1993 Paper 2 Question 8

Consider these ML functions for performing arithmetic in radix `k`, where `k` is an ML variable whose value is a positive integer.

```
fun value []              = 0
 |  value (x::xs)         = x + (k*value xs);

fun carry c []            = [c]
 |  carry c (x::xs)       = ((c+x) mod k) ::
                                    carry ((c+x) div k) xs;

fun sum c []  ys          = carry c ys
 |  sum c (x::xs) []      = carry c (x::xs)
 |  sum c (x::xs) (y::ys) = ((c+x+y)mod k) ::
                                    sum ((c+x+y)div k) xs ys;
```

(*a*) State and justify the rule of structural induction for lists.

(*b*) Your client would like you to prove the correctness of `sum`, expressed by the property

`value(sum 0 xs ys) = value(xs)+value(ys).`

Generalize this formula so that it permits a useful structural induction proof, explaining your reasons.

(*c*) Prove the base case of the structural induction.

(*d*) Prove the inductive step of the structural induction.

(*e*) What does the correctness proof say about the case where `k` equals 1? Discuss whether other properties are necessary to ensure correctness.

Proofs may assume the analogous correctness property for `carry` and standard mathematical laws. State these assumptions clearly.