1996 Paper 13 Question 14

Numerical Analysis II

Let

$$p_n(x) = a_n x^n + \cdots + a_1 x + a_0$$

have n_+ positive real roots. If Descartes' rule of signs is expressed in the form

$$0 \leqslant c - n_+ = 2k,$$

what do c and k represent? [4 marks]

How many *positive* real roots do the following polynomials have?

(a) $x^6 - x^4 - x - 2$

(b) $x^4 - 2x^3 - 235x^2 - 940x + 10200$, given that $x = 5$ is a root

How many *negative* real roots does the following polynomial have?

(c) $2x^3 - 53x^2 + 316x + 600$ [4 marks]

Given that the polynomial

$$343x^3 - 294x^2 + 32$$

has a double root, find all of its roots. [6 marks]

Müller's method uses the formula

$$x_{i+1} = x_i - \frac{2f(x_i)}{c_i \pm \sqrt{{c_i}^2 - 4b_i f(x_i)}}.$$

What is the advantage of having a square root in the formula? How is the sign chosen in the denominator? Describe briefly the idea underlying Müller's method (omitting algebraic details) and comment on the choice of starting values.

[6 marks]