

## 2002 Paper 7 Question 8

### Computer Systems Modelling

Consider an  $M/M/1$  queue and represent the state of the queue by the number of customers present.

- (a) Draw a state diagram for the Markov chain describing the state of the queue showing the possible states and transition rates. Briefly explain the diagram. [4 marks]
- (b) What is the condition for the existence of a steady-state equilibrium distribution  $p_k$  ( $k = 0, 1, 2, \dots$ ) for the number present? [2 marks]
- (c) Determine the steady-state average number of customers present. [4 marks]
- (d) Use Little's law to determine the steady-state average response time that a customer spends in the system. [2 marks]
- (e) Suppose that a single communication channel is used to carry data items sent by various sources connected to the channel. Assume that each source generates a stream of data items with inter-arrival times which are exponential at rate 2 items/second and that all sources are statistically independent. All the items wait in a single queue and are transmitted one at a time. The transmission times are exponentially distributed with mean 25 milliseconds and are statistically independent. Determine the largest number of sources that can be connected to the channel according to each of the following two criteria:
  - (i) The channel is not saturated.
  - (ii) The steady-state average response time for an item must not exceed 100 milliseconds.

[8 marks]