2006 Paper 13 Question 1

Data Structures and Algorithms

(*a*) Explain what the *heap* data structure is, state its defining properties and explain how to convert between the tree and vector representations of a heap. [2 marks]

(*b*) Describe an optimally efficient algorithm for transforming any random vector into a heap vector and explain why it works. [4 marks]

(*c*) Using the tree instead of the vector representation for clarity, apply this algorithm to the binary tree isomorphic to the letter vector "`P I S K T Z O P V N`", producing a frame-by-frame trace of the execution. For this answer, please use the sheet for part (*c*), supplied with the question paper, and use a new tree whenever any nodes change. [5 marks]

(*d*) Explain how to rearrange the heap after having extracted its top so that what remains is still a heap. Follow this procedure to extract the top three values, one by one, from the heap you built, producing a frame-by-frame trace as above. Please use the sheet for part (*d*). [5 marks]

(*e*) Describe a way to insert a new value into an existing heap in time $O(\log n)$ where n is the heap size. [4 marks]

Further copies of the sheets for parts (*c*) and (*d*) may be obtained from the Invigilator, if required. Ensure that you attach the sheets to the rest of your answer.