Overview of Natural Language Processing Part II & ACS L390

Gradient Descent and Neural Nets

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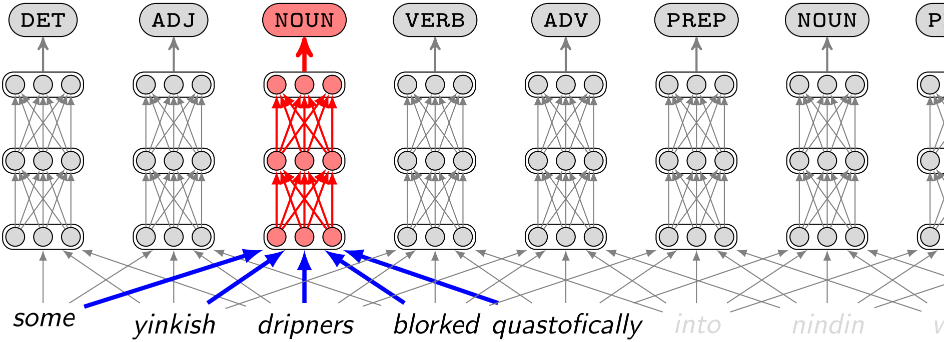
Michaelmas 2024/25



Gradient Descent and Neural Nets

1. Gradient Descent
2. Feedforward Neural Networks

Gradient Descent

Supervised learning

Assume there is a *good* annotated corpus

$$\mathcal{D} = \left\{ (x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \ldots, (x^{(l)}, y^{(l)}) \right\}$$

How can we get a *good* parameter vector?

Maximum-Likelihood Estimation

$$\hat{\theta} = \arg\max L(\theta)$$

where $L(\theta)$ is the log-likelihood of observing the data $\mathcal{D}$:

$$L(\theta) = \sum_{i=1}^{l} \log p(y^{(i)} | x^{(i)}; \theta)$$

Gradient Descent/Ascent

In general, finding a minimum/maximum is *hard*.

However, a simple idea that often works:

- Initialise θ with some value
- Iteratively improve θ

The derivative tells us whether to increase or decrease (but doesn't tell us how much to increase/decrease by):

$$\theta^{[t+1]} = \theta^{[t]} + \beta \frac{dL}{d\theta}(\theta^{[t]})$$

Gradient Descent for the Log-Linear Model

Assume we have a *parameter vector* θ.

$$p(y|x;\theta) = \frac{\exp(\theta^\top f(x,y))}{\sum_{y' \in \mathcal{Y}} \exp(\theta^\top f(x,y'))}$$

i.e.

$$p(y|x;\theta) \propto \exp(\theta^\top f(x,y))$$

$$\begin{aligned} L(\theta) &= \sum_{i=1}^{l} \log p(y^{(i)}|x^{(i)};\theta) \\ &= \sum_{i=1}^{l} \left(\theta^\top f(x^{(i)}, y^{(i)}) - \log \sum_{y' \in \mathcal{Y}} \exp(\theta^\top f(x^{(i)}, y')) \right) \end{aligned}$$

Gradient Descent for the Log-Linear Model

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Calculating gradients (chain rule)

$$\begin{aligned}
\frac{dL}{d\theta_k} &= \sum_{i=1}^{l} \left(f_k(x^{(i)}, y^{(i)}) - \frac{\sum_{y' \in \mathcal{Y}} \left(\exp(\theta^\top f(x^{(i)}, y')) f_k(x^{(i)}, y')\right)}{\sum_{y^* \in \mathcal{Y}} \exp(\theta^\top f(x^{(i)}, y^*))} \right) \\
&= \sum_{i=1}^{l} f_k(x^{(i)}, y^{(i)}) - \sum_{i=1}^{l} \sum_{y' \in \mathcal{Y}} f_k(x^{(i)}, y') \frac{\exp(\theta^\top f(x^{(i)}, y'))}{\sum_{y^* \in \mathcal{Y}} \exp(\theta^\top f(x^{(i)}, y^*))}
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&= \underbrace{\sum_{i=1}^{l} f_k(x^{(i)}, y^{(i)})}_{\textit{empirical counts}} - \underbrace{\sum_{i=1}^{l} \sum_{y' \in \mathcal{Y}} f_k(x^{(i)}, y') p(y' | x^{(i)}; \theta)}_{\textit{expected counts}}
\end{aligned}$$

Gradient Descent: Algorithm

Maximize $L(\theta)$ where

$$\frac{dL}{d\theta_k} = \underbrace{\sum_{i=1}^{l} f_k(x^{(i)}, y^{(i)})}_{\textit{empirical counts}} - \underbrace{\sum_{i=1}^{l} \sum_{y' \in \mathcal{Y}} f_k(x^{(i)}, y') p(y' | x^{(i)}; \theta)}_{\textit{expected counts}}$$

Initialize $\theta^{[0]} \leftarrow 0$
for $t = 1, \dots$
    calculate $\Delta = \frac{dL(\theta^{[t-1]})}{d\theta}$
    calculate $\beta_* = \arg\max_\beta L(\theta + \beta \Delta)$ ▷ line search
    update $\theta^{[t]} \leftarrow \theta^{[t-1]} + \beta_* \Delta$

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Challenges

- Go through every training sample to get Δ;
- Line search is another non-trival optimisation problem

Stochastic Gradient Descent

$$L(\theta) \quad = \quad \sum_{i=1}^{l} \log p(y^{(i)}|x^{(i)};\theta)$$

- Randomly use one or several training samples to get a suboptimal gradient Δ';
- Fix β.

Feedforward Neural Networks

Recap: about linear combination

$$p(y|x;\theta) = \frac{\exp(\theta^\top f(x,y))}{\sum_{y' \in \mathcal{Y}} \exp(\theta^\top f(x,y'))}$$

⋯	0	0	1	0	1	0	0	1	0	1	0	0	0	0	0	0	⋯

f_{1001}: `if` word$_{-2}$=*some* and tag=N

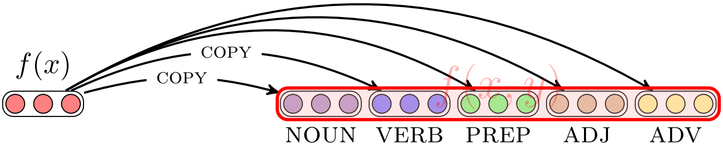
is θ_{1001} positive large?
vote for yes

$\theta^\top f(x,y)$ is a linear combination of θ. That is why a log-linear model is called a linear classifier.

Questions

Can we automate the design of features?

Is linear combination justified?

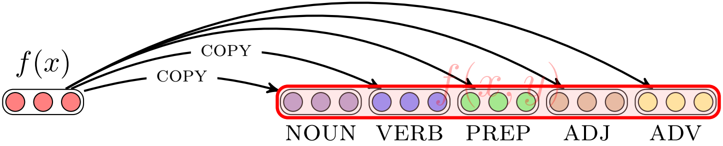


A simple way to define $f(x, y)$ based on $f(x)$ is "copy". We assumed that θ and $f(x, y)$ have DK dimensions:

- D – number of input features
- K – number of output classes

So we can also view θ as comprising K vectors with D dimensions:

$$p(y|x;\theta) \propto \exp(\theta_y^\top f(x))$$



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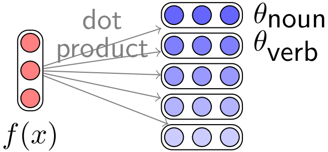

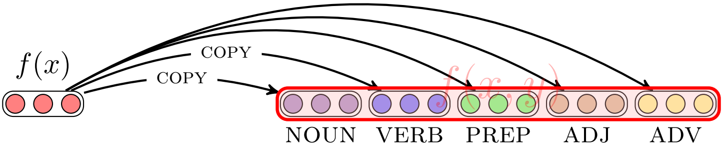

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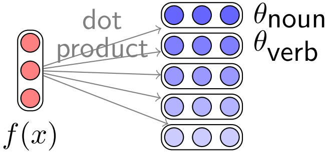
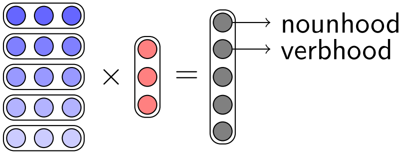
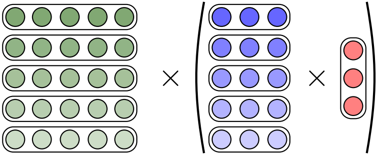




Now consider NER after POS tagging



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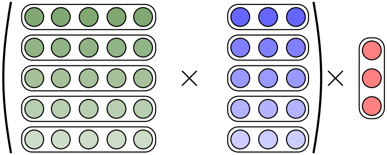

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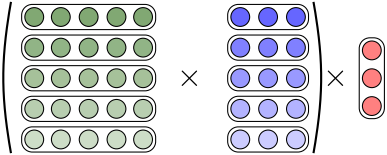

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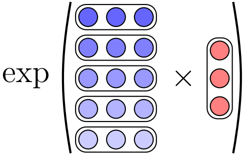
Introduce some non-linearity



$\exp / g$ is applied component-wise (to each dimension separately)

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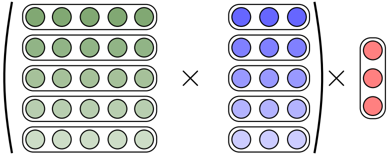

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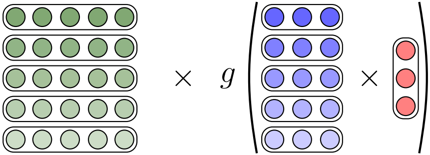

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Feedforward Neural Networks

Think about multi-class classificaition:

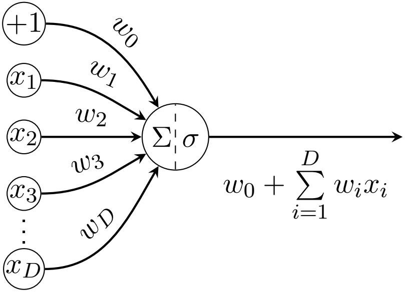
- D – number of features (input)
- K – number of classes (output)
- $\mathbf{x}$ – the input feature vector

Think about a particular class, say y_k. We describe the "friendship" between $\mathbf{x}$ and y_k in the following way:

$$\text{score\_function}(x, y_k) = w_0 + \sum_{i=1}^{D} w_i x_i$$

where $\mathbf{w}$ measures how much each feature w_i contributes to y_k.

Feedforward Neural Networks



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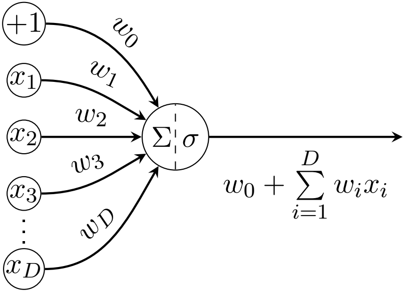
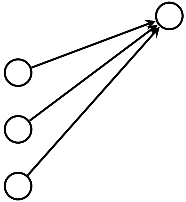
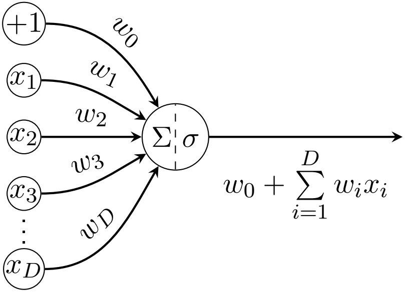
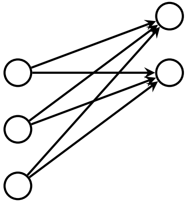




For each class y_k, we do the same thing.

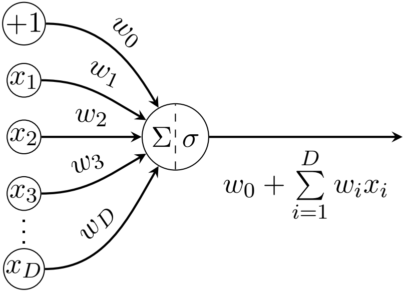
Feedforward Neural Networks

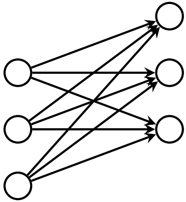




For each class y_k, we do the same thing. Again

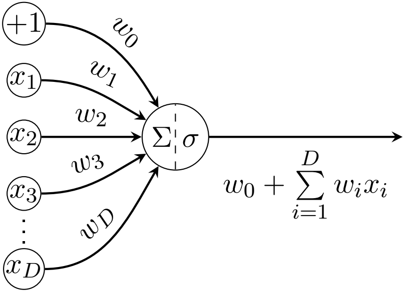
Feedforward Neural Networks

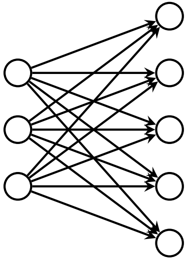




For each class y_k, we do the same thing. Again and again

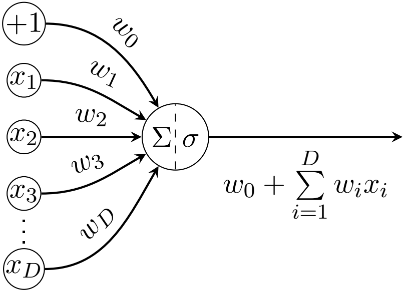
Feedforward Neural Networks

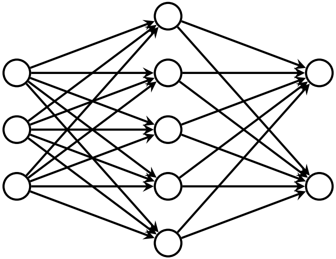




For each class y_k, we do the same thing. Again and again and again. This is called perceptron, which was invented by Frank Rosenblatt in 1958.

Feedforward Neural Networks





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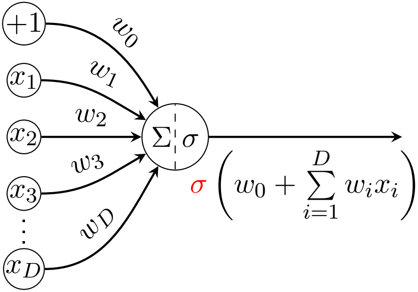
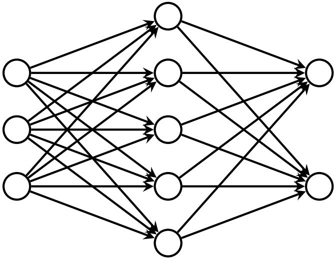
Things will be much more fun if we have a stack of perceptrons.

Feedforward Neural Networks





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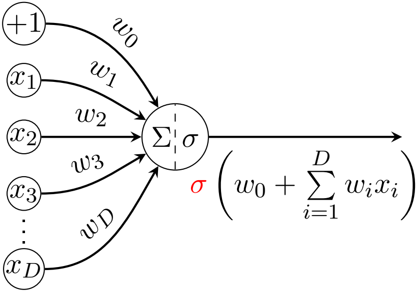
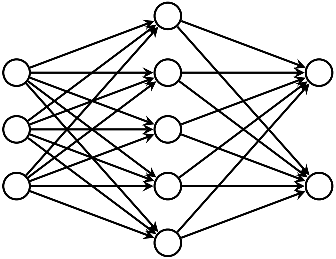
Sigmoid $\sigma(x) = \frac{1}{1+e^{-x}}$

Otherwise, simple matrix multiplication.

Feedforward Neural Networks





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Sigmoid $\sigma(x) = \frac{1}{1+e^{-x}}$

Otherwise, simple matrix multiplication. Now you can do non-linear classification.

Feedforward Neural Networks

A feedforward neural network is a composition of simple functions:

- 1 layer: $\exp(W_1 x)$ ▷ log-linear model
- 2 layers: $\exp(W_2 g(W_1 x))$
- 3 layers: $\exp(W_3 g(W_2 g(W_1 x)))$
- 4 layers: $\exp(W_4 g(W_3 g(W_2 g(W_1 x))))$
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Both $\exp$ and g are applied component-wise (to each dimension separately)

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The activation function g should be non-linear, e.g.:

- Rectified linear unit: $\text{ReLU}(z) = \max(0, z)$
- Hyperbolic tangent: $\tanh(z) = \frac{e^{2z}-1}{e^{2z}+1}$
- Sigmoid: $\sigma(z) = \frac{1}{1+e^{-z}}$

Gradient Descent for Neural Nets

Assume there is a good annotated corpus:

$$\mathcal{D} = \left\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \ldots, (x^{(l)}, y^{(l)})\right\}$$

Aim to maximise the log-likelihood (also called "cross entropy"):

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θ now contains parameters from many layers,
but we can still use gradient descent:

- Backpropagation: efficient application of chain rule
- Iterate many times over training set

Reading

- D Jurafsky and J Martin. *Speech and Language Processing* Chapter 6. `web.stanford.edu/~jurafsky/slp3/6.pdf`

* Essence of linear algebra `www.youtube.com/watch?v=fNk_zzaMoSs&list=PLZHQObOWTQDPD3MizzM2xVFitgF8hE_ab`