COMPUTER SCIENCE TRIPOS Part Ib

Thursday 6 June 1996 1.30 to 4.30

Paper 6

Answer **five** *questions.*
No more than **two** *questions from any one section are to be answered.*
Submit the answers in five **separate** *bundles each with its own cover sheet.*
Write on **one** *side of the paper only.*

SECTION A

1 Processor Architecture

Why do processors usually support supervisor and user modes of operation and what are they used for? [6 marks]

What is the difference between *interrupts*, *software interrupts* (initiated by a `SWI` instruction on the ARM) and *exceptions*? [8 marks]

What is an *imprecise exception* and why might a processor designer prefer it to a precise exception mechanism? [6 marks]

2 Computer Architecture

Give the circuit diagram and describe the operation of the following types of memory cell:

(*a*) bipolar memory cell (static) [5 marks]

(*b*) MOS memory cell (static) [5 marks]

Give the key characteristics of *each* of the following types of memory: DRAM, ROM, PROM, EPROM and EEPROM. [2 marks each]

3 Digital Communication I

Operations of similar functionality can be performed at different layers of a protocol stack. Discuss this in relation to

(*a*) routing [4 marks]

(*b*) multiplexing [4 marks]

(*c*) error recovery [4 marks]

(*d*) flow control [4 marks]

(*e*) synchronization [4 marks]

4 Graphics

What are *homogeneous coordinates*? [5 marks]

How can they be used in computer graphics to model

(*a*) translation? [5 marks]

(*b*) simple perspective? [10 marks]

SECTION B

5 Programming in C and C++

A grand debate is being planned by a society that has among its members a large number of computer professionals and working programmers. The motion to be put is "That the languages C and C++ should be consigned to outer darkness and their use banned in all serious computer projects". Prepare as your answer to this question a briefing document that could explain to people intending to attend the debate what the major points both for and against C and C++ will be, and the lines of arguments that are liable to be used to show how important they are. You are not required to come down either in favour of or against the languages (but may if you wish). [20 marks]

6 Compiler Construction

Programming languages are usually compiled into the machine code of the target computer, but sometimes an interpretive system is used. Discuss the relative merits of these two approaches. [7 marks]

Outline the key features of the design of an interpretive code that would be suitable for an implementation of the C programming language and describe the overall structure of an interpreter for it. [7 marks]

What techniques could be used to improve (a) the space efficiency, and (b) the time efficiency of interpretive codes? [6 marks]

7 Prolog for Artificial Intelligence

Describe how lists that are represented by difference lists may be concatenated (or "appended") in constant time. [6 marks]

Define a procedure `rotate(X,Y)` where both `X` and `Y` are represented by difference lists, and `Y` is formed by rotating `X` to the left by one element. [14 marks]

8 Databases

The international conference Extending Data Base Technology has arranged its next meeting in Avignon at the end of March, to coincide with the spring migration through the Camargue. Birders expect to add significant new sightings to the list of birds seen during a major database conference.

Delegates have been arguing for over a decade about the best relational model for the data. It is agreed that the following information should be recorded:

- the names of the major avifaunal regions: nearctic, western palæarctic, Australasia etc.
- for each birding delegate, a level of competence on a scale [1..5] within each avifaunal region
- good bird sites within each region (including all those visited during conferences), specifying the various habitat types occurring within each site
- observations made – by an observer, on a date, in a habitat type at some site, of a species belonging to a particular family of birds

Advise the delegates on a suitable relational schema for this database. [16 marks]

Explain what would be involved in processing the following queries in SQL.

(a) What species of the kingfisher family have been observed both in a waterside habitat and in arid grassland? [2 marks]

(b) Which delegate is recorded as having observed the greatest number of distinct species? [2 marks]

SECTION C

9 Foundations of Functional Programming

A new form of abstraction on combinators, $\lambda' x$, is proposed. It is to have the same defining equations as $\lambda^T x$, augmented with equations for the new combinators $\mathbf{B}'$, $\mathbf{C}'$ and $\mathbf{S}'$. The new equations should be applied instead of the existing ones if possible:

$$\begin{aligned}
\lambda' x.O\,P\,Q &\equiv \mathbf{B}'\,O\,P\,(\lambda' x.Q) && x \text{ not free in } O \text{ or } P\\
\lambda' x.O\,P\,Q &\equiv \mathbf{C}'\,O\,(\lambda' x.P)\,Q && x \text{ not free in } O \text{ or } Q\\
\lambda' x.O\,P\,Q &\equiv \mathbf{S}'\,O\,(\lambda' x.P)\,(\lambda' x.Q) && x \text{ not free in } O
\end{aligned}$$

The reduction rules for the new combinators are

$$\begin{aligned}
\mathbf{B}'\,O\,P\,Q\,R &\to_w O\,P\,(Q\,R)\\
\mathbf{C}'\,O\,P\,Q\,R &\to_w O\,(P\,R)\,Q\\
\mathbf{S}'\,O\,P\,Q\,R &\to_w O\,(P\,R)\,(Q\,R)
\end{aligned}$$

Here O, P, Q and R stand for combinatory terms.

Compare $\lambda' x$ with $\lambda^T x$ by applying both abstraction methods to the λ-term $\lambda x\,y\,z.z\,y\,x$. [4 marks]

Give graph reduction rules for the new combinators. [2 marks]

Prove $(\lambda' x.P)Q \twoheadrightarrow_w P[Q/x]$ by induction. (You need to discuss only the new combinators.) [6 marks]

The size of the result of translating $\lambda' x_1 \ldots x_n.P$ is linear in n. Give a convincing argument that this is true. [8 marks]

10 Logic and Proof

Briefly contrast the Davis–Putnam proof procedure with resolution. Illustrate your answer using proofs using both methods of

$$(P \to R) \wedge (\neg P \to \neg Q) \wedge (P \vee Q) \to (P \wedge R)$$ [8 marks]

A polynomial over the integers, using modulo-2 arithmetic, can be regarded as a Boolean formula under the correspondence $1 \mapsto$ **true** and $0 \mapsto$ **false**. Show how to translate an arbitrary propositional formula to an equivalent polynomial, describing the translations of $\neg A$, $A \wedge B$, $A \vee B$, $A \to B$ and $A \leftrightarrow B$. [5 marks]

Use this translation to show that $(A \wedge B) \leftrightarrow (B \wedge A)$ is a tautology. [2 marks]

Use this translation to give a rule for simplifying formulæ of the form

$$(\ldots((A \leftrightarrow A) \leftrightarrow A)\ldots \leftrightarrow A)$$ [5 marks]

11 Complexity Theory

One version of the algorithm that uses discrete Fourier Transforms when multiplying integers uses modular arithmetic for much of its internal working. The modulus involved will be chosen to be one greater than a power of two. Explain why this is the case, what power of two is involved, how this relates to the number of digits in the numbers being multiplied and how the basic operations of modular arithmetic are performed. Does it matter that the modulus used is usually not a prime number? [20 marks]

12 Semantics

If D is a complete partial order and $C \subseteq D$, then we say that C is a *closed* subset of D if it satisfies the following two conditions:

- For all $x \in D$ and $y \in C$, if $x \sqsubseteq y$ then $x \in C$.
- If $x_0 \sqsubseteq x_1 \sqsubseteq x_2 \sqsubseteq \ldots$ is an ω-chain such that $x_n \in C$ for all n, then $(\bigsqcup_{n \in \omega} x_n) \in C$.

Prove the following statements:

(a) If C_i is a closed subset of D for all $i \in I$, then $\bigcap_{i \in I} C_i$ is a closed subset of D. [4 marks]

(b) If $C_1, C_2, \ldots, C_N$ are closed subsets of D, then $\bigcup_{1 \leqslant i \leqslant N} C_i$ is a closed subset of D. [7 marks]

(c) If $x \in D$, then $\downarrow x \stackrel{\text{def}}{=} \{y \in D \mid y \sqsubseteq x\}$ is a closed subset of D. [4 marks]

(d) If $f : E \to D$ is a continuous function, then for all closed subsets C of D, $f^{-1}(C)$ is a closed subset of E. (Where $f^{-1}(C) \stackrel{\text{def}}{=} \{x \in E \mid f(x) \in C\}$.) [5 marks]