2003 Paper 2 Question 5

Probability

(a) If a continuous probability density function (p.d.f.) $f(x)$ is transformed by some transformation function $y(x)$ into a new p.d.f. $g(y)$, then:

$$g(y) = f\left(x(y)\right)\left|\frac{dx}{dy}\right|$$

What constraints are there on the function $y(x)$ and its inverse $x(y)$? What is the significance of the vertical bars round $\frac{dx}{dy}$? [4 marks]

(b) Suppose that X is a continuous random variable distributed Uniform(0,1). Its p.d.f. $f(x)$ is given by:

$$f(x) = \begin{cases} 1, & \text{if } 0 \leqslant x < 1 \\ 0, & \text{otherwise} \end{cases}$$

What four transformation functions are required to transform $f(x)$ into the following:

(i)

$$g(y) = \begin{cases} \lambda.e^{-\lambda y}, & \text{if } y > 0 \\ 0, & \text{otherwise} \end{cases}$$

[4 marks]

(ii)

$$g(y) = \begin{cases} \sin y, & \text{if } 0 \leqslant y < \frac{\pi}{2} \\ 0, & \text{otherwise} \end{cases}$$

[4 marks]

(iii)

$$g(y) = \begin{cases} \frac{1}{2}(2-y), & \text{if } 0 \leqslant y < 2 \\ 0, & \text{otherwise} \end{cases}$$

[4 marks]

(iv)

$$g(y) = \begin{cases} \frac{3}{8}(2-y)^2, & \text{if } 0 \leqslant y < 2 \\ 0, & \text{otherwise} \end{cases}$$

[4 marks]