

1998 Paper 8 Question 11

Information Theory and Coding

Consider a binary symmetric communication channel, having source alphabet $X = \{0, 1\}$ with probabilities $\{0.5, 0.5\}$. Its output alphabet is $Y = \{0, 1\}$ and its channel matrix is

$$\begin{pmatrix} 1 - \epsilon & \epsilon \\ \epsilon & 1 - \epsilon \end{pmatrix}$$

where ϵ is the probability of transmission error.

- (a) What is the entropy of the source, $H(X)$? [1 mark]
- (b) What is the probability distribution of the outputs, $p(Y)$, and the entropy of this output distribution, $H(Y)$? [3 marks]
- (c) What is the joint probability distribution for the source and the output, $p(X, Y)$, and what is the joint entropy, $H(X, Y)$? [4 marks]
- (d) What is the mutual information of this channel, $I(X; Y)$? [2 marks]
- (e) How many values are there for ϵ for which the mutual information of this channel is maximal? What are those values, and what then is the capacity of such a channel in bits? [3 marks]
- (f) For what value of ϵ is the capacity of this channel minimal? What is the channel capacity in that case? [2 marks]

The Fourier transform (whether continuous or discrete) is defined in the general case for complex-valued data, which gets mapped into a set of complex-valued Fourier coefficients. However, we are often concerned with purely real-valued data, such as sound waves or images, whose Fourier transforms we would like to compute. What simplification occurs in the Fourier domain as a consequence of having real-valued, rather than complex-valued, data? [5 marks]