2001 Paper 2 Question 5

Probability

Candidates for a certain first-year University Examination offer four subjects. At their first meeting the Examiners agree that, within each subject, they will sort the candidates into descending order of marks and those ranked in the top third will be awarded informal Firsts for that subject.

To save effort, the Examiners agree to mark each subject simply by throwing a fair die once for each candidate and awarding the outcome, a mark in the range 1 to 6. They reason, correctly, that 5 should be the threshold mark for a First since approximately one-third of the candidates are likely to score 5 or 6.

Noting that 5 is the threshold mark for a First in an individual subject and that each candidate offers four subjects, the Examiners agree further that 20 marks (4×5) should be the threshold mark for a First overall in the Examination. They reason, incorrectly, that using 20 as the threshold about one-third of the candidates will gain Firsts overall.

When they meet after the Examination, the Examiners are astonished to discover that, using a threshold of 20 marks, considerably fewer than one-third of the candidates gain Firsts.

(a) Assuming that dice throws are independent, what is the probability that a candidate obtains 20 or more marks overall?   [12 marks]

(b) To what value should the Examiners lower the threshold mark for a First overall to ensure that approximately one-third of the candidates gain Firsts?
                                                                                  [5 marks]

(c) What is the probability that a candidate obtains or exceeds this lower threshold?   [1 mark]

(d) Suppose the candidates were assessed more conventionally (on perceived ability) but still awarded integer marks in the range 1 to 6 for each subject. Suppose further that the threshold mark of 5 for a First again results in about one-third of the candidates in any individual subject being awarded a First. Discuss whether it would be reasonable now for the Examiners to assume that an overall threshold of 20 would lead to approximately one-third of the candidates gaining Firsts overall.   [2 marks]

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