

2000 Paper 5 Question 11

Logic and Proof

Given a propositional formula, we wish to test whether it is a tautology and, if it is not, to compute an interpretation that makes it false. Two techniques for doing this are the sequent calculus and ordered-binary decision diagrams. Give a brief outline of these techniques, applying both of them to the formulae

$$(A \rightarrow B) \rightarrow (B \rightarrow A) \quad \text{and} \quad (A \vee B) \rightarrow (\neg B \rightarrow A)$$

[7 + 7 marks]

It is proposed to replace the usual sequent calculus rule for disjunction on the left by this rule:

$$\frac{\Gamma, A \Rightarrow \Delta \quad \Gamma, B \Rightarrow \Delta, A}{\Gamma, A \vee B \Rightarrow \Delta}$$

Is this rule sound? Justify your answer.

[3 marks]

Give an example to show that using this rule instead of the usual one makes some proofs shorter.

[3 marks]