2007 Paper 9 Question 10

Digital Signal Processing

(*a*) Write an efficient microcontroller program (pseudocode) that outputs a continuous sine wave of frequency $f = 440$ Hz with values y_n in the range -1 to 1 at a sampling frequency $f_\mathrm{s} = 32$ kHz. The programming language you have available lacks complex-number arithmetic, the runtime environment offers only basic floating-point arithmetic (i.e., no trigonometric functions), addition is much faster than multiplication, and there is insufficient memory to store a precomputed waveform. [10 marks]

(*b*) The discrete sequence $y_n = \cos(2\pi n f_1/f_\mathrm{s}) + A \cdot \cos(2\pi n f_2/f_\mathrm{s})$ is fed into a (hypothetical) digital-to-analogue converter that outputs a constant voltage $y(t) = y_n$ during the time interval $n/f_\mathrm{s} \leq t < (n+1)/f_\mathrm{s}$ for all integers n.

(*i*) Explain how this behaviour of the digital-to-analogue converter affects the amplitude spectrum of the resulting signal. [5 marks]

(*ii*) What amplitude A has to be chosen for the second term such that the resulting amplitude spectrum shows equally high peaks at both $f_1 = 1$ kHz and $f_2 = 2$ kHz if the sampling frequency is $f_\mathrm{s} = 6$ kHz? [5 marks]