2006 Paper 13 Question 9

Numerical Analysis II

(a) With reference to solution of the differential equation $y' = f(x, y)$, explain the conventional notation x_n, $y(x_n)$, y_n, f_n. [3 marks]

(b) Explain the terms *local error*, *global error*, and *order* of a method. [3 marks]

(c) Without deriving any formulae, describe the general technique for deriving *multistep* formulae. [2 marks]

(d) Milne's method uses the multistep formulae

$$y_{n+1} = y_{n-3} + \frac{4h}{3}(2f_n - f_{n-1} + 2f_{n-2})$$
$$y_{n+1} = y_{n-1} + \frac{h}{3}(\tilde{f}_{n+1} + 4f_n + f_{n-1})$$

which each have local error $O(h^5)$. What is the meaning of the term $\tilde{f}_{n+1}$? Suggest a suitable starting procedure and explain how the Milne formulae are used. [6 marks]

(e) Let $x_0 = 0.2$, $y(x_0) = 1.67$, $h = 0.2$ and

$$f(x, y) = 1 + \frac{(y - x)(x + 2)}{x + 1}.$$

Suppose the following values of f_n have been generated by the starting procedure: 4.6, 5.6, 7.2 for $n = 1, 2, 3$. Calculate the first required value of $\tilde{f}_{n+1}$ to 2 significant digits. [3 marks]

(f) Contrast Milne's method with your starting procedure, commenting particularly on *stability*, *efficiency* and *step size* considerations. [3 marks]