2000 Paper 12 Question 11

Introduction to Functional Programming

Consider the following definitions of the functionals `foldl` and `foldr`:

```
fun foldl f e []     = e
  | foldl f e (h::t) = foldl f (f(h,e)) t;

fun foldr f e []     = e
  | foldr f e (h::t) = f(h, foldr f e t);
```

What is the type of `foldl`? [2 marks]

What is the type of the expression `foldr op/`? [2 marks]

For *each* of the following functions, write an ML definition using one of the functionals `foldl` or `foldr`.

(*a*) `product: (real list) -> real`, which given a list of real numbers gives their product.

(*b*) `exists: ('a -> bool) -> ('a list) -> bool`, which given a predicate p and a list l determines whether there is any element of l satisfying p.

(*c*) `length: ('a list) -> int` which determines the length of a list.

[9 marks]

Prove, by induction on lists, that for all lists of integers l, the following identity is true:

$$\texttt{foldl op+ 0 } l = \texttt{foldr op+ 0 } l$$

[7 marks]