*