2005 Paper 6 Question 9

Logic and Proof

(a) The propositional formula ϕ contains four propositional letters: P, Q, R and S. This formula evaluates to true in every case except when Q and R are false while S is true.

(i) What is the BDD for ϕ? [2 marks]

(ii) What is the BDD for $\neg P \to \phi$? [3 marks]

(iii) What is the BDD for the formula $P \wedge S \to R$? [2 marks]

(iv) What is the BDD for the formula $(P \wedge S \to R) \wedge \phi$? [4 marks]

Use alphabetic ordering for all BDDs.

(b) Use the DPLL procedure to determine whether or not the following set of clauses is satisfiable.

$$\{P, Q, R\} \quad \{\neg P, Q, R\} \quad \{P, \neg Q, \neg R\} \quad \{\neg P, \neg Q, \neg R\} \quad \{\neg Q, R\} \quad \{\neg P, Q, \neg R\}$$

[5 marks]

(c) Prove the formula $\forall x[\neg P(x) \to Q(x)] \wedge \exists x\, \neg Q(x) \to \exists x\, P(x)$ using the tableau calculus (with Skolemization). [4 marks]