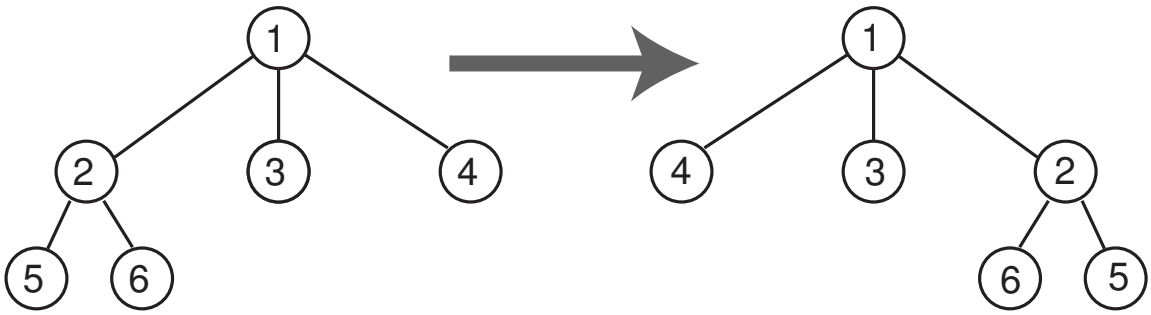
2002 Paper 1 Question 5

Foundations of Computer Science

This question concerns the following ML declaration of a tree datatype:

```
datatype 'a fan = Wave of 'a * ('a fan) list;
```

(*a*) Declare the function `flip`, which maps a tree to a mirror image of itself, as illustrated: [3 marks]



(*b*) Declare the curried function `paint f`, which copies a tree while applying the function `f` to each of its labels. [3 marks]

(*c*) Declare the function `same_shape`, which compares two trees and returns `true` if they are equal except for the values of their labels and otherwise returns `false`. [5 marks]

(*d*) State the types of functions `flip`, `paint` and `same_shape`. [3 marks]

(*e*) The function `paper` is declared in terms of the familiar functional `foldr`:

```
fun foldr f ([],    e) = e
  | foldr f (x::xs, e) = f(x, foldr f (xs,e));

fun paper (Wave(x,fs), q) = foldr paper (fs, q+1);
```

Describe the computation that results when `paper` is applied to a tree. [6 marks]