2002 Paper 12 Question 10

Introduction to Functional Programming

(*a*) Give a recursive definition of an ML datatype `'a tree` of binary trees consisting of nodes where data items are stored. Each such node is either a leaf or a branch node with left and right trees as branches. [3 marks]

(*b*) Write a recursive function `size` of type `'a tree -> int` that returns the number of nodes of a given tree. [4 marks]

(*c*) Write an iterative function `isize` of type `int * 'a tree -> int` which satisfies the following identity for all integers n and all trees t

$$\texttt{isize}(n, t) = \texttt{size}(t) + n \tag{1}$$

[6 marks]

(*d*) Prove, by structural induction, that the identity (1) holds for the two functions you defined. [7 marks]