2007 Paper 2 Question 3

Discrete Mathematics I

(a) Given $a, b \in \mathbb{N}$ with $a \geq b$ prove carefully that there are unique values $q, r \in \mathbb{N}$ such that $a = qb + r$ and $0 \leq r < b$. [6 marks]

(b) Prove further that the highest common factor of a and b is equal to the highest common factor of b and r. [2 marks]

(c) Derive Euclid's algorithm for finding the highest common factor of two numbers. [3 marks]

(d) Determine the algorithm's efficiency by finding a limit for the number of divisions required in its execution expressed as a function of a. [3 marks]

(e) Find all values $x, y \in \mathbb{Z}$ satisfying $72x + 56y = 40$. [3 marks]

(f) Find all values $z \in \mathbb{Z}$ satisfying $56z \equiv 24(\text{mod } 72)$. Express the answer in the form $z \equiv a(\text{mod } m)$. [3 marks]