1996 Paper 8 Question 11

Information Theory and Coding

Let X and Y represent random variables with associated probability distributions $p(x)$ and $p(y)$, respectively. They are not independent. Their conditional probability distributions are $p(x|y)$ and $p(y|x)$, and their joint probability distribution is $p(x, y)$.

(a) What is the *marginal entropy* $H(X)$ of variable X, and what is the *mutual information* of X with itself? [4 marks]

(b) In terms of the probability distributions, what are the *conditional entropies* $H(X|Y)$ and $H(Y|X)$? [4 marks]

(c) What is the *joint entropy* $H(X, Y)$, and what would it be if the random variables X and Y were independent? [4 marks]

(d) Give an alternative expression for $H(Y) - H(Y|X)$ in terms of the joint entropy and both marginal entropies. [4 marks]

(e) What is the *mutual information* $I(X; Y)$? [4 marks]