2004 Paper 6 Question 9

Logic and Proof

For each of the following statements, briefly justify whether it is true or false. In the following x, y, z are variables, and a, b, c are constants.

(a) Given any propositional logic formula ϕ that is a tautology, converting ϕ to CNF will result in $\mathbf{t}$.

(b) Executing the DPLL method on the clauses

$$\{P, Q, \neg S\} \quad \{\neg P, Q, \neg R\} \quad \{P\} \quad \{\neg Q, R\} \quad \{S, \neg Q\}$$

produces a result without needing any case split steps.

(c) The OBDD corresponding to the propositional logic formula $(P \vee Q) \wedge \neg P$ does not have any decision nodes for the propositional letter P.

(d) Skolemizing the first order logic formula $\exists x(\phi(x))$ results in a logically equivalent formula $\phi(a)$ (where a is a fresh constant).

(e) The Herbrand Universe that is generated from the clauses $\{P(a)\}$, $\{Q(x, b), \neg P(x)\}$ and $\{\neg Q(a, y)\}$ contains two elements.

(f) The two terms $f(x, y, z)$ and $f(g(y, y), g(z, z), g(a, a))$ can be unified.

(g) It is not possible to resolve the clauses $\{P(x)\}$ and $\{\neg P(f(x))\}$ because the *occurs check* prevents the literals being unified.

(h) The clause $\{P(x, x), P(x, a)\}$ can be factored to give the new clause $\{P(x, a)\}$.

(i) The empty clause can be derived from the clauses $\{P(x), P(a)\}$, $\{P(x), \neg P(a)\}$, $\{\neg P(b), Q\}$ and $\{\neg P(c), \neg Q\}$ using resolution.

(j) Because in the modal logic S4 the equivalence $\Box\Box\phi \simeq \Box\phi$ holds for every formula ϕ, it follows that $\Diamond\Diamond\phi \simeq \Diamond\phi$.

[2 marks each]