2005 Paper 5 Question 12

Foundations of Functional Programming

(a) Give a lambda expression that can be used to form the composition of two functions. [1 mark]

(b) Suppose that the lambda expression you have given above can be referred to using the name `B`. One way of representing the natural numbers as lambda expressions involves for instance having the number "3" represented by a term $\lambda f.\mathtt{B}f(\mathtt{B}ff)$ so that a numeral when applied to an argument f composes f with itself the given number of times.

In this scheme, write out lambda expressions that will serve as 0, 1 and 2. [3 marks]

(c) Present and explain lambda expressions that find the successor to a number represented as in part (b) and that add two numbers together. [6 marks]

(d) If m and n are two lambda expressions that both represent numbers in this style, what interpretation can be placed on the term $(m\ n)$? Explain and justify your claim. [4 marks]

(e) Explain how it is possible to produce a lambda expression that, given the representation of a non-zero number k, produces an expression that behaves like $k-1$. [6 marks]