2006 Paper 10 Question 6

Introduction to Functional Programming

(*a*) The *extension* of a list ℓ, denoted $\#\ell$, is the set of all its elements; that is, formally,

$$\#\texttt{[]} = \{\ \}$$
$$\#(h\texttt{::}t) = \{h\} \cup \#t$$

Thus, for instance, $\#\texttt{[0,1,2,3,1,2,3,2,3,3]} = \{\texttt{0}, \texttt{1}, \texttt{2}, \texttt{3}\}$.

You are asked to give four implementations of an (*extensional*) *remove* curried function

```
''a rm :  ''a -> ''a list -> ''a list
```

satisfying the following specification:

$$\#(\texttt{rm}\ x\ \ell) = (\#\ell) \setminus \{x\}$$

for all equality types α, and values x of type α and ℓ of type α `list`.

(*i*) The first implementation should use the ML built-in functional

```
'a filter :  ('a -> bool) -> 'a list -> 'a list
```

[2 marks]

(*ii*) The second implementation should use the ML built-in functionals

```
'a concat :  'a list list -> 'a list
('a, 'b) map :  ('a -> 'b) -> 'a list -> 'b list
```

[2 marks]

(*iii*) The third implementation should be a simple recursive function using only the `list` datatype constructors. [4 marks]

(*iv*) The fourth implementation should be a tail-recursive function using only the `list` datatype constructors. [6 marks]

(*b*) Rigorously argue for the correctness of either the third or the fourth of your implementations. [6 marks]