1999 Paper 4 Question 1

Computation Theory

Define the *primitive recursive* and *partial* ($\mu-$) *recursive* functions. [6 marks]

Suppose you are given a Turing machine with state set Q and k-symbol alphabet S whose action is defined by transition functions

$$
\begin{aligned}
q' = f(q,s) &\in Q \uplus \{H\} && \text{(disjoint union)} \\
s' = r(q,s) &\in S && \text{(replacement symbol)} \\
d' = d(q,s) &\in \{L,R,C\} && \text{(movement)}
\end{aligned}
$$

where the head moves to L or R on the tape unless $q' = H$, in which case $d' = C$ and the machine stops.

Extend the action of the machine by an additional state symbol D so that for all $s \in S$,

$$
\begin{aligned}
f(H,s) &= f(D,s) = D \\
r(H,s) &= r(D,s) = s \\
d(H,s) &= d(D,s) = C
\end{aligned}
$$

Show that the action of the Turing machine as extended in this way can be described by a primitive recursive function $T(t,x)$, where t is a step counter and x is a code specifying the initial configuration. [10 marks]

Hence show that computation by any Turing machine may be represented by a partial recursive function. [4 marks]