2000 Paper 2 Question 7

Regular Languages and Finite Automata

For each kind of regular expression over an alphabet Σ, define the language $L(\mathbf{r})$ of strings matching a regular expression $\mathbf{r}$ of that kind.   [4 marks]

Define the language $L(M)$ accepted by a deterministic finite automaton M.
                                                                                [2 marks]

Prove that for every deterministic finite automaton M with alphabet of input symbols Σ it is possible to construct a regular expression $\mathbf{r}$ over Σ satisfying $L(\mathbf{r}) = L(M)$.   [10 marks]

Illustrate your proof by constructing such an $\mathbf{r}$ for the deterministic finite automaton with state set $\{0, 1, 2\}$, alphabet of input symbols $\{a, b\}$, initial state 0, accepting states 1 and 2, and next-state function

$$(0, a) \mapsto 2, \quad (1, a) \mapsto 1, \quad (2, a) \mapsto 0,$$
$$(0, b) \mapsto 1, \quad (1, b) \mapsto 0, \quad (2, b) \mapsto 2.$$

                                                                                [4 marks]