2007 Paper 13 Question 9

Quantum Computing

(*a*) Consider a quantum finite automaton with two basis states, $|0\rangle$ being the start state and $|1\rangle$ the only accepting state. The automaton operates on a two-letter alphabet, with matrices $M_a = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix}$ and $M_b = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

What are the probabilities that the automaton accepts each of the following input strings?

    (*i*)   *a*   [3 marks]

    (*ii*) *aba*   [3 marks]

    (*iii*) *abb*   [3 marks]

(*b*) Give a complete description of the probabilities of acceptance associated with various possible input strings.   [5 marks]

(*c*) Prove that there is no *two-state* probabilistic automaton with the same behaviour as the automaton described in part (*a*).   [6 marks]