Foundations of Functional Programming

(a) Give λ-terms Y, K, T and I satisfying the following equalities for all terms M and N:

$$YM = M(YM)$$
$$KMN = M$$
$$TMN = NM$$
$$IM = M$$

[4 marks]

(b) A λ-term is *defined* if it has a head normal form. For each of the following terms, state whether or not it is defined, giving justification for your answer.

$$Y \qquad YK \qquad YT \qquad YI$$

[8 marks]

(c) A λ-term M is *solvable* if there exist variables $x_1, \ldots, x_m$ and terms $N_1, \ldots, N_n$ such that

$$(\lambda x_1, \ldots, x_m.M)N_1 \ldots N_n = I$$

For those terms in (b) that are solvable, exhibit the variables and terms that establish this. For those that are not, explain why they are not solvable.

[8 marks]