

1997 Paper 8 Question 11

Information Theory and Coding

The input source to a noisy communication channel is a random variable X over the four symbols a, b, c, d . The output from this channel is a random variable Y over these same four symbols. The joint distribution of these two random variables is as follows:

	$x = a$	$x = b$	$x = c$	$x = d$
$y = a$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{4}$
$y = b$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{16}$	0
$y = c$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{16}$	0
$y = d$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{16}$	0

- (a) Write down the marginal distribution for X and compute the marginal entropy $H(X)$ in bits. [3 marks]
- (b) Write down the marginal distribution for Y and compute the marginal entropy $H(Y)$ in bits. [3 marks]
- (c) What is the joint entropy $H(X, Y)$ of the two random variables in bits? [4 marks]
- (d) What is the conditional entropy $H(Y|X)$ in bits? [4 marks]
- (e) What is the mutual information $I(X; Y)$ between the two random variables in bits? [2 marks]
- (f) Provide a lower bound estimate of the channel capacity C for this channel in bits. [2 marks]
- (g) Draw a Venn diagram that describes the relationships among the quantities $H(X)$, $H(Y)$, $H(X|Y)$, $H(Y|X)$, $H(X, Y)$, and $I(X; Y)$. [2 marks]