

2006 Paper 7 Question 5

Computer Systems Modelling

(a) Describe the congruential methods for generating pseudo-random numbers from a Uniform $(0, 1)$ distribution. [3 marks]

(b) Let U be a Uniform $(0, 1)$ random variable. Show that for any continuous distribution function, $F(x)$, the random variable, X , defined by

$$X = F^{-1}(U)$$

has the probability distribution function $F(x)$. [3 marks]

(c) Apply the method of part (b) to generate random variables with the following distributions. In each case, specify the distribution function $F(x)$ that you use.

(i) Uniform distribution on the interval (a, b) , for $a < b$. [2 marks]

(ii) Exponential distribution with parameter λ . [2 marks]

(d) Define the Poisson process, $N(t)$, ($t \geq 0$) of rate λ . [2 marks]

(e) Show that for each fixed $t \geq 0$, $N(t)$ is a Poisson random variable with parameter λt . [3 marks]

(f) Show that the interarrival times of consecutive events in a Poisson process of rate λ are independent random variables each with the exponential distribution with parameter λ . Show how this leads to a method to simulate the events of a Poisson process. [5 marks]