2002 Paper 6 Question 11

Logic and Proof

(a) The formula A is converted to clauses, omitting the initial step of negating the formula. Resolution is attempted, but the empty clause cannot be derived. What can we conclude about A? [3 marks]

(b) The formula A is converted to clauses by the usual procedure, except that Skolemization is performed before negating the formula instead of afterward. Resolution is attempted and the empty clause is derived. What can we conclude about A? [3 marks]

(c) The formula A is converted to clauses by the usual procedure. The Davis–Putnam method is applied. In some of the case splits the empty clause is derived, but in others it is not. What can we conclude about A? [3 marks]

(d) For each of the following sequents, present either a formal proof or a falsifying interpretation. The modal logic is S4.

$$((\exists x \, P(x)) \to Q) \to \forall x \, (P(x) \to Q)$$
$$\Diamond \Box A \to \Diamond \Box \Diamond \Box A$$

[5 + 6 marks]

1