2004 Paper 8 Question 16

Computer Systems Modelling

Suppose that bus inter-arrival times, X, at a given bus stop have a probability density function $f_X(x)$ with mean $\mu = E(X)$ and variance $\sigma^2 = \mathrm{Var}(X) = E(X^2) - \mu^2$. Suppose that a randomly arriving customer arrives during a bus inter-arrival interval of length Y and suppose that the probability density of Y is $f_Y(y)$. It may be assumed that

$$f_Y(y) = Cyf_X(y)$$

for some constant C.

(a) Derive an expression for the constant C in terms of μ and σ^2. [7 marks]

(b) Derive an expression for the average waiting time as seen by a randomly arriving customer. [7 marks]

(c) For each of the following cases, calculate the average waiting time as seen by a randomly arriving customer.

(i) X is deterministic taking a value of 10. [2 marks]

(ii) X is exponentially distributed with mean $\mu = 10$. [2 marks]

(iii) X has a general distribution with mean $\mu = 10$ and variance $\sigma^2 = 500$. [2 marks]