2000 Paper 8 Question 11

Information Theory and Coding

(*a*) If a continuous signal is discretely sampled by multiplying it with a sequence of uniformly-spaced Dirac delta functions, having frequency f_s, what happens to the Fourier spectrum of the signal? [3 marks]

(*b*) What is the conditional probability $p(x|y)$, the probability of event x given that event y has occurred, provided that we know the following?

$p(x)$, the unconditional probability of event x
$p(y)$, the unconditional probability of event y
$p(y|x)$, the probability of event y given that event x has occurred

[3 marks]

(*c*) Consider a binary symmetric communication channel, whose input source is the alphabet $X = \{0, 1\}$ with probabilities $\{0.5, 0.5\}$; whose output alphabet is $Y = \{0, 1\}$; and whose channel matrix is

$$\begin{pmatrix} 1-\epsilon & \epsilon \\ \epsilon & 1-\epsilon \end{pmatrix}$$

where ϵ is the probability of transmission error.

(*i*) What is the entropy of the source, $H(X)$? [1 mark]

(*ii*) What is the probability distribution of the outputs, $p(Y)$, and the entropy of this output distribution, $H(Y)$? [3 marks]

(*iii*) What is the joint probability distribution for the source and the output, $p(X, Y)$, and what is the joint entropy, $H(X, Y)$? [3 marks]

(*iv*) What is the mutual information of this channel, $I(X; Y)$? [2 marks]

(*v*) How many values are there for ϵ for which the mutual information of this channel is maximal? What are those values, and what then is the capacity of such a channel in bits? [3 marks]

(*vi*) For what value of ϵ is the capacity of this channel minimal? What is the channel capacity in that case? [2 marks]