

2004 Paper 7 Question 9

Quantum Computing

- (a) You are given a qubit that is in one of two states: either $|\phi\rangle = |0\rangle$ or $|\psi\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$. If you measure the qubit in the computational basis, what is the probability that your measurement correctly identifies the state? [3 marks]
- (b) Draw a labelled schematic circuit diagram for:
- (i) the phase estimation algorithm; [4 marks]
 - (ii) Grover's algorithm. [4 marks]
- (c) Suppose a search problem has an unknown number M of marked states. Show how phase estimation and Grover's algorithm can be combined to estimate M to a high accuracy using $O(\sqrt{N})$ oracle calls. [Hint: The Grover iterate, G , has eigenvalues $e^{\pm i\theta}$ where $\sin^2(\theta/2) = M/N$.] [5 marks]
- (d) Suppose there is an algorithm which can determine the number M of marked states in an unsorted search space of size N using only $O(\log(N))$ oracle calls. Explain why this would allow us to solve NP-complete problems in polynomial time. [4 marks]