2007 Paper 9 Question 11

Computer Systems Modelling

(*a*) Consider a general birth–death process with birth rate λ_i and death rate μ_i in state i ($i = 0, 1, 2, \ldots$). What are the detailed balance equations for this process? [2 marks]

(*b*) Derive the steady-state distribution for the general birth–death process considered in part (*a*). What are the conditions for the steady-state distribution to exist? [4 marks]

(*c*) Describe the M/M/1 queue and give a stochastic model for the number of customers present. Find the steady-state distribution for the number of customers present and state the condition for it to exist. [4 marks]

(*d*) Derive the mean number of customers present in steady state in the M/M/1 queue. [4 marks]

(*e*) State Little's law and use it to derive the mean time spent in the M/M/1 queue in the steady state. [2 marks]

(*f*) Discuss what is meant by the traffic intensity for an M/M/1 and explain what happens to the mean number of customers present as the traffic intensity increases towards one. [4 marks]