

2003 Paper 6 Question 9

Logic and Proof

- (a) Define the concepts of a *true sequent*, a *valid sequent* and a *basic sequent*. You may take the concepts of true and valid formulæ as primitive. [3 marks]
- (b) Gentzen claimed that his proof systems performed natural logical reasoning. Consider the following sequent calculus rules:

$$\frac{A, \Gamma \Rightarrow \Delta \quad B, \Gamma \Rightarrow \Delta}{A \vee B, \Gamma \Rightarrow \Delta} \qquad \frac{A, \Gamma \Rightarrow \Delta}{\exists x A, \Gamma \Rightarrow \Delta}$$

State the second rule's proviso and explain the intuitions behind each rule.

[5 marks]

- (c) Choose one of these rules and give a rigorous argument for its soundness, using the concept of a valid sequent. [6 marks]
- (d) Precisely define the concept of the Most General Unifier (MGU) of two terms, giving examples. [6 marks]