1993 Paper 7 Question 10

Semantics

An imperative language has boolean expressions be, integer expressions ie, and commands C, whose abstract syntax is specified by:

$$\begin{aligned} ie &::= \underline{n} \mid X \mid ie + ie \mid ie - ie \\ be &::= \underline{b} \mid ie = ie \\ C &::= \mathsf{skip} \mid X := ie \mid C \,;\, C \mid \mathsf{if}\ be\ \mathsf{then}\ C\ \mathsf{else}\ C \mid \mathsf{while}\ be\ \mathsf{do}\ C \end{aligned}$$

where b is *true* or *false*, n is any integer, and X ranges over a fixed set of variables. Describe the operational semantics of the language in terms of inductively defined evaluation relations

$$be, S \Rightarrow b \qquad ie, S \Rightarrow n \quad \text{and} \quad C, S \Rightarrow S'$$

where S and S' are integer-valued functions on the set of variables. [5 marks]

In what sense are these evaluation relations *deterministic*? What is meant by the assertion that two commands are *semantically equivalent*? [3 marks]

For any choice of be, C and C', which of the following pairs of commands are semantically equivalent and which are not? Justify your answer in each case.

(a) $((\mathsf{while}\ be\ \mathsf{do}\ C)\ ;\ C)$ and $(\mathsf{if}\ be\ \mathsf{then}\ ((\mathsf{while}\ be\ \mathsf{do}\ C)\ ;\ C)\ \mathsf{else}\ C)$

(b) $(C\ ;\ (\mathsf{while}\ be\ \mathsf{do}\ C))$ and $(\mathsf{if}\ be\ \mathsf{then}\ (\mathsf{while}\ be\ \mathsf{do}\ C)\ \mathsf{else}\ C)$

(c) $(\mathsf{while}\ be\ \mathsf{do}\ (\mathsf{if}\ be\ \mathsf{then}\ C\ \mathsf{else}\ C'))$ and $(\mathsf{while}\ be\ \mathsf{do}\ C)$

[12 marks]