2001 Paper 11 Question 7

Mathematics for Computation Theory

(a) Let M be an N-state deterministic finite automaton over the finite alphabet S. Write $l(w)$ for the length of words $w \in S^*$. Suppose that M accepts the word $x \in S^*$, where $l(x) \geqslant N$.

(i) Show that x is a concatenation of words uvw, where $l(u) < N$, $1 \leqslant l(v) \leqslant N$, and M accepts the word $z_k = uv^k w$ for all natural numbers $k \in \mathbb{N}$. [8 marks]

(ii) Hence show that if M accepts an infinite set of words $x \in S^*$, it must accept some word $y \in S^*$ such that $N \leqslant l(y) < 2N$. [4 marks]

(b) A ternary integer representation is defined as follows using a variant of BNF:

```
    <zero> ::= 0
<non-zero> ::= 1 | 2
   <digit> ::= <zero> | <non-zero>
     <nzi> ::= <non-zero> | <nzi> <digit>
 <integer> ::= <zero> | <nzi> | - <nzi>
```

(i) Design a deterministic finite automaton with alphabet $S = \{-, 0, 1, 2\}$ that accepts precisely the valid integer representations. [6 marks]

(ii) Write down a regular expression that denotes the event recognised by your automaton. [2 marks]

[If you wish, you may refer to characters `-`, `0`, `1`, `2` by a, b, c, d respectively.]