2008 Paper 4 Question 5

Logic and Proof

(a) State (with justification) whether the following formula is satisfiable, valid or neither. Note that a and b are constants.

$$\Big[\forall x\,[q(x) \to r(x)] \wedge \neg r(a) \wedge \forall x\,[\neg r(x) \wedge \neg q(a) \to p(x) \vee q(x)]\Big] \to p(b) \vee r(b)$$

[13 marks]

(b) Attempt to prove the formula $[\exists x\,\forall y\,R(x,y)] \to \exists x\,\forall z\,R(x,f(z))$ by resolution, with brief explanations of each step, including the conversion to clause form.

[4 marks]

(c) Give a model for the following set of clauses, or prove that none exists.

$$\{\neg R(x,y), \neg R(y,x)\}$$

$$\{R(x,f(x))\}$$

$$\{\neg R(x,y), \neg R(y,z), R(x,z)\}$$

[3 marks]