

2005 Paper 5 Question 9

Logic and Proof

- (a) In order to prove the following formula by resolution, what set of clauses should be submitted to the prover? Justify your answer briefly.

$$\forall x [P(x) \vee Q \rightarrow \neg R(x)] \wedge \forall x [(Q \rightarrow \neg S(x)) \rightarrow (P(x) \wedge R(x))] \rightarrow \forall x S(x)$$

[7 marks]

- (b) Derive the empty clause using resolution with the following set of clauses, or give convincing reasons why it cannot be derived.

$$\{\neg P(x, x)\} \quad \{P(x, f(x))\} \quad \{\neg P(x, y), \neg P(y, z), P(x, z)\}$$

[6 marks]

- (c) Derive the empty clause using resolution with the following set of clauses, or give convincing reasons why it cannot be derived. (Note that a and b are constants.)

$$\{\neg P(a)\} \quad \{Q(a)\} \quad \{R(b)\} \quad \{S(b)\} \\ \{\neg Q(x), P(x), \neg R(y), \neg Q(y)\} \quad \{\neg S(x), \neg R(x), Q(x)\}$$

[7 marks]